# Linking Social and Personal Preferences: Theory and Experiment*

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September 25, 2019

**Abstract**

The goal of this paper is to link attitude toward risk over personal consumption with attitude toward risk over social consumptions. Because many everyday choices involve risk, these attitudes enter virtually every realm of individual decision-making. We provide necessary and sufficient conditions for deducing preferences over risky social choices (which have consequences both for the Decision Maker and for others) from risky personal choices (which have consequences only for the Decision Maker) and riskless social choices, and we offer an experimental test of the theory. The experiments generate a rich dataset that enables completely non-parametric revealed preference tests of the theory at the level of the individual subject. Many subjects behave as predicted by the theory but a substantial fraction do not.

**JEL Classification Numbers:** C91, D63, D81.
**Keywords:** social preferences, risk preferences, revealed preference, experiment.

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*We are grateful to Daniel Silverman, Bob Powell, Benjamin Polak, Daniel Markovitz, Edi Karni, Douglas Gale, Raymond Fisman and Chris Chambers for helpful discussions and encouragement and for suggestions from a number of seminar audiences. Financial support was provided by the National Science Foundation, the Research Council of Norway, and the Peder Sather Center for Advanced Study. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of any funding agency. Experiments reported in this paper were conducted by the Choice Lab at the Centre for Experimental Research on Fairness, Inequality and Rationality (FAIR) at NHH Norwegian School of Economics.

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# 1 Introduction

Many individuals must make choices that have consequences only for themselves – choices in the *personal domain* – and choices that have consequences both for themselves and for others – choices in the *social domain*. Many of these choices involve risk, so a full understanding of choice behavior in these domains requires an understanding of both the individual's preferences over consequences and the individual's attitude toward risk. It is natural to ask: Is there a connection between an individual's attitude toward risk in the personal domain and that individual's attitude toward risk in the social domain? This paper offers formalizations of this question, theoretical answers to this question, and an experimental test of the theory.

Our motivation for asking (and answering) this question arises not only from intellectual curiosity but also from pragmatism because we often choose – or at least influence – which individuals will be in a position to make choices that have consequences both for themselves and for us: Chairs, Deans, Mayors, Governors, Congresspersons, Senators – even Presidents. And we certainly care not only whether the President prefers peace to war, but what actions the President would be prepared to take to alter the risks of peace or war. Blockading Soviet ships bound for Cuba (as John Kennedy did) risks war, and putting forward both tax reform and civil rights legislation simultaneously (as Kennedy also did) risks accomplishing neither.[1]

But can we draw any inferences at all about the President's risky choices in the social domain from the fact that the President chooses to conduct an illicit affair or smoke in secret or invest aggressively or exaggerate his accomplishments (to mention just a few personal choices that have made headlines in recent memory)? What these personal choices have in common is that they involve (personal) risk – to the President's marriage, health, finances, reputation. Drawing inferences about (present and future) risky social choices from knowledge of a (past and present) risky personal choices would seem useful, and it would seem possible as well – provided that there is a *linkage* between attitude toward risk in the personal domain and attitude toward risk in the social domain.

In this paper, we give a precise formalization of these ideas, establish a theoretical linkage between preferences and risk attitudes in the social domain and in the personal domain, and provide an experimental test of the theory. One important qualification needs

[1] And in fact, Kennedy accomplished neither tax reform nor civil rights legislation; both were pushed through by Lyndon Johnson after Kennedy's assassination.

to be remembered when interpreting our results. As the examples above illustrate, there is a question that seems both puzzling and important: what is the meaning of an individual's "attitude toward risk" in an environment in which the consequences are not monetary – or more generally, involve consequences other than consumption? We confess that we have no answer to offer to this question; indeed we suspect it has no entirely satisfactory answer as it requires a particular spatial model of choices and specific functional forms for preferences. Keeping this in mind, we formulate an abstract model that avoids this issue entirely (we explain below) but we construct our experiment so that the consequences are *monetary* (for the subject and for one other).

Revealed preference theory tells us that choices determine preferences and vice versa, so we formulate our problem abstractly by assuming that the individual $\mathcal{DM}$ ($\mathcal{DM}$) has preferences over risky choices in the social domain but that that we can observe only preferences over risky choices in the personal domain and preferences over non-risky choices in the social domain. (It is convenient to view the personal domain as a subset of the social domain in which the consequences for others are fixed.) We ask whether it is possible, on the basis of these observations, to *deduce* preferences over risky choices in the social domain. Our theoretical results provide necessary and sufficient conditions that such deductions be possible. The required conditions depend on what is observed and also about the $\mathcal{DM}$'s degree of rationality.

The formal model considers a $\mathcal{DM}$, characterized by a fixed preference relation $\succeq$ over the set $L(\Omega)$ of (finite) lotteries on a set $\Omega$ of *social states*. (To make the analysis simpler and sharper, we assume that $\Omega$ is finite. This avoids subtle issues about the topology of $\Omega$ and the continuity of $\succeq$.) A subset $P \subset \Omega$ of these social states have consequences only for the $\mathcal{DM}$ – these are *personal states* – while the others have consequences both for the $\mathcal{DM}$ and for others. We do not observe the entire preference relation $\succeq$ on $L(\Omega)$ but only some portion of it. In our main theoretical result, and in our experimental work, we assume that we can observe the restriction $\succeq_0$ of $\succeq$ to $[L(P) \times L(P)] \cup [\Omega \times \Omega]$. That is, we can observe comparisons between social states – including personal states – and comparisons between personal lotteries, but we *cannot* observe comparisons between social states and personal lotteries.[2]

We ask: in what circumstances it is possible to *deduce* the entire preference relation $\succeq$ from restriction $\succeq_0$? In other words, in what circumstances does $\succeq_0$ admit a *unique*

[2]For completeness, we do provide theoretical analysis of the setting in which we can observe comparisons between social states and personal lotteries – that is, we can observe the restriction $\succeq_1$ of $\succeq$ to $[\Omega \cup L(P)] \times [\Omega \cup L(P)]$ – but we do not have any experimental evidence in that setting.

extension to the preference relation $\succeq$ over the full domain of lotteries $L(\Omega)$ on social states? If we make the (relatively weak) assumption that the $\mathcal{DM}$'s preferences obey the usual axioms of individual choice under uncertainty – Completeness, Transitivity, Continuity, Reduction of Compound Lotteries and the Sure Thing Principle – together with a natural axiom that we call State Monotonicity, then a necessary and sufficient condition that it be possible to deduce the entire preference relation $\succeq$ from the sub-preference relation $\succeq_0$ is that the $\mathcal{DM}$ finds every social state to be indifferent to some personal state.[3]

The theoretical result seems clean and satisfying but it is another question entirely whether is also descriptive of reality. To address this latter question, we designed and executed an experiment in which subjects were confronted with choices in three domains:

- PERSONAL RISK The objects of choice are risky personal choices (equiprobable binary lotteries whose consequences are monetary outcomes for *self* alone).
- SOCIAL The objects of choice are riskless social choices (deterministic divisions of money between *self* and one *other*).
- SOCIAL RISK The objects of choice are risky social choices (equiprobable binary lotteries whose consequences are divisions of money between *self* and *other*).

In the experimental setting, each decision problem is presented as a choice from a two-dimensional budget line using a graphical interface developed by Choi, Fisman, Gale, and Kariv (2007b). The PERSONAL RISK domain is identical to the (symmetric) risk experiment of Choi, Fisman, Gale, and Kariv (2007a). The SOCIAL domain is identical to the (linear) two-person dictator experiment of Fisman et al (2007).[4] The SOCIAL RISK

[3]As we discuss below, State Monotonicity is a *much* weaker assumption than Independence because it compares only lotteries whose outcomes are primitives – social states – rather than lotteries whose outcomes are themselves lotteries. Almost all decision-theoretic models that have been proposed as alternatives to Expected Utility obey State Monotonicity.

[4]Ahn, Choi, Gale, and Kariv (2014) extended the work in Choi et al. (2007a) on risk to settings with ambiguity. Choi, Kariv, Müller, and Silverman (2014) investigated the correlation between individual behavior and demographic and economic characteristics in the CentERpanel (a representative sample of Dutch households). These datasets have also been analyzed by others, including Halevy, Persitz, and Zrill (2018) and Pollison, Quah, and Renou (2018). Fisman, Jakiela, Kariv, and Markovits (2015b), Fisman, Jakiela, and Kariv (2015a), Fisman, Jakiela, and Kariv (2017) and Li et al (2017) build on the work in Fisman, Kariv, and Markovits (2007) to study distributional preferences. Since all experimental designs share the same graphical interface, we are building on expertise we have acquired in previous work.

domain is new and intended to represent the choice problem over lotteries over pairs of consumption for *self* and for *other*.[5]

The approach has a number of advantages over earlier approaches. First, the choice of a bundle subject to a budget constraint provides more information about preferences than a typical binary choice. Second, because the interface is extremely user-friendly, it is possible to present each subject with *many* choices in the course of a single experimental session, yielding much larger data set. This makes it possible to analyze behavior at the level of the individual subject, without the need to pool data or assume that subjects are homogenous. And third, it is easy to make direct comparisons of choices across the three domains using an attractive non-parametric econometric approach that builds on classical revealed preference analysis.

For some of our subjects, the experimental data does not provide testable implications of our theory unless we make additional assumptions about the form, parametric or otherwise, of the underlying utility function. However, for two classes of subjects, the predictions of our theory *are* testable:

- For subjects who are *selfish* – those who, in the Social domain give nothing to *other* – the theory predicts that choice behavior in the Personal Risk domain should *coincide* with choice behavior in the Social Risk domain.
- For subjects who are *egalitarian* – those who, in the Social domain, treat *other* symmetrically to *self* – the theory predicts that choice behavior in the Social domain should *coincide* with choice behavior in the Social Risk domain. (That is, risk attitude should be irrelevant in the Social Risk domain.)[6]

The theory of revealed preference allows us to provide strong and completely non-

[5]It is of course possible that presenting choice problems graphically biases choice choice behavior in some particular way – and that is a useful topic for experiment – but there is no evidence that this is the case. For instance: behavior in the Social domain elicited graphically Fisman et al. (2007) is quite consistent with behavior elicited by other means (Camerer, 2003), and behavior in the Personal Risk domain elicited graphically Choi et al. (2007b) is quite consistent with behavior elicited by other means (Holt and Laury, 2002).

[6]The objects of choice in the Social domain are payout pairs $(x, y)$ where $x$ is the payout to other and $y$ is the payout to *self*. In the experimental setting, we offer choices from linear budget sets $px + qy \leq w$. We identify behavior in the Social domain to be selfish if the choice subject to the budget constraint $px + qy \leq w$ is always of the form $(0, y)$ (payout to *other* is 0). We identify behavior in the Social domain to be egalitarian if $(a, b)$ is chosen subject to the budget constraint $px + qy \leq w$ if and only if $(b, a)$ is chosen subject to the mirror-image budget constraint $qx + py \leq w$.

parametric tests of these predictions. Among our subjects, we find many who are completely (or at least extremely) selfish and a number who are completely (or at least extremely) egalitarian; for many of these, the theoretical predictions are well supported by the experimental data, but for a number of subjects the theoretical predictions are rejected. We might say that, according to the theory, these subjects' preferences are *not consistent* across the various choice domains. Of course this does not mean that Theorems is false, but only that these subjects do not satisfy all the hypotheses of the Theorem. Most obviously, these subjects might have preferences over lotteries that do not obey State Monotonicity.

The analysis is complicated by the fact that individual choices frequently involve at least some errors: subjects may compute incorrectly, or execute intended choices incorrectly, or err in other less obvious ways. Because of these "mistakes" subjects' preferences need not be consistent *within* a choice domain – their choices do not conform perfectly with the Generalized Axiom of Revealed Preference (GARP) – so these "mistakes" must be taken into account testing for consistent *across* the various choice domains. (The discussion in Section 6 explains how we do this.)

The remainder of the paper is organized as follows. Section 2 provides the theoretical framework for our analysis. Section 3 contains our main theoretical result and proof. Section 4 provides the transition from the general theory to the implications for the experimental setting. Section 5 describes the experiment and the data and Section 6 presents the tests of the theory. Section 7 presents an extension of the theoretical analysis to the setting in which we can observe comparisons between social states and personal lotteries. Section 8 describes how the paper is related to prior research and provides some concluding remarks.

## 2 Framework

We consider a $\mathcal{DM}$ and a given set of outcomes $\Omega$ with a distinguished proper subset $P \subset \Omega$. For convenience, we refer to elements of $\Omega$ as *social states* and to elements of $P$ as *personal states*. In the interpretation discussed in the Introduction, a social state has consequences for society (of which the $\mathcal{DM}$ is a member) as a whole; a personal state has consequences only for the $\mathcal{DM}$. We stress, however, that this is only an interpretation: our abstract formalization is quite general and encompasses many other interpretations. We are agnostic about the specific natures of $\Omega, P$ in part because outside observers may differ with respect to their knowledge of the relevant social states and personal states and with respect to what they observe.[7]

We assume $\Omega$ is finite; this avoids subtle issues about the topology of $\Omega$ and the continuity properties of preferences. We also assume that $P \subset \Omega$ contains at least two states that the $\mathcal{DM}$ does not find indifferent (there are states $A, B \in P$ with $A \succ B$); this avoids degeneracy. For any subset $\Theta \subset \Omega$, we write $L(\Theta)$ for the set of finite lotteries over states in $\Theta$. We frequently write $\sum_i p_i \omega_i$ for the lottery whose outcome is the state $\omega_i$ with probability $p_i$. We refer to lotteries in $L(P)$ as *personal lotteries* and to lotteries in $L(\Omega)$ as *social lotteries*.

We assume that the $\mathcal{DM}$ has a preference relation $\succeq$ on $L(\Omega)$ that satisfies the familiar requirements: Completeness, Transitivity, Continuity, Reduction of Compound Lotteries and the Sure Thing Principle. These imply that we may – and do – identify the lottery $\sum_i p_i \omega$ with the certain state $\omega$. Throughout, we also assume that $\succeq$ obeys the following requirement, which we call *State Monotonicity*.

