

Postponing Retirement, Endogenous Fertility, Social Security and Welfare

Gen Zhixiang

Tanggang Yuan

Abstract Based on an extended overlapping generations model with endogenous fertility, this paper studies the effects of raising the mandatory retirement age on the fertility rate, social security benefit, and welfare in a neoclassical growth model. The study shows that a small amount of the postponement of the retirement age can improve the fertility rate to a minor extent in the long run. When the output elasticity of capital is no less than 0.5, the postponement of the retirement age will have a negative effect on social security benefit and also a negative effect on welfare in the long run. When, however, the output elasticity of capital is small enough, both of the social security benefit and welfare increase in delaying retirement age in the long run.

Keywords Retirement •Fertility• PAYG social security •Welfare

1 Introduction

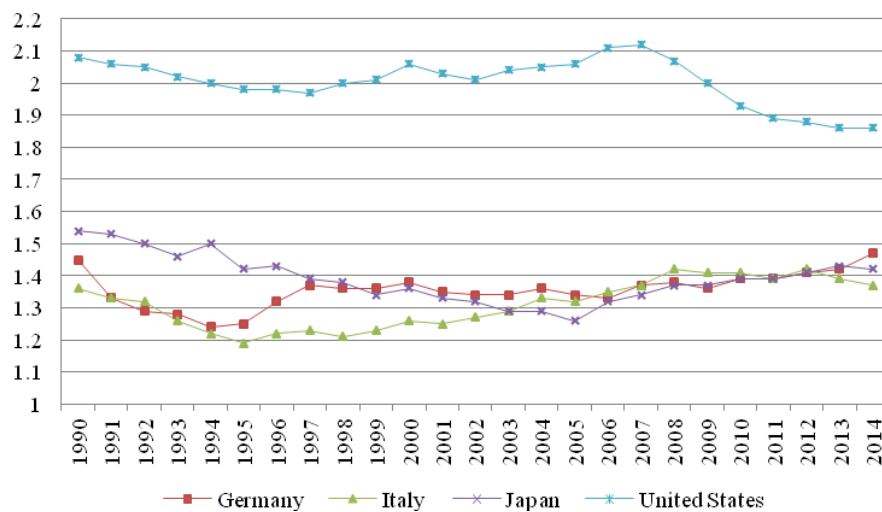
Many developing and developed countries are experiencing rapid population aging and a decline in labor supply. Due to population aging, the sustainability of existing social security systems is increasingly challenged, especially with the pay-as-you-go (PAYG) pension system. In order to reduce the burden of pension payouts, the usual practice is to cutting per-capita pension benefits, increasing contribution rates or postponing retirement. Many experts propose to postpone the mandatory retirement

age, and it is supported that postponing retirement may become feasible because of the political push of aging (Galasso and Profeta, 2004; Galasso, 2008). Currently, many countries are prepared to raise or have raised the mandatory retirement age or the official pension age. For example, in 1983, the United States Congress implemented an increase in the Normal Retirement Age (NRA) of 2 months per year, an increase that started in 2000 (Mastrobuoni, 2009). Japan will postpone the retirement age from 60 to 65 in 2006, Germany passed the law in 2006 that the retirement age would be extended to 67 years from 2012 to 2029, Italy took flexible policy of extending the retirement age both in 1995 and 2004, and China will raise the mandatory retirement age in progressive steps in the next few years.

Through the policy of postponing the retirement age in progressive steps, some countries expect to alleviate the strain of the government budget without decreasing the social security benefit to increasing the contribution rate. However, some studies show that the postponement of the retirement age may be harmful for social security benefit in the long run (Miyazaki 2014; Fanti 2014). But in these studies, they tend to take the fertility rate as exogenously given. In reality, the fertility rate is affected by many factors, such as wage income, time and money cost of child-rearing etc., as a consequence, it changes over time. Therefore, it is likely to produce inaccurate conclusions under the assumption of exogenous fertility. Most of the studies have been conducted with endogenous fertility (Becker and Barro 1988; Barro and Becker 1989; Zhang and Zhang 1998; Wigger 1999; Omori 2009; Miyazaki 2013; Wang 2015; Sommer, 2016).¹In the model with endogenous fertility, much attention has been paid to the relationship among social security, fertility, income and growth (Wang et al., 1994; Zhang, 1995; Blackburn and Cipriani, 1998; Kremer and Chen, 1999; Groezen et al., 2003; Rosati, 2004; Fanti and Gori, 2012; Varvarigos and Zakaria, 2013). To the best of our knowledge, there are no relevant literature to investigate the effects of raising the mandatory retirement age on the fertility, PAYGO social security benefit, and welfare together in the long run. In this paper, we will study the effects of endogenous fertility in a neoclassical growth model.