**State Monotonicity** If $\omega_i, \omega'_i \in \Omega$ for $i = 1, \ldots k$, $\omega_i \succeq \omega'_i$ for each $i$ and $p = (p_1, \ldots, p_k)$ is a probability vector, then

$$\sum_{i=1}^{k} p_i \omega_i \succeq \sum_{i=1}^{k} p_i \omega'_i$$

State Monotonicity is *equivalent* to a condition that (Grant, Kajii, and Polak, 1992) call Degenerate Independence.[8] For ease of comparison, recall that the familiar (von Neumann and Morgenstern, 1944) Independence Axiom is

**Independence** If $W_i, W'_i \in L(\Omega)$ for $i = 1, \ldots k$, $W_i \succeq W'_i$ for each $i$ and $p =$

[7] In earlier versions of this paper we were more specific about the nature of $\Omega, P$. We assumed that $\Omega$ has a product structure: $\Omega \subset X \times Z$; where, given a state $\omega = (x, z)$ the component $x$ represents the personal component of $\omega$ and $z$ represents the social component of $\omega$. We assumed a given reference social component $z_0 \in Z$ and identified $P$ with the product $X \times \{z_0\}$. However, because the particular structure played no role in the actual analysis, we prefer to use the more abstract formulation given here.

[8] As we do here, Grant et al. (1992) asks to what extent *all* preference comparisons can be deduced from a *subset* of preference comparisons. However, because our intent is different from Grant et al. (1992) we face quite different issues so the differences are greater than the similarities. The discussion in the Conclusion elaborates on that.

$(p_1, \ldots, p_k)$ is a probability vector, then

$$\sum_{i=1}^{k} p_i W_i \succeq \sum_{i=1}^{k} p_i W_i'$$

Notice that the difference between these two Axioms is precisely that the Independence Axiom posits comparisons between *lotteries over lotteries*, while State Monotonicity only posits comparisons between *lotteries over states*.[9] As we discuss below, the difference is enormous. We note that almost all decision-theoretic models that have been proposed as alternatives to Expected Utility of which we are aware obey State Monotonicity, including Weighted Expected Utility (Dekel, 1986; Chew, 1989), Rank Dependent Utility (Quiggin, 1982, 1993), and (much of) Prospect Theory (Tversky and Kahneman, 1992).

To see what State Monotonicity does and does not imply, suppose that $\Omega$ consists of three mutually non-indifferent states $\Omega = \{A, X, B\}$; without loss, assume that $A \succ X \succ B$ and picture the familiar Marschak-Machina triangle in which each point represents a lottery $(p_A, p_X, p_B)$ over the states $A, X, B$ ($p_A = 0$ on the horizontal edge, $p_X = 0$ on the hypotenuse, and $p_B = 0$ on the vertical edge). Continuity requires that $X$ be indifferent to some lottery over $A, B$; say $X \sim \frac{1}{2}A + \frac{1}{2}B$.[10] Assuming the other axioms, Independence implies that the preference relation $\succeq$ admits an Expected Utility representation, so that the indifference curves in the triangle are parallel straight lines. Hence knowledge that $X \sim \frac{1}{2}A + \frac{1}{2}B$ completely determines $\succeq$ on the entire triangle. In particular, $\frac{1}{2}A + \frac{1}{2}X \sim \frac{3}{4}A + \frac{1}{4}B$, $\frac{1}{2}X + \frac{1}{2}B \sim \frac{1}{4}A + \frac{3}{4}B$ and so forth – see Figure 1A.

In Weighted Expected Utility, Betweenness, which is a weaker axiom than Independence, implies that all indifference curves are again straight lines but they need not be parallel; in particular, it may be that $\frac{1}{2}A + \frac{1}{2}X \sim \frac{1}{8}A + \frac{7}{8}B$ and $\frac{1}{2}X + \frac{1}{2}B \sim \frac{1}{10}A + \frac{9}{10}B$ – see Figure 1B. In this example, the indifference curves "fan out," becoming steeper (corresponding to higher risk aversion) when moving northeast in the triangle. In Rank Dependent Utility and Prospect Theory, indifference curves can change slopes and also "fan out" and "fan in," especially near the triangle boundaries – see Figure 1C.

To see what State Monotonicity implies, consider a lottery $aA + xX + bB$ ($0 \leq a, x, b \leq 1$

[9] We have formulated State Monotonicity in terms of weak preference, rather than indifference, because the two are not generally equivalent. We have formulated the Independence Axiom in terms of weak preference, rather than indifference, only to highlight the difference between State Monotonicity and Independence.

[10] Some caution must be exercised here. In the absence of the Independence Axiom, which we do not assume, the Continuity Axiom is stronger than the Archimedean Axiom.

and $x = 1 - a - b$) and a lottery $(a+\varepsilon)A + (x-\varepsilon)X + bB$ obtained by shifting shifting $\varepsilon$ of the probability mass from $X$ to $A$. State Monotonicity – together with Transitivity and the Sure Thing Principle – entails that

$$\begin{aligned} aA + xX + bB &\sim aA + (x-\varepsilon)X + \varepsilon X + bB \\ &\preceq aA + \varepsilon A + (x-\varepsilon)X + bB \\ &\sim (a+\varepsilon)A + (x-\varepsilon)X + bB. \end{aligned}$$

Thus State Monotonicity implies that the preference relation $\succeq$ is increasing (from bottom to top) along vertical lines. Similarly, $\succeq$ is also decreasing (from left to right) along horizontal lines.[11] Hence the indifference curves of $\succeq$ must be "upward sloping" (pointing northeast in the triangle) but can otherwise be quite arbitrary – see Figure 1D.

The $\mathcal{DM}$'s preference relation $\succeq$ over *all* lotteries $L(\Omega)$ is fixed, but not known (to us). We seek to *deduce* $\succeq$ but must base this deduction on observation/inference of only a *subset* of all comparisons. Our main focus is on a setting in which we observe only the $\mathcal{DM}$'s comparisons between social states and the $\mathcal{DM}$'s comparisons between personal lotteries – but not the $\mathcal{DM}$'s comparisons between social states and personal lotteries. That is, we observe the *sub-preference relation* $\succeq_0$whose graph is:

$$\text{graph}(\succeq_0) = \text{graph}(\succeq) \cap ([L(P) \times L(P)] \cup [\Omega \times \Omega])$$

We also consider in Section 7 the setting in which we observe the $\mathcal{DM}$'s comparisons between social states and personal lotteries. That is, we observe the *sub-preference relation* $\succeq_1$whose graph is:

$$\text{graph}(\succeq_1) = \text{graph}(\succeq) \cap ([\Omega \cup L(P)] \times [\Omega \cup L(P)])$$

Notice that observing $\succeq_0$ is just the same as observing the restriction of $\succeq$ to $L(P)$ *and* the restriction of $\succeq$ to $\Omega$, whereas observing $\succeq_1$ is just the same as observing the restriction of $\succeq$ to $\Omega$ *and* $L(P)$. Our main focus is on the setting in which we observe $\succeq_0$ rather than $\succeq_1$ because that setting is more realistic and because it can more easily be presented in an experimental setting.[12] To see the difference between observing $\succeq_0$ and

[11] Put differently, State Monotonicity implies that if the lottery $W'$ is formed from the lottery $W$ by *only* shifting probability mass from lower-ranked states to higher-ranked states, then $W' \succeq W$.

[12] When choosing a President, for instance, the voter observes/infers comparisons between social states on the basis of policy statements the candidate makes, *and* observes/infers comparisons between personal lotteries through the candidate's observed choices and media reports. But the voter is unlikely to observe/infer the candidate's comparisons between social states *and* personal lotteries.

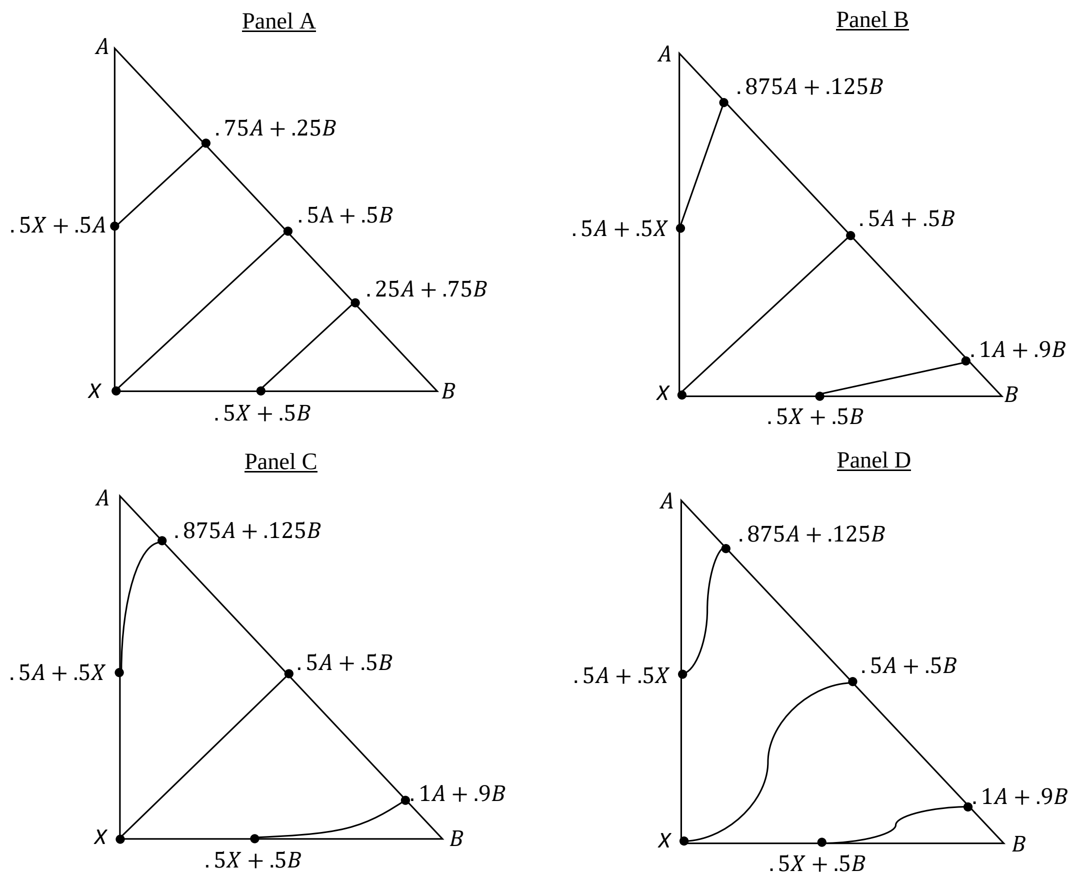



Figure 1: Independence, Betweenness, Rank-dependence, and State Monotonicity in the Marschak-Machina triangle

$\succeq_1$, consider once again the setting in which $\Omega$ consists of the three states $A, X, B$ with $A \succ X \succ B$ and picture the Marschak-Machina triangle in which each point represents a lottery $(p_A, p_X, p_B)$ over the states $A, X, B$. Suppose that $A, B$ are personal states and $X$ is a social state (and hence is not equivalent to any personal state).

If we observe $\succeq_0$ we observe the ordering $A \succ_0 X \succ_0 B$ and the ordering of lotteries between $A, B$ – but no others – shaded gray in Figure 2A. State Monotonicity assures us that from these observations we can *infer* the ordering of lotteries between $A, X$ and lotteries between $X, B$ – see Figure 2B. Continuity assures us that $X$ is indifferent to *some* lottery $aA + (1-a)B$ – but we do not observe *which lottery*. If we observe $\succeq_1$ then we *do* observe which lottery – but that is all – see Figure 2C. However, if we observe $\succeq_1$ and we assume that $\succeq$ obeys Independence – and hence has an Expected Utility representation – then observing *which lottery* completely determines $\succeq$ – see Figure 2D.

This illustrates how deducing the $\mathcal{DM}$'s entire preference relation $\succeq$ from a sub-preference relation depends on the amount that can be observed/infrared about the $\mathcal{DM}$'s preferences as well as on the degree of rationality we ascribe to the $\mathcal{DM}$, and in particular on whether the observer believes/assumes that the $\mathcal{DM}$'s preferences obey the axioms of Expected Utility or believes/assumes only that the $\mathcal{DM}$'s preferences obey some weaker criteria. We next show that if we assume only State Monotonicity, then a necessary and sufficient condition that it be possible to deduce the entire preference relation $\succeq$ from $\succeq_0$ is that the $\mathcal{DM}$ finds every social state to be indifferent to some personal state.

## 3 Deducing Preferences

The question we have in mind can now be formulated in the following way: If we observe the sub-preference relation $\succeq_0$ can we *deduce* the entire preference relation $\succeq$? In different words: is $\succeq$ the *unique* entire preference relation that extends the sub-preference relation $\succeq_0$ and obeys the same axioms (Completeness, Transitivity, Continuity, Reduction of Compound Lotteries, the Sure Thing Principle and State Monotonicity)? We next provide necessary and sufficient conditions that this be the case. When these necessary and sufficient conditions are *not* satisfied, there will be *many* lotteries in $L(\Omega)$ over which the preference ordering of the $\mathcal{DM}$ $\succeq$ *cannot* be deduced from $\succeq_0$.

**Theorem 1** *Assume that the $\mathcal{DM}$'s preference relation $\succeq$ satisfies Completeness, Transitivity, Continuity, Reduction of Compound Lotteries, the Sure Thing Principle and State*

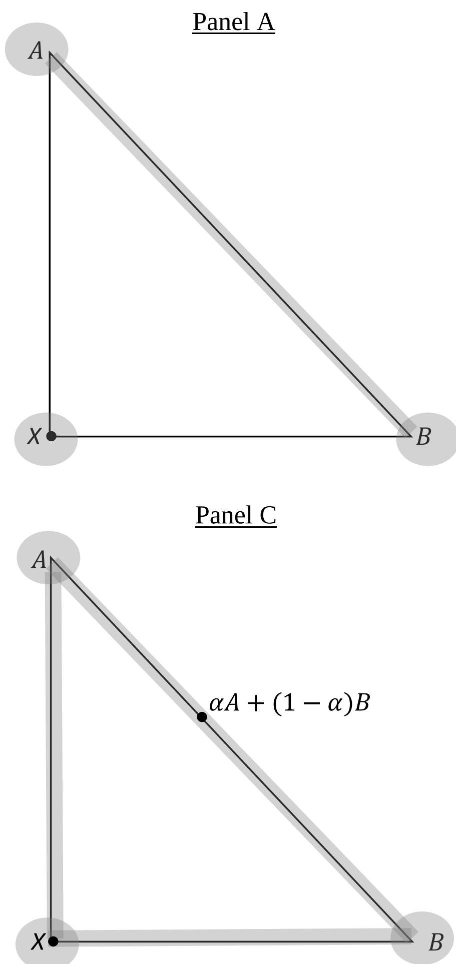



Figure 2: The difference between observing and in the Marschak-Machina triangle

*Monotonicity. In order that $\succeq$ can be deduced from $\succeq_0$ it is necessary and sufficient that the $\mathcal{DM}$ finds every social state $\omega \in \Omega/P$ to be indifferent to some personal state $\tilde{\omega} \in P$.*

**Proof.** To see that this condition is sufficient, assume that every social state $\omega$ admits a personal state equivalent $\tilde{\omega}$. State Monotonicity implies that $\sum p_i\omega_i \sim \sum p_i\tilde{\omega}_i$ for every lottery $\sum p_i\omega_i \in L(\Omega)$. Hence given two lotteries $\sum p_i\omega_i, \sum q_j\omega_j$ it follows from transitivity that

$$\sum p_i\omega_i, \succeq \sum q_j\omega_j \Leftrightarrow \sum p_i\tilde{\omega}_i \succeq \sum q_j\tilde{\omega}_j \Leftrightarrow \sum p_i\tilde{\omega}_i \succeq_0 \sum q_j\tilde{\omega}_j.$$

That is, $\succeq$ can be deduced from $\succeq_0$.

To see that this condition is necessary, we suppose that there is some social state $X$ that the $\mathcal{DM}$ does *not* find indifferent to any personal state and construct a preference relation that agrees with $\succeq$ on $L(P)$ and on $\Omega$ but not on all of $L(\Omega)$. We will need to take some care in order to be sure that the preference relation we construct obeys Continuity and State Monotonicity.

Because $\succeq$ is continuous (by assumption) and $L(\Omega)$ can be identified with a finite-dimensional simplex, which is a separable metric space, we can use Debreu's representation theorem (Debreu 1954) to find a utility function $u : L(\Omega) \to \mathbb{R}$ that represents $\succeq$, that is

$$\forall \Gamma, \Gamma' \in L(\Omega) : \Gamma \succeq \Gamma' \Leftrightarrow u(\Gamma) \geq u(\Gamma').$$

Without loss, assume that the range of $u$ contained in the interval $[0, 1]$. We construct a new utility function $U : L(\Omega) \to \mathbb{R}$ that agrees with $u$ on $L(P)$ and induces the same ordering as $u$ on $\Omega$ but does not induce the same ordering as $u$ on $L(\Omega)$.

To make the remainder of the proof more transparent, consider again three mutually non-indifferent states $\Omega = \{A, X, B\}$, two of which are personal states $P = \{A, B\}$, and assume without loss that $A \succ B$. As our earlier discussion of State Monotonicity suggests, in this setting it is easy construct a utility function $U$ on $L(\Omega)$ with the desired properties – but it is a little less easy to do so in a way that generalizes to the general setting with finitely many states. It is convenient to distinguish three cases: (i) $A \succ X \succ B$, (ii) $X \succ A \succ B$, and (iii) $A \succ B \succ X$:

- **Case (i) $\mathbf{A \succ X \succ B}$:** Continuity guarantees that there is some $\gamma \in (0, 1)$ such that $X \sim \gamma A + (1 - \gamma)B$; equivalently, $u(X) = u(\gamma A + (1 - \gamma)B)$. We construct a continuous utility function $U$ on $L(\Omega)$ that agrees with $u$ on $L(P)$ and for which

$U(A) > U(X) > U(B)$ – so that $U$ induces the same ordering as $u$ on $\Omega$ – but $U(X) \neq u(X) = u(\gamma A + (1-\gamma)B)$ – so that the preference relation $\succeq_U$ induced by $U$ does not agree with $\succeq$ on $L(\Omega)$.

To understand the idea behind the construction, consider once again the Marschak-Machina triangle – see Figure 1D. In order that $U$ satisfy State Monotonicity, $U$ must be strictly increasing (from bottom to top) along vertical lines and strictly decreasing (from left to right) along horizontal lines. To construct $U$ we first define two auxiliary functions $f, g$:

$$\begin{aligned} f(aA + xX + bB) &= u(aA + xA + bB) \\ g(aA + xX + bB) &= u(aA + xB + bB) \end{aligned}$$

for every lottery $aA + xX + bB \in L(\Omega)$. Because $u$ is continuous, both $f$ and $g$ are continuous. Moreover, $f$ is constant on vertical lines and strictly decreasing on horizontal lines, while $g$ is strictly increasing on vertical lines and constant on horizontal lines.

Hence for every $\lambda \in (0,1)$ the convex combination $\lambda f + (1-\lambda)g$ is strictly increasing on vertical lines and strictly decreasing on horizontal lines. Choose $\lambda$ so that

$$\lambda f(X) + (1-\lambda)g(X) = \lambda u(A) + (1-\lambda)u(B) \neq u(X),$$

define the utility function $U = \lambda f + (1-\lambda)g$, and let $\succeq_U$ be the preference relation induced by $U$. It is evident that $\succeq_U$ satisfies Completeness, Transitivity, Reduction of Compound Lotteries and the Sure Thing Principle. Because $u, f, g$ are continuous, so is $U$; hence $\succeq_U$ satisfies Continuity. By construction, $U$ is strictly increasing along vertical lines and strictly decreasing along horizontal lines, so $\succeq_U$ satisfies State Monotonicity. Finally, note that $U$ agrees with $u$ on $L(P)$ and

$$U(A) = u(A) > U(X) = \lambda u(A) + (1-\lambda)u(B) > u(B) > U(B)$$

so that $\succeq_U$ is an extension of $\succeq$. Finally, because we have chosen $\lambda$ so that $U(X) \neq u(X) = u(\gamma A + (1-\gamma)B)$ it follows that $\succeq_U \neq \succeq$, so the argument is complete.

- **Case (ii) $\mathbf{X} \succ \mathbf{A} \succ \mathbf{B}$:** This is easier than Case (i). Continuity guarantees that there is some $\nu \in (0,1)$ for which $A \sim \nu X + (1-\nu)B$. Choose $\lambda > 0$ for which $u(\nu A + (1-\nu)B) + \lambda\nu \neq u(A)$ and set

$$U(xX + aA + bB) = u(xA + aA + bB) + \lambda x$$

By construction, $U$ agrees with $u$ on $L(P)$ and $U(X) = u(A) + \lambda x > u(A) = U(A)$ so the preference relation $\succeq_U$ represented by $U$ is indeed an extension of $\succeq$. It easily checked that $\succeq_U$ satisfies all the required axioms; since $U(\nu X + (1-\nu)B) = u(\nu A + (1-\nu)B) + \lambda\nu \neq u(A)$ we conclude that $\succeq_U \neq \succeq$.

- **Case (iii) $\mathbf{A} \succ \mathbf{B} \succ \mathbf{X}$:** The argument is almost the same as in Case (ii): we simply interchange the roles of $A, B$ and change the sign of the linear term. Continuity guarantees that there is some $\eta \in (0,1)$ for which $B \sim \eta A + (1-\eta)X$. Choose $\lambda > 0$ for which $u(\eta A + (1-\eta)B) - \lambda\eta \neq u(B)$ and set

$$U(aA + bB + xX) = u(aA + bB + xB) - \lambda x$$

By construction, $U$ agrees with $u$ on $L(P)$ and $U(X) = u(B) - \lambda x < u(B) = U(B)$ so the preference relation $\succeq_U$ represented by $U$ is indeed an extension of $\succeq$. It easily checked that $\succeq_U$ satisfies all the required axioms; since $U(\eta A + (1-\eta)X) = u(\eta A + (1-\eta)B) - \lambda\eta \neq u(B)$ we conclude that $\succeq_U \neq \succeq$.

We now turn to the general setting. We must take account of the possible presence of more states and of the possible differences in the ranking of the distinguished social state with respect to personal states, but the main idea remains the same. Suppose that there is some social state $X$ that the $\mathcal{DM}$ does *not* find indifferent to any personal state. Write $\mathcal{A}$ for the set of states that are strictly preferred to $X$ according to $\succeq$, $\mathcal{B}$ for the set of states that are strictly dis-preferred to $X$, and $\mathcal{X}$ for the set of states that are indifferent to $X$. Because $X$ is not indifferent to any personal state, no member of $\mathcal{X}$ is indifferent to any personal state; moreover, at least one of $\mathcal{A}, \mathcal{B}$ is non-empty.

If $\mathcal{A} \neq \emptyset$, let $A$ be any $\succeq$-minimal element of $\mathcal{A}$; if $\mathcal{B} \neq \emptyset$ let $B$ be any $\succeq$-maximal element of $\mathcal{B}$. (Such minimal and maximal elements exist because $\Omega$ is finite.) For each lottery $\Gamma = \sum p_i\omega_i \in L(\Omega)$ write

$$\Gamma_{\mathcal{A}} = \sum_{\omega_i \in \mathcal{A}} p_i\omega_i \ ; \quad \Gamma_{\mathcal{B}} = \sum_{\omega_i \in \mathcal{B}} p_i\omega_i \ ; \quad \Gamma_{\mathcal{X}} = \sum_{\omega_i \in \mathcal{X}} p_i\omega_i \ .$$

Evidently, $\Gamma = \Gamma_{\mathcal{A}} + \Gamma_{\mathcal{B}} + \Gamma_{\mathcal{X}}$.

We now distinguish three cases that are parallel to the cases considered above and carry through constructions that are parallel to the constructions used in those cases:

- **Case (i) $\mathcal{A} \neq \emptyset$ and $\mathcal{B} \neq \emptyset$:** Continuity guarantees there is some $\gamma \in (0,1)$ such that $X \sim \gamma A + (1-\gamma)B$. As before, define auxiliary functions $f, g : L(\Omega) \to \mathbb{R}$ by

$$\begin{aligned} f(\Gamma) &= u(\Gamma_{\mathcal{A}} + x(\Gamma)A + \Gamma_{\mathcal{B}}) \\ g(\Gamma) &= u(\Gamma_{\mathcal{A}} + x(\Gamma)B + \Gamma_{\mathcal{B}}) \end{aligned}$$

where $x(\Gamma) = \sum_{\omega_i \in \mathcal{X}} p_i$. Choose $\lambda$ so that $\lambda f(X) + (1-\lambda)g(X) \neq u(\gamma A + (1-\gamma)B$ and define $U = \lambda f + (1-\lambda)g$. It is easily checked that the preference relation $\succeq_U$ induced by $U$ satisfies all the desired axioms; because $U(X) \neq U(\gamma A + (1-\gamma)B)$ we conclude that $\succeq_U \neq \succeq$.

- **Case (ii) $\mathcal{A} = \emptyset$ and $\mathcal{B} \neq \emptyset$:** Because there are at least two inequivalent personal states, we can choose $B' \in \mathcal{B}$ with $B \succ B'$. Continuity guarantees there is some $\nu \in (0,1)$ for which $B \sim \nu X + (1-\nu)B'$. Choose $\lambda > 0$ so that $u(\nu B + (1-\nu)B') - \lambda\nu \neq u(B)$ and set

$$U(\Gamma) = u(x(\Gamma)B + \Gamma_{\mathcal{B}}) - \lambda x(\Gamma)$$

  It is easily checked that the preference relation $\succeq_U$ induced by $U$ satisfies all the desired axioms; because $U(\nu X + (1-\nu)B') = u(\nu B + (1-\nu)B') - \neq u(B)$ we conclude that $\succeq_U \neq \succeq$.

- **Case (iii) $\mathcal{A} \neq \emptyset$ and $\mathcal{B} = \emptyset$:** Because there are at least two inequivalent personal states, we can choose $A' \in \mathcal{A}$ for which $A' \succ A$. Continuity guarantees there is some $\eta \in (0,1)$ for which $A \sim \eta A' + (1-\eta)X$. Choose $\lambda > 0$ for which $u(\eta A' + (1-\eta)A) + \lambda\eta \neq u(A)$ and set

$$U(\Gamma) = u(\Gamma_{\mathcal{A}} + x(\Gamma)A) + \lambda x(\Gamma)$$

  It is easily checked that the preference relation $\succeq_U$ induced by $U$ satisfies all the desired axioms; because $U(\eta A' + (1-\eta)X) = u(\eta A' + (1-\eta)A) + \lambda\eta \neq u(A)$ we conclude that $\succeq_U \neq \succeq$.

In each case we have constructed a preference relation that extends $\succeq_0$, satisfies all of our axioms and differs from $\succeq$, so the proof is complete. $\blacksquare$

## 4 Testable Implications

This section provides a bridge between the general theory described above and our experiment, designed to test the implications of the theory. We first define a special setting that serves as the domains/environments in the experimental design and describe their theoretical properties. Then we develop a number of theoretical results in this setting that are testable on the basis of observed choices.

In order to connect the theoretical predictions of the Theorem with our experimental data, it is convenient to isolate the argument for sufficiency and extend it to a setting in which we consider only a particular set of lotteries. To this end, fix a non-empty set $\Pi$ of probability vectors. For each non-empty subset $\Theta \subset \Omega$, let $L_\Pi(\Theta)$ be the set of lotteries of the form $p_1\theta_1 + \ldots + p_k\theta_k$, where $(p_1, \ldots, p_k) \in \Pi$ and $\theta_1, \ldots, \theta_k \in \Theta$. In view of the Sure Thing Principle, we may identify the lottery $p_1\theta + \ldots + p_k\theta$ with $\theta$ itself, so $\Theta \subset L_\Pi(\Theta)$.