¹ Malthus was the first economist who studied demography and took fertility as an endogenous variable.

As an alternative to the traditional way of relying on children, the postponement of the retirement age is considered to decrease the fertility rate because it reduces the parents' material dependence on children. However, it is found that, in some OECD countries, the fertility rate is not decreasing over time after extending the retirement age. For instance, Fig.1 shows the total fertility rates of several countries from OECD. From the Fig, we can see a little rise in total fertility rates after postponing the retirement age.² Therefore, the decreasing fertility due to delaying retirement needs further study.



Source: OECD Data (fertility rates)

Fig.1. Total fertility rates.

The aim of this paper is thus to investigate how the postponement of the mandatory retirement age affects the fertility rate, PAYG social security benefit, and welfare in the long run. We use the extended overlapping generations (OLG) model by Diamond (1965) in a neoclassical growth framework, and find that a small amount of the postponement of the mandatory retirement age decreases precautionary saving, and thus increases disposable income, which encourages the young to have more children. Therefore, it can improve the fertility rate in the long run, but the degree of the improvement is very limited. It is shown that the fertility rate is mainly affected by the cost of child-rearing.

² The fertility rate of the United States to decrease for other reasons in 2008. For example, the financial crisis had greatly reduced the wealth of people and had risen the unemployment rate, which decreased the fertility desire among people (Lovenheim and Mumford, 2013).

As to social security benefit, in the steady state, postponing retirement has two effects on the agent's social security benefit. One is that it can improve the fertility. Hence, there are more young workers to support old pensioners, so it increases the agent's social security benefit. Moreover, a small amount of the postponement of the retirement age increases the contribution rates, which also increases the social security benefit. The other is that the rising fertility rate and delaying retirement decrease the capital of per worker, thus decreases the real wage. As a consequence, it decreases the agent's social security benefit. When the output elasticity of capital is no less than 0.5, the latter dominates the former, so the agent's social security benefit decreases in a small amount of the postponement of the retirement age in the long run. When the output elasticity of capital is less than a given value, which is the function of discount factor and preference weight for the number of children relative to the young agent's consumption, the former dominates the latter, so the agent's social security benefit increases in the long run. When the output elasticity of capital is more than the given value and less than 0.5, at a low payroll tax rate, the agent's social security benefit decreases in the long run, while at a high payroll tax rate, the agent's social security benefit increases in the long run.

An agent's welfare depends on a young agent's consumption, an old agent's consumption after retirement, and the number of children in the steady state. We find that a small amount of the postponement of the retirement age decreases the young agent's consumption. When the output elasticity of capital is less than or equal to 0.5, the old agent's consumption increases in the long run. The agent's welfare increases in a small amount of the postponement of the retirement age when the output elasticity of capital is small, otherwise it decreases in the long run.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 analyzes the effects of the postponement of the mandatory retirement age on the Fertility, PAYG social security benefit, and welfare in the long run. Section 4 concludes.

2 The model

2.1 Agents

Time is discrete and continues forever, $t=1,2,\dots$. Consider a two-period general equilibrium OLG model in a closed economy, the adulthood of an agent is separated into two periods: youth and old age. Each period is one unit of time. Young agents in period t in elastically supply one unit of labor and earn a wage of w_t while paying a social security tax, which is denoted by $\tau \in (0,1)$. In the second period, $t+1$, the postponement of the mandatory retirement age is x , which is a policy variable determined by the government, then s/he has to work a fraction $x \in [0,1)$ of the time and pays the tax $x\tau w_{t+1}$. For the rest of her or his time, $1-x$, she or he is retired. When $x=0$ implies that the agent works full time in the first period of life and retires in the whole second period of life, which is the traditional assumption of the OLG model of Diamond (1965). The relationship between populations of adjacent generations is linked by the endogenous fertility as $n_t = N_{t+1} / N_t$. If a young agent at date t has n_t children, we assume that the physical cost of rearing them is $\delta n_t w_t$, where $\delta \in (0,1)$.³ Thus, a young agent's budget constraint is