If $\Pi$ is the set of all probability vectors then $L_\Pi(\Omega) = L_\Pi(\Omega)$. In that context, recall that observing $\succeq_0$ is just the same as observing the restrictions of $\succeq$ to $\Omega$ and to $L(P)$. Hence the following proposition generalizes the sufficient condition in Theorem 1.

**Proposition 2** *Let $\Pi$ is a non-empty set of probability vectors and let $\succeq$ be a preference relation on $L_\Pi(\Omega)$ that satisfies Completeness, Transitivity, Continuity, the Sure Thing Principle and State Monotonicity. In order that $\succeq$ can be deduced from its restrictions $\succeq_\Omega$ to $\Omega$ and $\succeq_{L_\Pi(P)}$ to $L_\Pi(P)$, it is sufficient that the $\mathcal{DM}$ finds social state $\omega \in \Omega/P$ to be indifferent to some personal state $\tilde{\omega} \in P$.*

**Proof.** By assumption, every social state $\omega \sim_\Omega \tilde{\omega}$ for some personal state $\tilde{\omega} \in P$. (We do not require that $\tilde{\omega}$ be unique.) State Monotonicity implies that if $(p_1, \ldots, p_k) \in \Pi$ and $\omega_1, \ldots, \omega_k \in \Omega$ then

$$\sum p_i\omega_i \sim \sum p_i\tilde{\omega}_i$$

Hence given two lotteries $\sum p_i\omega_i, \sum q_j\omega_j \in L_\Pi(\Omega)$ it follows from State Monotonicity and Transitivity that

$$\sum p_i\omega_i \succeq \sum q_j\omega_j \Leftrightarrow \sum p_i\tilde{\omega}_i \succeq \sum q_j\tilde{\omega}_j \Leftrightarrow \sum p_i\tilde{\omega}_i \succeq_{L_\Pi(P)} \sum q_j\tilde{\omega}_j.$$

That is, $\succeq$ can be deduced from $\succeq_\Omega$ and $\succeq_{L(P)}$, as asserted. ■

In the experiment there is a subject *self* (the $\mathcal{DM}$) and an (unknown) *other* so the set of social states $\Omega$ consists of *monetary* payout pairs $(a, b)$, where $b \geq 0$ is the payout for *self* and $a \geq 0$ is the payout for *other*. Because the set of lotteries we can present to (human) subjects is limited, we restrict the set $L(\Omega)$ of lotteries on the set $\Omega$ of social states to binary lotteries with equal probabilities $\frac{1}{2}(a, b) + \frac{1}{2}(c, d)$. In the framework of the Proposition 2, we are restricting the set of lotteries to $\Pi = \{(\frac{1}{2}, \frac{1}{2})\}$ . To simplify notation, let $\mathbb{L}$ be the set of all such equiprobable lotteries. Within $\mathbb{L}$ we distinguish three subsets:

$$\begin{aligned}\mathcal{PR} &= \{\tfrac{1}{2}(0,b) + \tfrac{1}{2}(0,d)\} = \mathbb{L}(P)\\ \mathcal{SC} &= \{\tfrac{1}{2}(a,b) + \tfrac{1}{2}(a,b)\} = \Omega\\ \mathcal{SR} &= \{\tfrac{1}{2}(a,b) + \tfrac{1}{2}(b,a)\} = \mathbb{L}(\mathrm{Perm}(\Omega))\end{aligned}$$

where $\mathrm{Perm}(\Omega)$ is the set of permutations of $\Omega$. Notice that $\mathbb{L}(\Omega)$ is a 4-dimensional convex cone, which cannot be presented to subjects in an experiment in an obvious way. But the three subsets are 2-dimensional sub-cones that can be presented using our graphical experimental interface (more below).

We can interpret choice in each domain/environment – Personal Risk, Social Choice, Social Risk – as choice in one of these subsets above by making an obvious identification: in the Personal Risk domain $\langle x,y\rangle \mapsto \frac{1}{2}(0,x)+\frac{1}{2}(0,y)$; in the Social Choice domain $(x,y) \mapsto \frac{1}{2}(x,y) + \frac{1}{2}(x,y)$; in the Social Risk domain $[x,y] \mapsto \frac{1}{2}(x,y) + \frac{1}{2}(y,x)$. Hence in the Personal Risk domain, the objects of choice are equiprobable personal lotteries (*other* receives nothing); in the Social Choice domain, the objects of choice are deterministic payout pairs for *self* and *other* (in view of the Sure Thing Principle); in the Social Risk domain, the objects of choice are equiprobable social lotteries over symmetric pairs of payouts for *self* and for *other*. Note we have used the different brackets $\langle x,y\rangle$, $(x,y)$, $[x,y]$ as a reminder that we are thinking of the pair $x,y$ as representing a personal lottery in $\mathcal{PR}$, a social state in $\mathcal{SC}$, a social lottery in $\mathcal{SR}$, respectively.

Let $\succeq$ be a preference relation on $\mathbb{L}$ and write $\succeq_{\mathcal{PR}}$, $\succeq_{\mathcal{SC}}$, $\succeq_{\mathcal{SR}}$ for its restrictions (sub-preference relations) to $\mathcal{PR}$, $\mathcal{SC}$, $\mathcal{SR}$, respectively. The restriction $\succeq_{\mathcal{PR}}$ of $\succeq$ to $\mathcal{PR}$ prescribes preferences over personal lotteries and the restriction $\succeq_{\mathcal{SC}}$ of $\succeq$ to $\Omega$ prescribes preferences over social states. Thus to observe $\succeq_0$ in this setting is *exactly* to observe both $\succeq_{\mathcal{PR}}$ *and* $\succeq_{\mathcal{SC}}$ so Proposition 2 provides a sufficient condition that $\succeq$ – and, in particular, the restriction $\succeq_{\mathcal{SR}}$ to $\mathcal{SR}$ – be deducible from $\succeq_{\mathcal{PR}}$ and $\succeq_{\mathcal{SC}}$. That is, if we observe $\succeq_{\mathcal{PR}}$ and $\succeq_{\mathcal{SC}}$, and if every social state is indifferent to some personal state (a condition that is determined completely by $\succeq_{\mathcal{SC}}$), then we can deduce $\succeq_{\mathcal{SR}}$. But to test this implication, it is not enough to know just the *fact* that every social state is indifferent to some personal state; we need to know, for each social state, a *particular* personal state to which the social state is indifferent. For some of our subjects, this would require making additional assumptions about the form, parametric or otherwise, of the underlying preferences. However, for two classes of subjects – *selfish* and *impartial* – we can construct a formal non-parametric test.

We say that preferences $\succeq_{\mathcal{SC}}$ in the Social Choice domain are *selfish* if $(x,y) \sim_{\mathcal{SC}} (0,y)$ and *impartial* if $(x,y) \sim_{\mathcal{SC}} (y,x)$ for all $(x,y) \in \Omega$. For selfish $\mathcal{DM}$, we show that choice behavior in the Personal Risk domain *coincides* with choice behavior in the Social Risk domain; For impartial $\mathcal{DM}$, we show that choice behavior in the Social Choice domain *coincides* with choice behavior in the Social Risk domain (so an impartial $\mathcal{DM}$ is immune to social risk).

In our experiments, we present subjects with a sequence of standard consumer decision problems: selection of a bundle of commodities from a standard budget set:

$$\mathcal{B} = \{(x, y) \in \Omega : p_x x + p_y y = m\}$$

where $p_x, p_y, m > 0$. A choice of the allocation $(x, y)$ from the budget line represents an allocation between accounts $x, y$ (corresponding to the usual horizontal and vertical axes). The actual payoffs of a particular choice in a particular domain/environment are determined by the allocation to the $x$ and $y$ accounts, according to the particular domain – Personal Risk, Social Choice, Social Risk. With these preliminaries in hand, we can state the theoretical predictions for selfish preferences and impartial preferences in the experimental setting (but these results hold for arbitrary choice sets):

**Proposition 3** *If preferences $\succeq_{\mathcal{SC}}$ in the* Social Choice *domain are selfish then preferences $\succeq_{\mathcal{PR}}$ in the* Personal Risk *domain coincide with preferences $\succeq_{\mathcal{SR}}$ in the* Social Risk *domain. In particular: if preferences $\succeq_{\mathcal{SC}}$ in the* Social Choice *domain are selfish then choice behavior in the* Personal Risk *domain and choice behavior in the* Social Risk *domain coincide – so if $\mathcal{B}$ is a budget set then $[x, y] \in \arg\max(\mathcal{B})$ in the* Personal Risk *domain if and only if $\langle x, y\rangle \in \arg\max(\mathcal{B})$ in the* Social Risk *domain.*

**Proof.** $\langle x, y\rangle \in \arg\max(\mathcal{B})$ in the Personal Risk domain if and only if $\langle x, y\rangle \succeq_{\mathcal{PR}} \langle \hat{x}, \hat{y}\rangle$ for every $\langle \hat{x}, \hat{y}\rangle \in B$. Unwinding the notation, this means that

$$\langle x, y\rangle = \frac{1}{2}(0, x) + \frac{1}{2}(0, y) \succeq_{\mathcal{PR}} \frac{1}{2}(0, \hat{x}) + \frac{1}{2}(0, \hat{y}) = \langle \hat{x}, \hat{y}\rangle$$

for every $\langle \hat{x}, \hat{y}\rangle \in B$. If preferences $\succeq_{\mathcal{SC}}$ are selfish $(x, y) \sim_{\mathcal{SC}} (0, y)$ for all $(x, y)$ this will hold if and only if

$$[x, y] = \frac{1}{2}(y, x) + \frac{1}{2}(x, y) \succeq_{\mathcal{SR}} \frac{1}{2}(\hat{y}, \hat{x}) + \frac{1}{2}(\hat{x}, \hat{y}) = [\hat{x}, \hat{y}]$$

for every $[\hat{x}, \hat{y}] \in B$. We conclude that preferences $\succeq_{\mathcal{PR}}$ in the Personal Risk domain coincide with preferences $\succeq_{\mathcal{SR}}$ in the Social Risk domain. Since preferences in the two domains coincide, choice behavior coincides as well. ■

**Proposition 4** *If preferences $\succeq_{\mathcal{SC}}$ in the* Social Choice *domain are impartial then $\succeq_{\mathcal{SC}}$ coincide with preferences $\succeq_{\mathcal{SR}}$ in the* Social Risk *domain. In particular: if preferences $\succeq_{\mathcal{SC}}$ in the* Social Choice *domain are impartial then choice behavior in the* Social Choice *domain and choice behavior in the* Social Risk *domain coincide – so if $\mathcal{B}$ is a budget set then $(x, y) \in \arg\max(\mathcal{B})$ in the* Social Choice *domain if and only if $[x, y] \in \arg\max(\mathcal{B})$ in the* Social Risk *domain.*

**Proof.** Assume preferences $\succeq_{\mathcal{SC}}$ in the Social Choice domain are impartial. Consider two choices $[x, y]$ and $[\hat{x}, \hat{y}]$ in the Social Risk domain and suppose $[\hat{x}, \hat{y}] \succ_{\mathcal{SR}} [x, y]$. When we express this explicitly in terms of lotteries, this means

$$\tfrac{1}{2}(\hat{x}, \hat{y}) + \tfrac{1}{2}(\hat{y}, \hat{x}) \succ_{\mathcal{SR}} \tfrac{1}{2}(x, y) + \tfrac{1}{2}(y, x)$$

State Monotonicity implies that either $(\hat{x}, \hat{y}) \succ_{\mathcal{SC}} (x, y)$ or $(\hat{y}, \hat{x}) \succ_{\mathcal{SC}} (y, x)$; Impartiality implies that if either of these is true then they both of these are true. Hence we conclude that if $[\hat{x}, \hat{y}] \succ_{\mathcal{SR}} [x, y]$ in the Social Risk domain then $(\hat{x}, \hat{y}) \succ_{\mathcal{SC}} (x, y)$ in the Social Choice domain. Conversely if $(\hat{x}, \hat{y}) \succ_{\mathcal{SC}} (x, y)$ in the Social Choice domain then

$$\tfrac{1}{2}(\hat{x}, \hat{y}) + \tfrac{1}{2}(\hat{y}, \hat{x}) \succ_{\mathcal{SR}} \tfrac{1}{2}(x, y) + \tfrac{1}{2}(y, x)$$

That is: $[\hat{x}, \hat{y}] \succ_{\mathcal{SR}} [x, y]$ in the Social Risk domain. Putting these together we conclude that preferences $\succeq_{\mathcal{SC}}$ in the Social Choice domain coincide with preferences $\succeq_{\mathcal{SR}}$ in the Social Risk domain. Since preferences in the two domains coincide, choice behavior coincides as well. ■

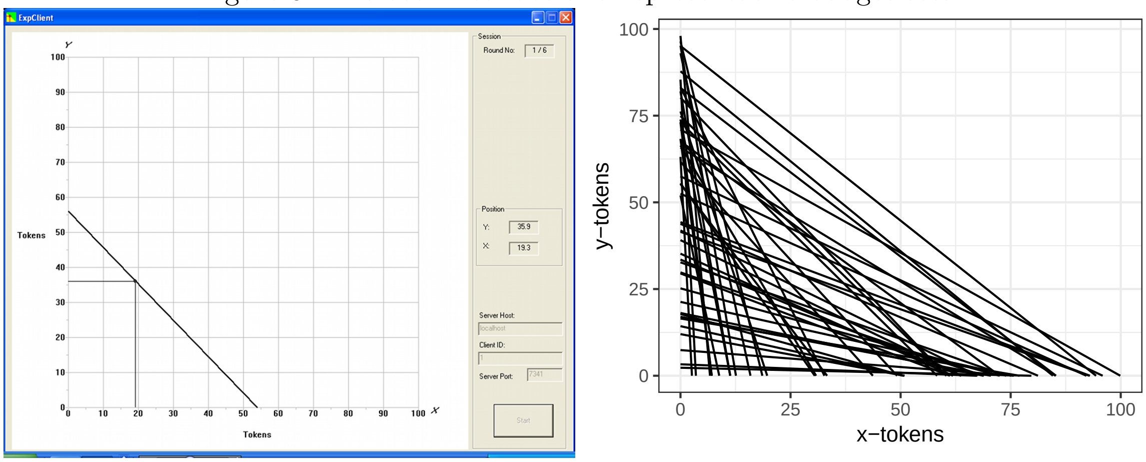
# 5 Experimental Design and Sample

In this section, we present the experimental design and the sample participating in the experiment.