$$c_t^y + s_t = (1 - \tau - \delta n_t) w_t. \quad (1)$$

Where c_t^y and s_t are the young agent's consumption and savings, respectively. When she or he becomes old at $t+1$, she or he receives the interest income, $R_{t+1} s_t = (1 + r_{t+1}) s_t$, where r_{t+1} is the net interest rate. In addition, she or he receives a net wage income of $(1 - \tau) x w_{t+1}$, and a social security benefit of $(1 - x) P_{t+1}$. Therefore, in the second period, $t+1$, an old agent's budget constraint is

$$c_{t+1}^o = R_{t+1} s_t + (1 - \tau) x w_{t+1} + (1 - x) P_{t+1}. \quad (2)$$

³We do not consider the time spent rearing one child, because we can look for a person to look after our children in the labor market and pay the wage, the time cost can be converted into money cost to an extent. This setting will not affect our results.

Where P_{t+1} is the per unit of time pension. An agent cares about not only over consumption but also in relation to the number of children she or he has. Thus an agent born to date t has the following lifetime utility:

$$U_t = \ln c_t^y + \beta \ln c_{t+1}^o + \gamma \ln n_t. \quad (3)$$

Where $\beta \in (0,1)$ is a subjective discount factor and $\gamma > 0$ is a preference weight to have children relative to c_t^y . Under the constraints (1) and (2), one can maximize the lifetime utility and achieve the following functions:

$$\begin{aligned} s_t &= \frac{\beta(1-\tau)}{1+\beta+\gamma} w_t - \frac{(1+\gamma)[(1-\tau)xw_{t+1} + (1-x)P_{t+1}]}{(1+\beta+\gamma)R_{t+1}} \\ n_t &= \frac{\gamma(1-\tau)}{\delta(1+\beta+\gamma)} + \frac{\gamma[(1-\tau)xw_{t+1} + (1-x)P_{t+1}]}{(1+\beta+\gamma)\delta w_t R_{t+1}}. \end{aligned} \quad (4)$$

2.2 Firms

As regards production sector, we assume that firms are identical and act competitively. At each date t , a firm uses aggregate capital K_t and total labor supply L_t to produce consumption goods, where $L_t = N_t + xN_{t-1}$. The production technology is Cobb-Douglas function, and the total factor productivity (TFP) is normalized to 1. Therefore, the production function is $Y_t = F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$, where $\alpha \in (0,1)$ is the capital share of total output. We suppose that the capital stock per worker and output per worker are denoted by $k_t = K_t / L_t$ and $y_t = Y_t / L_t$, respectively, so the intensive form of production function may be written as $y_t = k_t^\alpha$. For simplicity, it is assumed that physical capital totally depreciates at the end of each period. ⁴ Consequently, under the competitive market, we can get the following marginal conditions for capital and labor in equilibrium:

$$R_t = 1 + r_t = \alpha k_t^{\alpha-1}, w_t = (1-\alpha)k_t^\alpha. \quad (5)$$

⁴ Since one period in this model corresponds to about 30 years, it is reasonable to assume that physical capital depreciates fully within one period.

2.3 Government

We assume that the Following PAYG social security budget is balanced by the government in every period:

$$\tau N_{t+1} w_{t+1} + \tau x N_t w_{t+1} = (1-x) N_t P_{t+1} \quad (6)$$

Where the left-hand side of the above equation represents the social security tax receipts and the right-hand side represents the pension expenditure. Furthermore, we can write the equality as follows:

$$\tau w_{t+1} (n_t + x) = (1-x) P_{t+1}. \quad (7)$$

2.4 Equilibrium

The physical capital accumulation of a firm at every period t comes from the previous agents' saving, so the market-clearing condition in goods as well as in capital markets is expressed by the identity: $K_{t+1} = N_t s_t$. We can write it into the following form in terms of capital stock per worker:

$$k_{t+1} = \frac{s_t}{n_t + x}. \quad (8)$$

Exploiting (4), (5), (7) and (8), we can obtain the following dynamic evolution equation:

$$k_{t+1} = \frac{\alpha \beta (1-\alpha)(1-\tau)}{\alpha(1+\beta+\gamma)(x+n_t) + (1-\alpha)(1+\gamma)(x+\tau n_t)} k_t^\alpha \quad (9)$$

and

$$n_t = \frac{\gamma \alpha (1-\tau) k_t^\alpha + \gamma x k_{t+1}}{(1+\beta+\gamma) \delta \alpha k_t^\alpha - \gamma \tau k_{t+1}}. \quad (10)$$

Plugging equation (9) into equation (10) with arrangement, we have

$$n_t = \frac{\gamma(1-\tau)[\alpha(1+\beta+\gamma)(x+n_t) + (1-\alpha)(1+\gamma)(x+\tau n_t)] + \gamma x \beta (1-\alpha)(1-\tau)}{\delta(1+\beta+\gamma)[\alpha(1+\beta+\gamma)(x+n_t) + (1-\alpha)(1+\gamma)(x+\tau n_t)] - \gamma \tau \beta (1-\alpha)(1-\tau)}. \quad (11)$$

The equation (11) is quadratic in n_t and it has at most two real roots. According to the lemma 9 in Abel (2003), the positive and higher root is stable. Then the fertility rate converges to the following steady state n^* after one period.

$$n^* = \frac{\gamma(1-\tau)(\tau - \tau\alpha + \alpha) - \delta x(1 + \alpha\beta + \gamma) + \sqrt{[\gamma(1-\tau)(\tau - \tau\alpha + \alpha) - \delta x(1 + \alpha\beta + \gamma)]^2 + 4\delta[\alpha(1 + \beta + \gamma) + (1 - \alpha)(1 + \gamma)\tau]\gamma(1-\tau)x}}{2\delta[\alpha(1 + \beta + \gamma) + (1 - \alpha)(1 + \gamma)\tau]}.$$

(12)

Equation (9) guarantees a unique steady state, and the capital stock per worker converges to the

following steady state k^* , we have

$$k^* = \left(\frac{\alpha\beta(1-\alpha)(1-\tau)}{\alpha(1+\beta+\gamma)(x+n^*) + (1-\alpha)(1+\gamma)(x+\tau n^*)} \right)^{\frac{1}{1-\alpha}}.$$

(13)

Therefore, the social security benefit of the agent in equilibrium is

$$p^* = (1-x)P^* = \tau(1-\alpha)(n^* + x)k^{*\alpha}.$$

(14)

Where P^* represents the social security benefit in equilibrium.

3 Fertility, PAYG social security benefit, and welfare in the long run

3.1 Fertility

Proposition 3.1 The capital stock per worker decreases as the retirement age increases, and private savings decreases as well in the long run. A small amount of the postponement of the mandatory retirement age x can improve the fertility rate in the long run.

Proof We know that all variables are continuous in the parameter values α, β, γ and τ . Taking the derivative of n^* with respect to x and rearranging it at $x=0$, we have

$$\begin{aligned}\frac{\partial n^*}{\partial x}\Big|_{x=0} &= \frac{-\delta(1+\alpha\beta+\gamma)(\tau-\tau\alpha+\alpha)+\delta[\alpha(1+\beta+\gamma)+(1-\alpha)(1+\gamma)\tau]}{\delta[\alpha(1+\beta+\gamma)+(1-\alpha)(1+\gamma)\tau](\tau-\tau\alpha+\alpha)} \\ &= \frac{\alpha\beta(1-\alpha)(1-\tau)}{[\alpha(1+\beta+\gamma)+(1-\alpha)(1+\gamma)\tau](\tau-\tau\alpha+\alpha)} > 0.\end{aligned}$$

Therefore, this implies that a small amount of the postponement of the mandatory retirement age can increase the fertility rate in the long-run.