## 5.1 Experimental Design

At the beginning of the session, the participants received general instructions and were told that the experiment would consist of four parts. The instructions were read aloud by an experimenter.

Figure 3: The user interface and representative budget sets

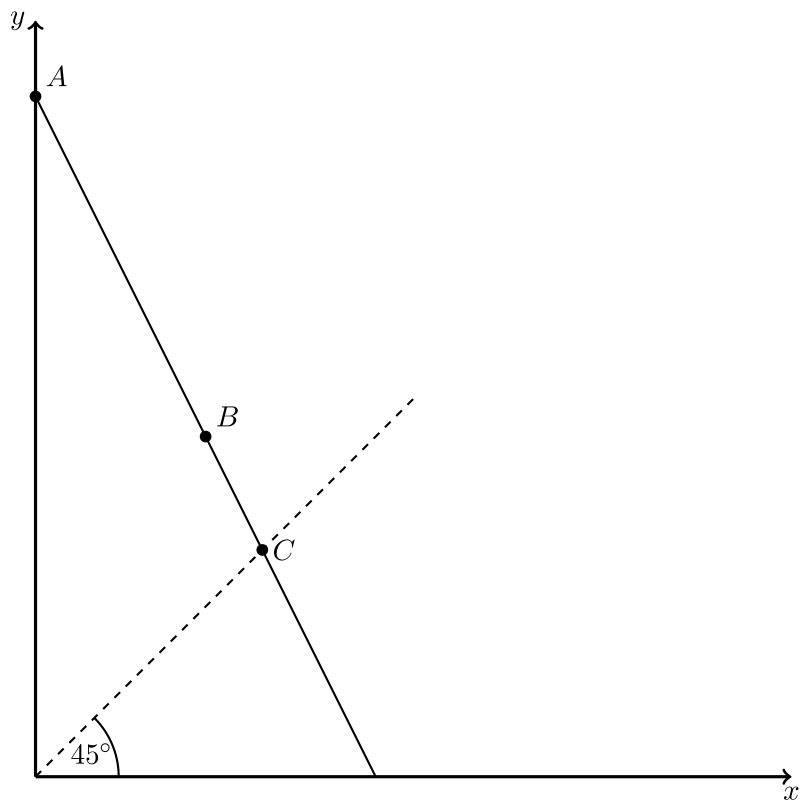



Note: The left panel shows the graphical user interface used in the lab. the right panel shows an example of the budget sets that a participant faced in each of the three domains.

Each experimental session lasted about one and a half hours. In each part of the experiment, the participants made 50 choices, where in each choice problem they were asked to allocate tokens between two accounts (Arrow securities), labeled $x$ and $y$. The two accounts were equally likely to be chosen. The left panel in Figure 3 illustrates how a choice problem was presented for the participants on the computer screen, while the right panel shows the variation in budget lines presented to the participants. The $x$ account corresponds to the $x$-axis and the $y$ account corresponds to the $y$-axis. The intersections of the budget line with the $x$-axis and the $y$-axis show the maximum number of tokens that could be allocated to the $x$ account and $y$ account, respectively. Each choice problem started by having the computer select a budget line randomly from the set of lines that intersect at least one axis at or above the 50 token level and intersect both axes at or below the 100 token level. The allocation task was to choose a point on the budget line, where the subjects used the mouse or the arrows on the keyboard to move the pointer on the computer screen to the desired allocation. For each participant, 50 budget lines were drawn at random, as illustrated in the right panel. Each individual made 50 choices in three different domains, as we denote the RISK, SOCIAL, and SOCIAL RISK domains.[13] The participants faced the same set of budget sets in all three domains, with a random within-domain sequence.

[13]The participants also made choices in a domain where they had no personal stake, but since the theory does not cover this setting, we do not include it in the present paper.

Figure 4: Example of allocation decisions




Note: Three examples of allocations, $A$, $B$, and $C$, that are discussed in the main text.

Figure 4 provides examples of allocation decisions for a given budget line. In the Risk domain, the point $C$, which lies on the 45 degree line, corresponds to the safe allocation with a certain payoff $y = x$. This allocation is consistent with infinite risk aversion. By contrast, point $A$ represents an allocation in which all income is allocated in the cheaper security $x = 0$ and $y = 1/p_y$, consistent with risk neutrality. Point $B$, which lies in the middle of the budget line, corresponds to the allocation with equal expenditures, $p_x x = p_y y$, consistent with a logarithmic von Neumann-Morgenstern utility function. In general, unless the slope of the budget line is equal to -1, expected monetary payoff, but also risk exposure, is maximized by putting all tokens in the cheaper account. Thus, in the Risk domain, the participants were placed in situations where they had to make a trade-off between expected monetary payoff and the riskiness of the allocation.

In the Social domain, the $y$ account defined money to self and the $x$ account defined money to the other person. Thus, the point $C$ would represent an equal division of the money, while point $A$ represents the selfish choice where all the money is allocated to the decision maker. In this domain, the participants had to make a trade-off between allocating money to self and to the other person, where the price of allocating money to the other person depends on the slope of the budget line. A selfish person would always allocate everything in the $y$ account, whereas an impartial person would allocate an equal amount of tokens to both accounts if they had the same price.

Finally, in the Social Risk domain, the two accounts represent two positions that the two individuals in the pair would occupy with equal probability. Thus, in this domain, a selfish individual would assign everything to the cheaper domain if he was entirely risk neutral, but always divide equally if he was infinitely risk averse. A concern for efficiency or equality would moderate the choices in the social risk domain, where a concern for efficiency would pull towards assigning more to the cheapest asset and a concern for equality would pull towards dividing equally.

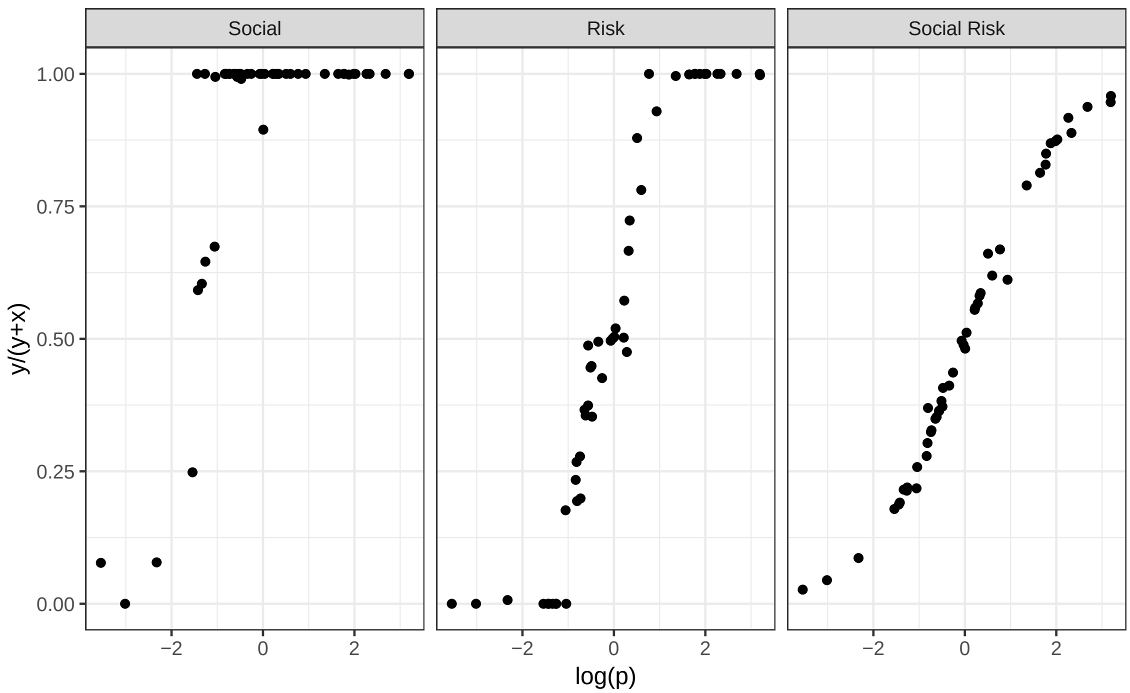
In Figure 5, we show an example of the individual level data generated by the experiment. In the Social domain, this individual allocates everything to him- or herself when the relative price is larger than one. However, he or she allocates a share to the other when the relative price makes it sufficiently cheap to give away money. In the Risk domain, the individual is close to risk neutral and largely allocates all the money to the cheapest account. Finally, in the Social Risk domain, we observe that the individual is more risk averse than in the Risk domain, which is suggestive of a concern for equality also influencing the choice in the Social Risk domain.

### 5.2 Sample

We conducted sessions at the University of Bergen and the business school NHH Norwegian School of Economics, in total we had 276 students participating.[14] The average age is 22.8 years (standard deviation: 3.5 years), with a majority of males (59%). At the end of the experiment, we asked them about their income and to report where they see themselves in the political landscape: 23% report to be liberal/left-wing, 26% report that they are moderate, and 51% percent report right leaning political preference.

[14] Additionally, we had one session where, due to a configuration mistake, the budget sets were not the same across domains. This session is not part of the data set we consider.

Figure 5: Data available from a single participant




Note: This figure shows the data available for a single individual from the experimental design. For each of 50 budget sets (characterized by a relative price, $p = p_x/p_y$), we see the share allocated to each of the assets.

## 6 Descriptive statistics on choices

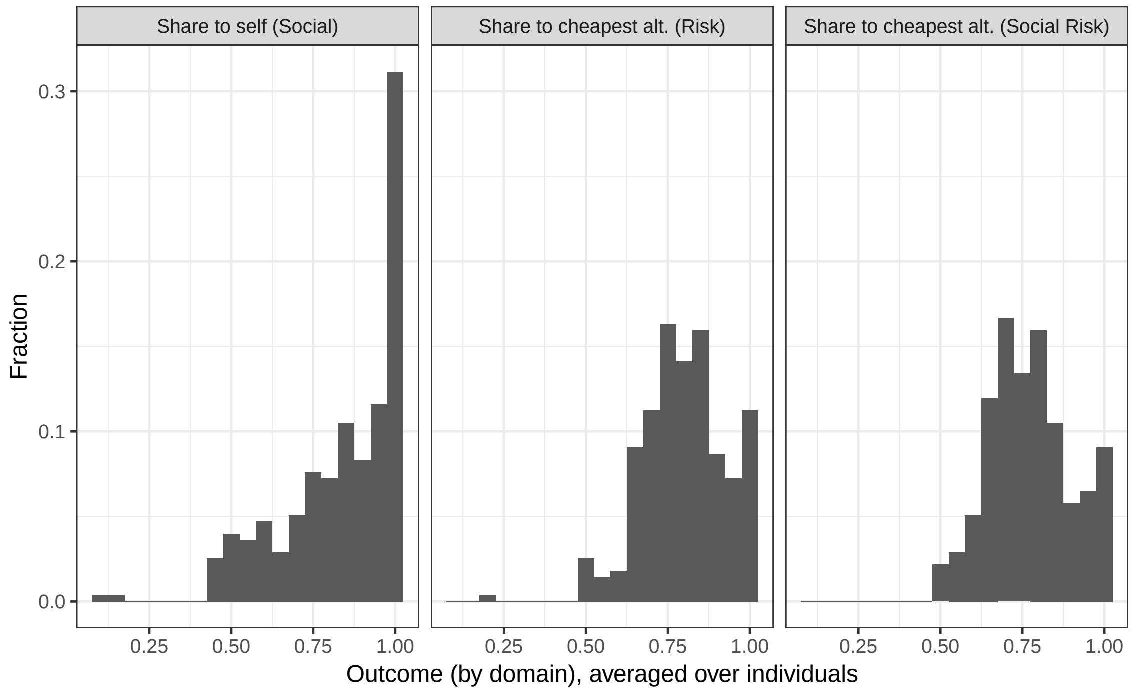
We here provide an overview of the choices in the different domains. Figure 6 shows the distribution of the average share of tokens allocated to account $y$. The left panel shows the amount kept by the decision-maker as a fraction of the tokens allocated to both accounts $x, y$ in the SOCIAL domain. We observe that about a third of the participants (37%) are selfish, in the sense of keeping at least 95% of the total amount of tokens in the allocation. Very few participants allocate less than 50% to themselves, which is in line with individuals largely assigning more weight to own income than to the income of others.

From the middle panel, we observe that about 10% of the individuals are risk neutral and always assign all the tokens to the cheapest account in the RISK domain, whereas very few individuals are infinitely risk averse and always assign the same amount to both tokens. The right panel reports from the SOCIAL RISK domain, where we observe a similar pattern. About 10% of the individuals allocate everything in the cheapest account, which is consistent with both risk neutrality and a strict preference for efficiency. At the same time, very few individuals allocate the same amount to both accounts, which would be consistent with both infinite risk aversion and a strict preference for equality.

In all three panels, we observe that there are very few individuals who allocate less to the cheapest account, which implies that there are very few violations of the first order stochastic dominance principle in our data. Let us now look more generally at how consistent the choice data is with individual utility maximization in each of the three domains, following the approach of Choi et al. (2007a). Since the budget sets are linear in our experiment, classical revealed preference theory provides a direct test: choices in a finite collection of budget sets are consistent with maximizing a well-behaved (that is, piecewise linear, continuous, increasing, and concave) utility function if and only if they satisfy the Generalized Axiom of Revealed Preference (GARP). Hence, in order to decide whether the choices are consistent with utility-maximizing behavior, we only need to check whether they satisfy GARP.

Although testing conformity with GARP is conceptually straightforward, there is an obvious difficulty: GARP provides an exact test of utility maximization – either the data satisfy GARP or they do not – but individual choices frequently involve at least some errors: subjects may compute incorrectly, or execute intended choices incorrectly, or err in other less obvious ways. To account for the possibility of errors, we assess how nearly individual choice behavior complies with GARP by using the *Critical Cost Efficiency Index*

Figure 6: Distribution of (individual mean) experimental outcomes by condition

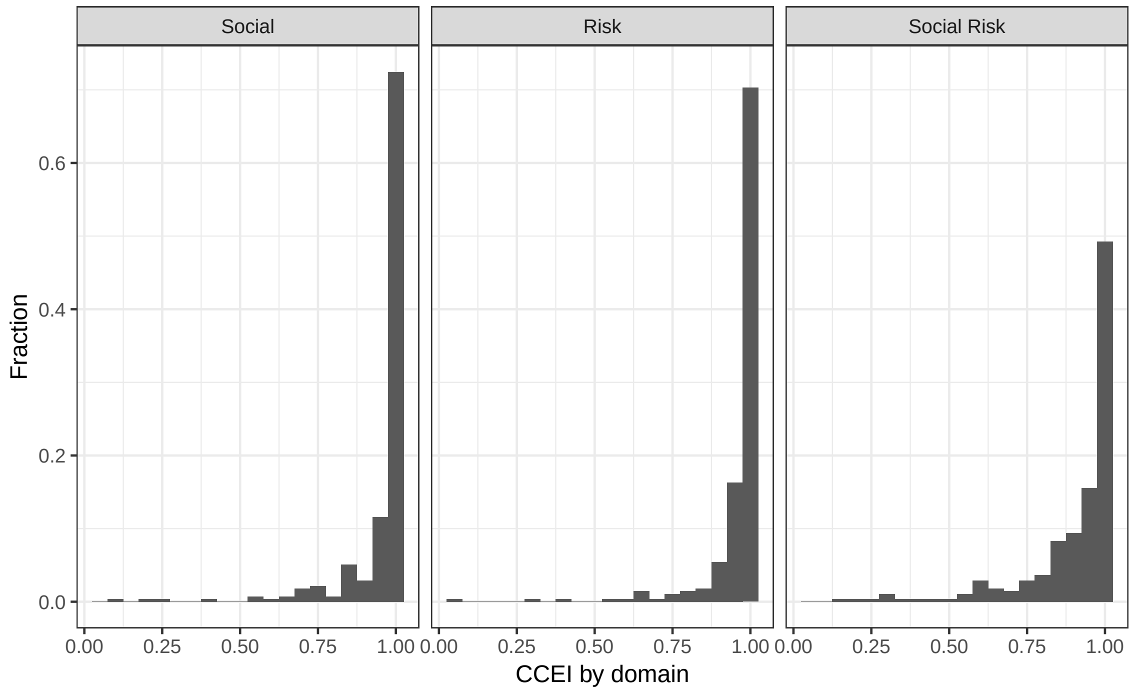



Note: The histogram shows the distribution the individual-level averages,

$$\overline{y_{i\cdot}/(x_{i\cdot}+y_{i\cdot})}.$$

The first panel shows the distribution of average share that participants allocate to themselves (averaged over the 50 budget sets). The second panel shows the distribution of average share allocated to the cheapest asset in the risk domain, and the third panel shows to distribution of the average share allocated to the cheapest asset in the social risk domain.

Figure 7: Histograms of the CCEI in the three domains




(CCEI), which measures the fraction by which each budget constraint must be shifted in order to remove all violations of GARP. By definition, the CCEI is between 0 and 1; indices closer to 1 mean the data are closer to perfect consistency with GARP and hence to perfect consistency with utility maximization.

In Figure 7, we report the distribution of the CEEI in each of the three domains. We observe that the individual choices *within each domain* are largely consistent with utility maximization. In all three domains, 70% or more of the participants have a $CCEI > 0.9$. Interestingly, we also note that the average CCEI is lower in the Social Risk domain, which may suggest that this domain represents a harder choice problem for the participants.

# 7 Linkages across the different domains

The theory part of the paper establishes that under given conditions, we should observe linkages across the different domains. In particular, the theory provides sharp predictions for individuals identified as selfish or impartial in the Social domain: preferences in the Social Risk domain should be the same as in the Risk domain for selfish individuals, and the same as in the Social domain for impartial individuals. In this section, we study empirically the linkages in the choice data. First, we study the aggregate choice patterns with a regression approach; second, we offer a novel non-parametric test of the theory at the individual level.

### 7.1 Regression analysis

The main focus of the regression analysis is the estimation of the following equation:

$$sr_i = \alpha + \beta^1 s_i + \beta^2 r_i + \beta^3 s_i r_i + \beta^4 \mathbf{X}_i + \epsilon_i, \tag{1}$$

where $sr_i$ is the average share allocated by individual $i$ to the cheapest account in the Social Risk domain, $s_i$ is an indicator variable taking the value one if individual $i$ allocated on average at least 95% to his or her own account in the Social domain, $r_i$ is the average share allocated to the cheapest account by of individual $i$ in the Risk domain, and $\mathbf{X}_i$ is a vector of background variables (age, gender, school, political affiliation).

Our main interest in the regression analysis is study the linkages between the choices in the Social Risk and the Risk domain differ for selfish and non-selfish individuals at the aggregate level, as captured by $\beta^3$. The theory predicts that selfish individuals make the same choices in the Social Risk and in the Risk domain, but is consistent with non-selfish individuals making different choices in these two domains. At the aggregate level, we would therefore expect that the linkage between the two domains is at least strong for selfish individuals as for non-selfish individuals, that is, $\beta^3 \geq 0$.

Table 1 reports regressions of choices in the Social Risk domain on choices in the Social domain and the Risk domain, where column (5) provides the estimates for equation (1). We observe that the aggregate data strongly suggest that there are linkages across the three domains. As seen from columns (1)-(3), the choice behavior both in the Social domain and in Risk domain is highly predictive of the choice behavior in the Social Risk domain. In columns (4)-(5), we establish that these linkages differ between selfish and non-selfish individuals. The estimated interaction term $\beta^3$ is positive, large and highly significant ($p < 0.01$), which shows that there is a much stronger linkage between the choices in the Social Risk and the Risk domains for the selfish individuals than for the non-selfish individuals. For the selfish individuals, we observe that on average, allocating one percent more to the cheapest account in the RISK domain is associated with allocating 0.8% more to the cheapest account in the SOCIAL RISK domain. Also for the non-selfish individuals do we observe a highly significant association between these two domains, but for these individuals it appears that equality concerns reduces their willingness to take risk in the SOCIAL RISK domain.

### 7.2 Non-parametric analysis: An individual level test

To provide a more direct test of the theory prediction, we propose a test, at the individual level, of the null hypothesis that the choices we have from two domains (on the same budget lines) could have been generated by the same preferences.

Consider first an individual who faces the same choice problem (budget set) a number of times in a given domain $d$. It seems natural to model the choices of such an individual as a random variable $X_b^d$, where the subscript $b$ indicates the budget set at which choices were made and the superscript $d$ the domain in which decisions were made. If choices are completely deterministic, the distribution of $X_b^d$ would be degenerate, but in practice it is likely that there would be some variation in realized choices. Such variation could come about for at least two distinct reasons. First, the preference ordering might give rise to a choice correspondence that does not uniquely select a singleton choice, but there is a range of alternatives that maximize utility. Since the decision maker is indifferent within this range, he or she might randomize over that range. Second, individuals might implement decisions with a trembling hand or some other source of randomness in implementation. We assume that these two sources compose and characterize actual choices as iid realizations of $X_b^d$, draws made with the distribution function $F_b^d$.

Consider now the experimental setting in which participants face decisions in two different domains – the budget sets are the same in terms of "tokens," but the meaning of these tokens depend on the domain. According to our theory, for some individuals, preferences in these two domains are equal. We extend the concept of two preference orderings being the same in domain $i$ and $j$, $\succeq_i = \succeq_j$, to the restriction that conditional on the budget set $b$, the distribution of choices are the same in the two domains, $F_b^i = F_b^j$.[15]

Assume that the budget sets belong in the space $\mathcal{B}$ and allocations in $\mathcal{X}$. Now introduce

[15] There might be other ways to model decision making that also arrive at such a characterization (e.g. random utility models), and our test would work equally well for those.)

Table 1: Linkages across domains: Regression analysis

| | Social risk – share to cheapest account | | | | |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) |
| Selfish (d) | 0.040*** | | 0.023** | −0.157** | −0.156** |
| | (0.015) | | (0.010) | (0.064) | (0.065) |
| Personal risk | | 0.745*** | 0.738*** | 0.654*** | 0.617*** |
| | | (0.039) | (0.038) | (0.048) | (0.050) |
| Selfish (d) × Personal risk | | | | 0.223*** | 0.210*** |
| | | | | (0.078) | (0.079) |
| Female (d) | | | | | −0.013 |
| | | | | | (0.010) |
| Business school (d) | | | | | 0.032*** |
| | | | | | (0.011) |
| Right leaning (d) | | | | | −0.007 |
| | | | | | (0.010) |
| Constant | 0.760*** | 0.178*** | 0.176*** | 0.242*** | 0.263*** |
| | (0.009) | (0.031) | (0.031) | (0.038) | (0.040) |
| Observations | 276 | 276 | 276 | 276 | 271 |
| $R^2$ | 0.024 | 0.577 | 0.585 | 0.597 | 0.606 |

*Note:* The dependent variable is the average share allocated to the cheapest account in the Social Risk domain. The explanatory variables are an indicator variable for the individual being selfish in the Social domain (mean share allocated to self is at least 95%), the share of tokens allocated to the cheapest account in the Risk domain, an interaction term, and a set of background variables. "Female" is an indicator variable taking the value one if the individual is a female, "Business school" is an indicator variable taking the value one if the individual is a student at the business school, and "Right leaning" is as an indicator variable for self-reporting being "slightly right wing," "right wing," or "very right wing." Five individuals chose not to report their demographics. Standard errors in parentheses ($^{*}: p < 0.10$, $^{**}: p < 0.05$, $^{***}: p < 0.01$).

a **revealed preference statistic** $R$. If $N$ is the number of observations, we can define $R$ as a function that maps a data set of budgets and choices to a real interval

$$R : \left(\times_1^N \mathcal{B}\right) \times \left(\times_1^N \mathcal{X}\right) \to [0,1],$$

such that $R(\boldsymbol{b}, \boldsymbol{x}) = 1$ if and only if the dataset $(\boldsymbol{b}, \boldsymbol{x})$, satisfy a revealed preference restriction ($\boldsymbol{b}$ is an $N$-vector of budget sets and $\boldsymbol{x}$ is an $N$-vector of realized choices). We will primarily have the Afriat Critical Cost Efficienciy Index (CCEI) as our candidate for such revealed preference statistic.

If we knew that $X_b^d$ reflects $\succeq_d$ and that any (potential) randomness in choices arises from mixing over the non-unique values of a deterministic choice correspondence, the realization of $R(\boldsymbol{b}^d, \boldsymbol{x}^d)$ would be degenerate at the value 1 (where the superscript index the domain). For two such data sets from different domains, $(\boldsymbol{b}^i, \boldsymbol{x}^i)$ and $(\boldsymbol{b}^j, \boldsymbol{x}^j)$, we would evaluate the revealed preference statistic on the concatenated data set, and if $R((\boldsymbol{b}^i, \boldsymbol{b}^j), (\boldsymbol{x}^i, \boldsymbol{x}^j)) < 1$, we conclude that $\succeq_i \neq \succeq_j$. Often statistical tests will be defined on differences in test statistics between two populations. This is not possible in our setting, since we explicitly want to allow individuals to have preferences that satisfy GARP within each domain (hence $R(\boldsymbol{b}^i, \boldsymbol{x}^i) = R(\boldsymbol{b}^j, \boldsymbol{x}^j) = 1$), but this does not establish that the two consistent preference orderings are also equal.[16]

In practice, it is not a productive avenue to rely on an assumption that the only source of randomness in choices arise from mixing over regions of indifference. First, we know from our data that many individuals have realized revealed preference statistics that are less than 1 even within domain. Second, even for data sets from individuals that have revealed preference statistics equal to 1 within a domain, we do not know for sure if this is because the test on the finite data set is too weak.[17] We can, however, rely on our characterization of choices as random variables to come up with an alternative statistical test for equality of preferences. In each domain, realizations of choices are drawn iid conditional on budget sets. This gives rise to a distribution of the revealed preference statistic that is a functional of the distribution functions for each of the choices (within domain), $F_R^d(F_{b_1}^d, F_{b_2}^d, \ldots, F_{b_n}^d)$.

[16] We could potentially have considered measures of distance between choices in two domains that are not revealed preference statistics, but the advantage of revealed preference statistics is that they respond only to those differences in choices that indicate violations of GARP, not to those that arise from mixing over non-unique choice correspondences.

[17] Since $R((\boldsymbol{b}^i, \boldsymbol{b}^j), (\boldsymbol{x}^i, \boldsymbol{x}^j)) \leq \min\{R(\boldsymbol{b}^i, \boldsymbol{x}^i), R(\boldsymbol{b}^j, \boldsymbol{x}^j)\}$ for some popular revealed preference statistics (such as the CCEI), relying on tests on concatenated data is also unwise as a basis for comparing the *magnitude* of violations in the concatenated data with those in each of the two domains. The concatenated data would provide more opportunities for violations, biasing the joint CCEI downwards from the domain-specific CCEIs.

If $F_b^i = F_b^j$ for all budget sets $b$, and choices in each domain are realized on the same vector of budget sets $\boldsymbol{b}$, $R(\boldsymbol{b}, \boldsymbol{x}^i)$ and $R(\boldsymbol{b}, \boldsymbol{x}^j)$ will be iid draws from the same same distribution. This sampling distribution refers to independent draws of complete vectors of choices, $\boldsymbol{x}$ conditional on $\boldsymbol{b}$. Since $X_b^i$ and $X_{b'}^i$ are independent, but not identically distributed when $b \neq b'$, characterizing the sampling distribution of $R$ will be hard. In the experiment, each of the $N$ budget sets are drawn at random, so $(b_k^d, X_{b_k^d}^d)$ and $(b_l^d, X_{b_l^d}^d)$ are iid when $k \neq l$, but since our experimental design fixes $b_l^i = b_l^j$ for all $l = 1, \ldots, N$, we cannot aim to estimate the sampling distribution of $R$ by the bootstrap or other resampling methods.

However, since $X_b^i$ and $X_b^j$ are iid under the null for all $b \in \mathcal{B}$, we can think of the allocation of "domain labels" to each of them as being random. Random reshuffling of "domain labels" would define a randomization distribution of the revealed preference statistic. Our two actual revealed preference statistics (one from each domain) should, under the null, be random draws from this randomization distribution. Formalizing this notion, for two $N$-long choice vectors $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$, define the set of combinations:

$$C(\boldsymbol{x}^1, \boldsymbol{x}^2) = \left\{\boldsymbol{x} : x_n \in \{x_n^1, x_n^2\}\right., \ n = 1, \ldots, N\}. \tag{2}$$

This set has cardinality $2^N$. Our two actual vectors of choices, $\boldsymbol{x}^i$ and $\boldsymbol{x}^j$, are elements of this set. Given the common vector of budget sets $\boldsymbol{b}$ (with elements $b_n$) and the null hypothesis that $F_{b_n}^i = F_{b_n}^j$ for all $n = 1, \ldots, N$, $R(\boldsymbol{b}, \tilde{\boldsymbol{x}})$ has a distribution determined by $\tilde{\boldsymbol{x}}$ being drawn from a uniform distribution on $C(\boldsymbol{x}^i, \boldsymbol{x}^j)$. This distribution, let us define its distribution function (evaluated at $r$) as $F_R(r; \boldsymbol{b}, \boldsymbol{x}^i, \boldsymbol{x}^j)$, can in principle be calculated exactly based on the available data $(\boldsymbol{b}, \boldsymbol{x}^i, \boldsymbol{x}^j)$. Our two actual draws are iid from this randomization distribution. Given two actual data sets, we can calculate this randomization distribution. We can then evaluate how likely realizations that are at least as extreme as our two empirical revealed preference statistics are, under the null hypothesis that the preferences are the same in the two domains. The natural alternative hypothesis is one-sided; if preferences are not the same in the two domains being tested, violations of GARP should occur more often in the randomizing distributions than in single-domain data sets.

We are in the position that for our test, under the null we have two realizations from the randomization distribution $F_R$. Let these be labelled $R^i$ and $R^j$. There are many ways to aggregate this information, the two corner cases are the minimum and the maximum of the realized values. Consider first the minimum, $R^- = \min\{R^i, R^j\}$. Under the null, the $p$-value that we want to calculate is $P(R^- \geq t) = 1 - P(R^- < t)$. The distinction between strict and weak inequality matters, since the distribution $F_R$ might have discrete mass points (and it will have this for the CCEI). We can express $P(R^- < t) = \lim_{s \uparrow t} F_{R^-}(s)$, with $F_{R^-}$ being the distribution function of the minimum of two draws from $F_R$. Standard probability calculus provides $F_{R^-}(t) = 1 - (1 - F_R(t))^2$, and we can calculate our $p$-value as

$$\begin{aligned} p^- &= P(R^- \geq t), \\ &= 1 - (1 - (1 - \widehat{F}_R(t - \epsilon))^2), \\ &= (1 - \widehat{F}_R(t - \epsilon))^2, \end{aligned}$$

with $\widehat{F}_R$ being an estimate of the randomization distribution function $F_R$, and $\epsilon$ taken as a small positive number.

Alternatively, we could aim to combine the two draws by taking the maximum of the two, $R^+ = \max\{R^i, R^j\}$. The corresponding $p$-value can be calculated as

$$\begin{aligned} p^+ &= P(R^+ \geq s), \\ &= 1 - P(R^+ < s), \\ &= 1 - (\widehat{F}_R(s - \epsilon))^2, \end{aligned}$$

also for a small $\epsilon$.

Not wanting to choose between these two, we could take the minimum of these two $p$-values, do a Bonferroni-correction, and use

$$\min\{2 \min\{p^-, p^+\}, 1\} \tag{3}$$

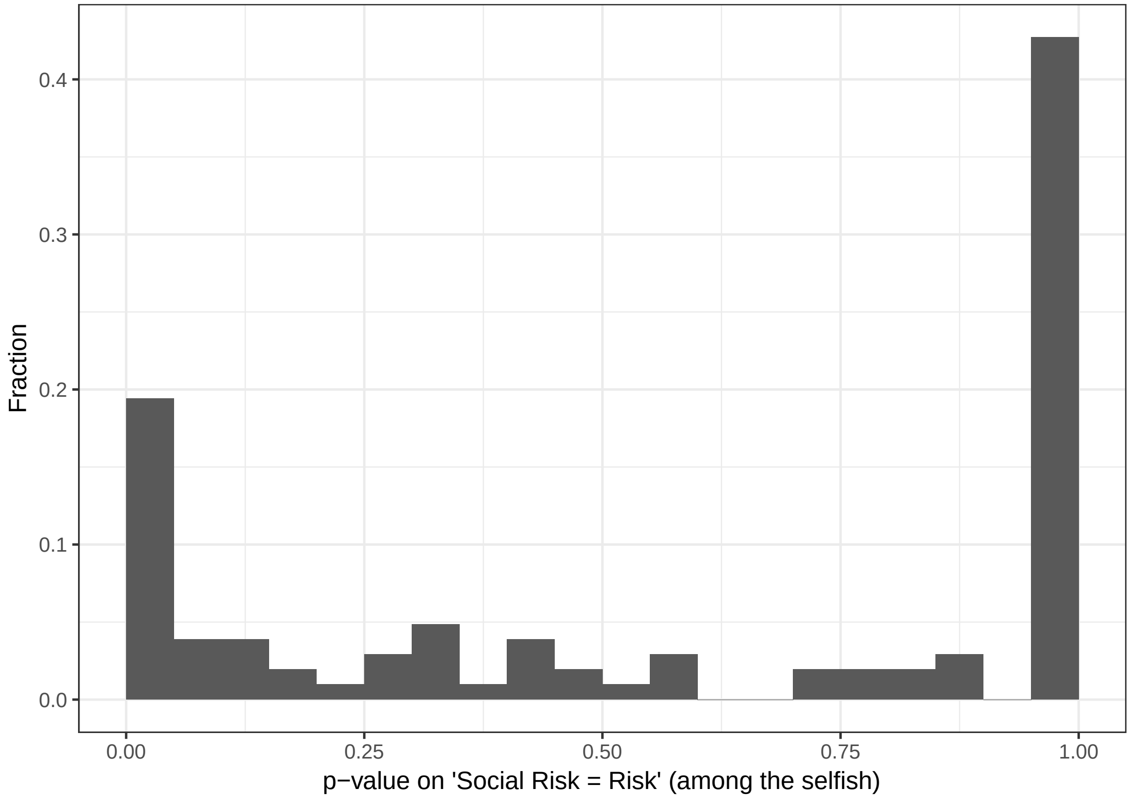
as our final $p$-value on our null hypothesis that preferences are the same in the two domains.

### 7.3 Non-parametric analysis: Empirical results

For testing the predictions from the theory section, we need an operational definition of "selfish" and "impartial" individuals. We define these groups based on the average share of tokens allocated to self in the Social domain, $\overline{y/(y+x)}$, where we allow for a band of 5 percentage points. We find 103 selfish individuals (37.3% of the total) with $\overline{y/(y+x)} \geq 0.95$, and 19 impartial individuals (6.9% of the total) with $0.45 \leq \overline{y/(y+x)} \leq 0.55$.[18]

[18] In Figure **??** and **??** in the online appendix, we show the correlation of choices on the different budget sets for the individuals classified as selfish and impartial, respectively.

Figure 8: Distribution of $p$-values for test of prediction for selfish individuals

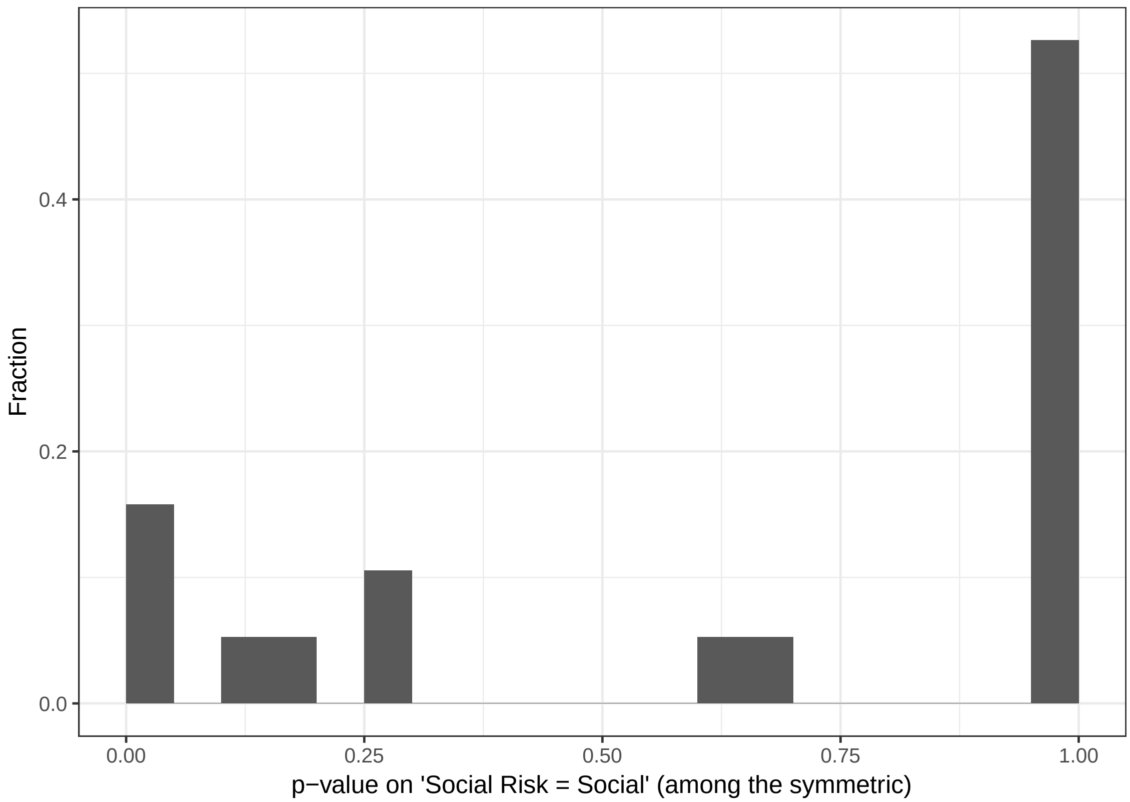



Note: Based on 1000 random permutations and a Bonferroni correction for the two test-statistics per each selfish individual ($N = 103$).

In Figure 8, we present results from the test of the hypothesis for the selfish individuals. The figure shows the histogram of $p$-values calculated according to equation ((3)). We observe that for the large majority of selfish participants (79.6%), we cannot reject that their choices in the Social Risk and Risk domains are generated by the same preferences; only for a small minority, we can reject the prediction for $p < 0.01$.

In Figure 9, we present the corresponding results for the impartial individuals. Again, we find little evidence against the theoretical prediction. For the large majority of impartial spectators (84.2%), we cannot reject that their choices in the Social Risk and Social domains are generated by the same preferences; only for a small minority, we can reject the prediction for $p < 0.01$.

Figure 9: Distribution of $p$-values for test of hypothesis X among the symmetric




Note: Based on 1000 random permutations and a Bonferroni correction for the two test-statistics per each symmatric individual ($N = 19$).

Our analysis suggests that the null hypothesis of the theory cannot be rejected for most selfish and impartial individuals. However, an important question is whether this is due to the power of our test being limited. One way to study this question would be with respect to parameterized demand systems. Within the literature on consumer demand, however, a different approach has been popular, based on how Becker (1962) pointed out that even completely random behavior that satisfies the budget constraint would in some respects look similar to utility maximizing behavior. Bronars (1987) took this idea and formalized a benchmark test for revealed preference statistics by asking what distribution of CCEI statistics we would expect given uniform random choices on the budget lines.

Our examination of the power of the nonparametric test builds on the idea of (Bronars, 1987). We ask the question: if a *fraction* of decisions are made uniformly random on the budget lines, how would our nonparametric test $p$-values react? In Figure 10 we examine the effect of increasing the fraction of uniform random decisions mixed into the $\tilde{\boldsymbol{x}}$ vector that is drawn from $C(\boldsymbol{x}^1, \boldsymbol{x}^2)$ (equation (2)) and replacing elements that are actual choices for the calculation of $F_R$. Since we are looking at decisions which under the null could also reasonably be expected to respect first order stochastic dominance, we only draw random decisions from the non-dominated part of the budget lines.

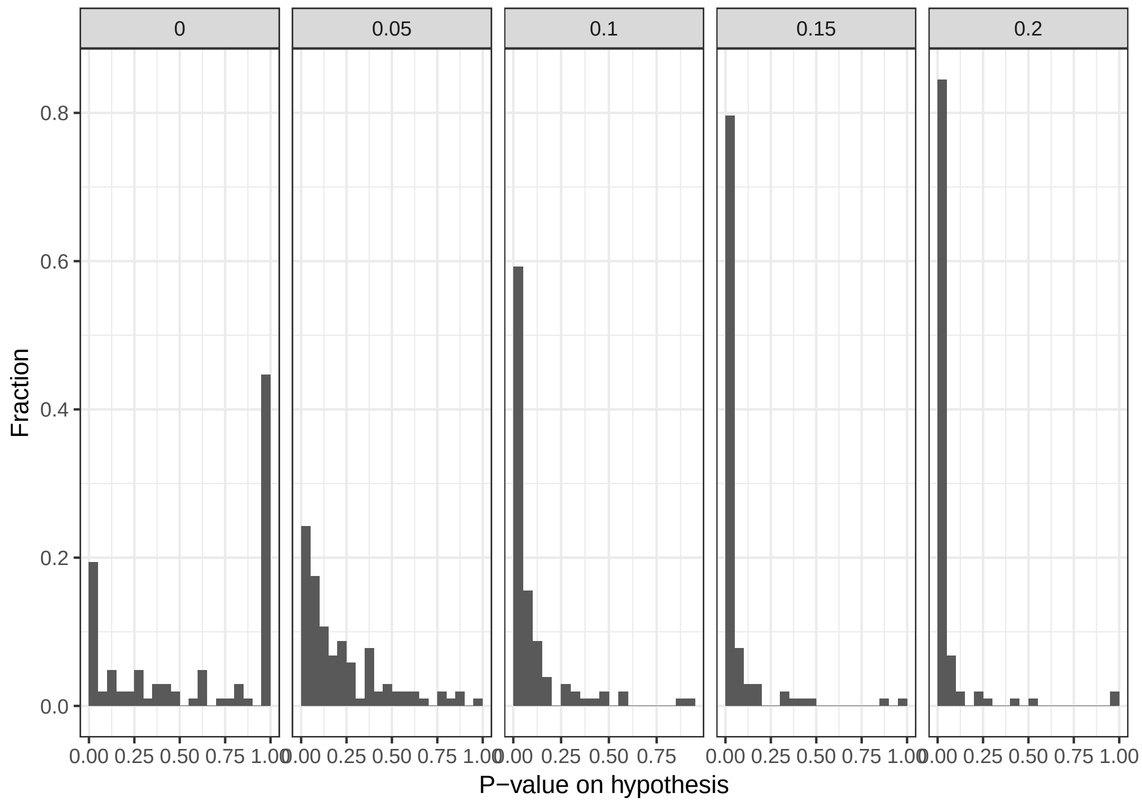
From the second column and onwards, we observe that even a small proportion (10% in the third column) of such random decisions lead to rejection of equality for a large fraction, and we thus conclude that our test has power.

Overall, we therefore find strong empirical evidence for the choice behavior in the experiment being in line with the theory predictions.

# 8 Observing More

We have found necessary and sufficient conditions that $\succeq$ can be deduced from $\succeq_0$. If we observe comparisons between social states and personal lotteries we observe more information, and it might be hoped that this would make it easier to deduce $\succeq$ from what is observed. However, a very small twist on the previous argument will show that this hope is mistaken: the additional information is not useful. On the Other hand, if we know – or are willing to assume – that the Decision Maker's true preferences $\succeq$ obey the Independence Axiom and so admit an expected utility representation – then, this extra information does indeed become useful.

Figure 10: Power of our non-parametric test




Note: Based on 1000 replication in which a certain fraction of actual decisions are replaced by decisions that are drawn uniformly random over the part of the budget lines that are not first order stochastically dominated. In the left-most panel, we report Figures 8 for reference, in the other panels we increase the proportion of random decisions (indicated by column headings).

To formalize this, define $\succeq_1$ to be the restriction of $\succeq$ to $[\Omega \cup L(P)] \times [\Omega \cup L(P)]$; i.e. the preference relation whose graph is

$$\text{graph}(\succeq_1) = \text{graph}(\succeq) \cap ([\Omega \cup L(P)] \times [\Omega \cup L(P)])$$

In contrast to observing $\succeq_0$, which entails observing comparisons between social states and comparisons between personal lotteries, observing $\succeq_1$ entails observing comparisons between social states, comparisons between personal lotteries, and comparisons between social states *and* personal lotteries.

**Theorem 5** *Assume that the Decision Maker's preferences satisfy Completeness, Transitivity, Continuity, Reduction of Compound Lotteries, the Sure Thing Principle and State Monotonicity. In order that $\succeq$ can be deduced from $\succeq_1$ it is necessary and sufficient that the Decision Maker finds every social state $\omega \in \Omega$ to be indifferent to some personal state.*

**Proof.** In view of Theorem 1, this condition is sufficient that $\succeq$ can be deduced from $\succeq_0$, so it is certainly sufficient that $\succeq$ can be deduced from $\succeq_0$, which provides more information.

To see that it is necessary we use the same argument as in the proof of Theorem 1, but with a small twist. suppose that there is a social state $X$ that is not indifferent with any personal state. . First choose a continuous utility representation $u$of $\succeq$ whose range lies in $[0, 1]$. Define $\mathcal{A}, \mathcal{B}, \mathcal{X}$ to be the sets of social states that are strictly preferred to $X$, strictly dis-preferred to $X$ and indifferent to $X$, respectively. If $\mathcal{A} \neq \emptyset$, let $A \in \mathcal{A}$ be a minimal element; if $\mathcal{B} \neq \emptyset$, let $B \in \mathcal{B}$ be a maximal element. For each lottery $\Gamma \in L(\Omega)$, define $\Gamma_{\mathcal{A}}, \Gamma_{\mathcal{B}}, \Gamma_{\mathcal{X}}, x(\Gamma)$ exactly as before. We distinguish the same three cases as in Theorem 1; the arguments for Case (ii) and Case (iii) are identical to those in the proof of Theorem 1, but the argument for Case (i) requires a little small twist because we must be careful to construct the utility function $U$ to preserve the relationship between the social state $X$ and personal *lotteries*, as well as personal states. (No additional care is required in Case (ii) because $X$ is preferred to all personal lotteries, or in Case (iii) because $X$ is dis-preferred to all personal lotteries.)

Suppose therefore that neither $\mathcal{A}$ nor $\mathcal{B}$ is empty. Use Continuity to choose $\gamma, \zeta \in (0, 1)$ such that $X \sim \gamma A + (1 - \gamma)B$ and $(1/2)A + (1/2)X \sim \zeta A + (1 - \zeta)B$. As before, define auxiliary functions $f, g : L(\Omega) \to \mathbb{R}$ by

$$\begin{aligned} f(\Gamma) &= u(\Gamma_{\mathcal{A}} + x(\Gamma)A + \Gamma_{\mathcal{B}}) \\ g(\Gamma) &= u(\Gamma_{\mathcal{A}} + x(\Gamma)B + \Gamma_{\mathcal{B}}) \end{aligned}$$

We are no longer free to choose $U$ to be an arbitrary convex combination $U = \lambda f + (1-\lambda)g$ because we require $U(X) = U(\gamma A + (1-\gamma)B) = u(\gamma A + (1-\gamma)B)$, which would completely determine $\lambda$ – and for this $\lambda$ it might happen that the preference relation $\succeq_U$ coincides with $\succeq$. However, there is an easy work-around. Let $H : [0,1] \times [0,1] \to [0,1]$ be a function that is strictly increasing in each variable separately and is the identity on the diagonal; set

$$U_H(\Gamma) = H(f(\Gamma), g(\Gamma))$$

Because $H$ is the identify on the diagonal it follows that $U_H$ agrees with $u$ on $L(\mathcal{A} \cup \mathcal{B})$; because $H$ is strictly increasing in each variable it follows that $U_H$ obeys State Monotonicity. Because $f(X) = u(A) > u(X) = u(\gamma A + (1-\gamma)B > u(B) = g(X)$ we can choose $H$ so that $U_H(X) = H(f(X), g(X)) = u(X)$; because $f(X) = u(A) > u((1/2)A + (1/2)X) > u((1/2)A + (1/2)B) = g(X) > u(B)$, we can also choose $H$ so that $U_H((1/2)A + (1/2)X) = H(u(A), u((1/2)A + (1/2)B)) \neq U(\zeta A + (1-\zeta)B) = u((1/2)A + (1/2)X)$. For such a choice of $H$ it is easily checked that the preference relation $\succeq_{U_H}$ is an extension of $\succeq_1$ and differs from $\succeq$, as desired. ■

However if we assume that the Decision Maker's preferences have an expected utility representation we can do more.

**Theorem 6** *Assume that the Decision Maker's preferences satisfy Completeness, Transitivity, the Archimedean Axiom, Reduction of Compound Lotteries, the Sure Thing Principle and Independence (and hence admit an expected utility representation). In order that $\succeq$ can be deduced from $\succeq_1$ it is necessary and sufficient that the Decision Maker finds every social state to be indifferent to some personal state.*

**Proof.** To see that this condition is sufficient, consider lotteries $\sum p_i \omega_i$ and $\sum q_j \omega_j$. (a) guarantees we can find personal lotteries $\alpha_i$ and $\beta_j$ such that $\omega_i \sim \alpha_i$ and $\omega_j \sim \beta_j$ for each $i, j$. Independence guarantees that $\sum p_i \omega_i \sim \sum p_i \alpha_i$ and $\sum q_j \omega_j \sim \sum q_j \beta_j$ so

$$\sum p_i \omega_i \succeq \sum q_j \omega_j \iff \sum p_i \alpha_i \succeq \sum q_j \beta_j \iff \sum p_i \alpha_i \succeq_1 \sum q_j \beta_j$$

Thus $\succeq$ can be deduced from $\succeq_1$.

To see, that this condition is necessary, assume there is a social state $X \in \Omega$ that is not indifferent to any personal lottery. If there were personal states $A, A' \in P$ such that $A \succeq X \succeq A'$ then Continuity would imply that $X \sim \gamma A + (1-\gamma)A'$ for some $\gamma \in (0,1)$, which would contradict the assumption that $X$ is not indifferent to any personal lottery.

It follows that either $X \succ A$ for all personal states $A \in P$ or $X \prec A$ for all personal states $A \in P$.

Choose a continuous utility function $u : \Omega \to \mathbb{R}$ that yields an Expected Utility representation of $\succeq$ and write $Eu$ for the expected utility extension of $u$ to $L(\Omega)$. Note that the range $Eu(L(P))$ of $Eu$ on $L(P)$ is precisely the convex hull of the range $u(P)$ of $u$ on $P$. Let $f : \mathbb{R} \to \mathbb{R}$ be any strictly increasing function that is the identity on $Eu(L(P))$. Then $f \circ u$ defines a utility function $u_f$ on; write $Eu_f$ for its expected utility extension to $L(\Omega)$. Because $f$ is the identify on $Eu(L(P))$, $Eu_f$ agrees with $Eu$ on $L(P)$; because $f$ is strictly increasing, the ordering induced by $u_f$ on $\Omega$ agrees with the ordering induced by $u$. Hence the preference ordering $\succeq_f$represented by $Eu_f$ agrees with $\succeq$ on $[\Omega \cup L(P)] \times [\Omega \cup L(P)]$; that is, $\succeq_f$ is an extension of $\succeq_1$. Because $X$ does not have a personal lottery equivalent , $u(X) \notin Eu(L(P))$; because $Eu(L(P))$ is convex – hence an interval – this means either $u(X) < u(A)$ for all personal states $A \in P$ or $u(X) > u(A)$ for all personal states $A \in P$. In what follows we treat the case in which $u(X) > u(A)$ for all $A \in P$ (i.e., $X$ is preferred to every personal state); the argument in the reverse case is similar and left to the reader.

By assumption there are personal states $A, B \in P$ with $A \succ B$, so that $u(A) > u(B)$. Choose $f$ so that $f(u(X)) \neq u(X)$, and $\lambda = [u(A) - u(B)]/[u(X) - u(B)]$; note that $\lambda \in (0, 1)$. The expected utility property guarantees that

$$Eu(\lambda X + (1 - \lambda)B) = \lambda u(X) + (1 - \lambda)u(B) = u(A)$$

and also that

$$Eu_f(\lambda X + (1 - \lambda)B) = \lambda u_f(X) + (1 - \lambda)u_f(B) = \lambda u_f(X) + (1 - \lambda)u(B)$$

Since $f(u(X)) \neq u(X)$ we conclude that $Eu_f(\lambda X + (1 - \lambda)B) \neq u(A) = u_f(A)$. Thus, $A \sim \lambda X + (1 - \lambda)B$ but $A \not\sim_f \lambda X + (1 - \lambda)B$; in particular, $\succeq_f \neq \succeq$. This completes the proof. ■

# 9 Concluding remarks

Our paper has shown how economic analysis can inform our understanding of the nature of social risk preferences. We have established under which condition social risk preferences can be deduced from social preferences and personal risk preferences. We have also shown that the theory has descriptive support in a experimental study where individuals make choices in each of the three domains. Our results suggest a number of interesting applications in economics. In particular, it shows how we can shed light on people's policy preferences for redistribution and taxation by observing their personal risk choices and social choices. But these insights apply much more generally. An example that initially motivated this paper was that campaigns for political office, and especially for the U.S. Presidency, often place a great deal of emphasis on the personal character of the candidate. But why? The present paper suggests one possibility: in line with our theory, people use observations of the candidate's personal risk choices and social choices to deduce the social risk preferences of the candidate, which likely play an important role in many Presidential decisions.

We believe that the paper opens up several research avenues. There are a number of interesting questions related to the theoretical framework. For example, the paper establishes linkages across the domains under the assumption that we have complete information about both the risk preferences and the social preferences. But in many cases, we only know some part of the preference structure in these domains, since we only observe a finite set of choices, and it is important to better understand theoretically how much we can infer from these partial preference structures. Another interesting theoretical question is how to develop the theory to account for inconsistencies in individual choices, as observed in the experiment.

The paper also calls for more empirical analysis. It is of great interest to study choice behavior in the different domains in a broader set of situations than in the present experiment. It would also be interesting to extend the empirical analysis to nationally representative samples of individuals, to better understand how the social risk preferences relate to more general policy preferences and to get a better understanding of the relative importance of personal risk preferences and social preferences in people's social risk preferences.

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