Obviously, according to the equation (13), we have

$$\frac{\partial k^*}{\partial x} < 0.$$

From the equation (8), in the steady state, we have

$$s^* = (n^* + x)k^*.$$

Taking the derivative of s^* with respect to x and rearranging it by using the first order conditions at $x=0$, we have

$$\frac{\partial s^*}{\partial x}\Big|_{x=0} = \left(\frac{\alpha\beta(1-\alpha)(1-\tau)}{\alpha(1+\beta+\gamma)+(1-\alpha)(1+\gamma)\tau} \right)^{\frac{1}{1-\alpha}} n^*\Big|_{x=0}^{\frac{1}{1-\alpha}} \frac{(1-\alpha)^3(1+\gamma)\tau^2 - (1-\alpha)(1-2\alpha+\alpha^2)(1+\gamma)\tau - \alpha[(1+\gamma)(1-\alpha+\alpha^2) + \alpha\beta]}{(1-\alpha)[\alpha(1-\tau) + \tau][\alpha(1+\beta+\gamma) + (1-\alpha)(1+\gamma)\tau]}.$$

Therefore, according to the above equation, the sign of $\frac{\partial s^*}{\partial x}\Big|_{x=0}$ depends on the sign of the function $h(\tau)$, which is the numerator of the third term on the right-hand side.

Thus, we have

$$h(\tau) = (1-\alpha)^3(1+\gamma)\tau^2 - (1-\alpha)(1-2\alpha+\alpha^2)(1+\gamma)\tau - \alpha[(1+\gamma)(1-\alpha+\alpha^2) + \alpha\beta].$$

Notice that

$$h(0) = -\alpha[(1+\gamma)(1-\alpha+\alpha^2) + \alpha\beta] < 0,$$

and

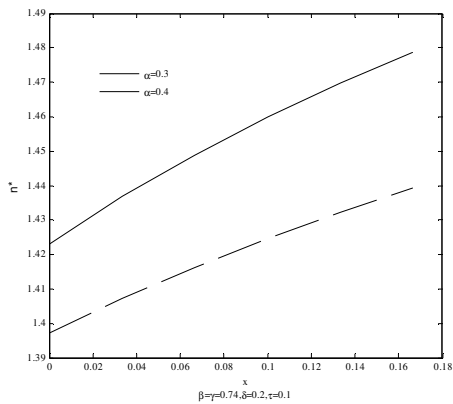
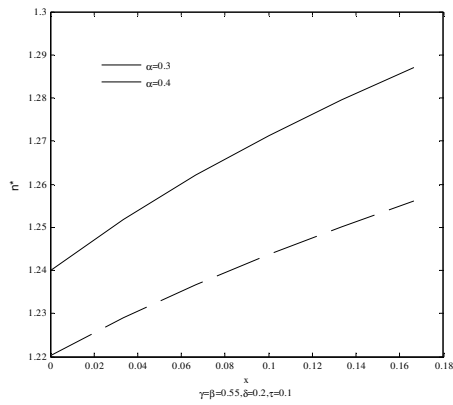
$$h(1) = -\alpha(1+\gamma)(1-\alpha+\alpha^2) < 0.$$

Because $h(\tau)$ is quadratic in τ , it is a parabola, which is opening to the top

because of $(1-\alpha)^3(1+\gamma) > 0$, and the axis $\tau = \frac{(1-\alpha)(1-2\alpha+\alpha^2)}{2(1-\alpha)^3} > 0$. This implies

that $h(\tau) < 0$ for all $\tau \in (0,1)$. Therefore, a small amount of the postponement of the mandatory retirement age x decreases private savings.

In order to study the impact of raising the mandatory retirement age on the fertility, we use different parameters to verify its robustness of result. Here, we enumerate several parameter settings. We assume that the length of each period is 30 years.⁵ The retirement age is increased from 1 to 5 years old, so x is 1/30 to 1/6. Other parameters are set as follows: $\alpha = 0.3, 0.4$, $\beta = 0.98^{30} = 0.55$ or $\beta = 0.99^{30} = 0.74$, $\gamma = \beta$, $\delta = 0.2, 0.25$, $\tau = 0.1, 0.15$.



⁵According to Feldstein (1985), a generation is 30 years.

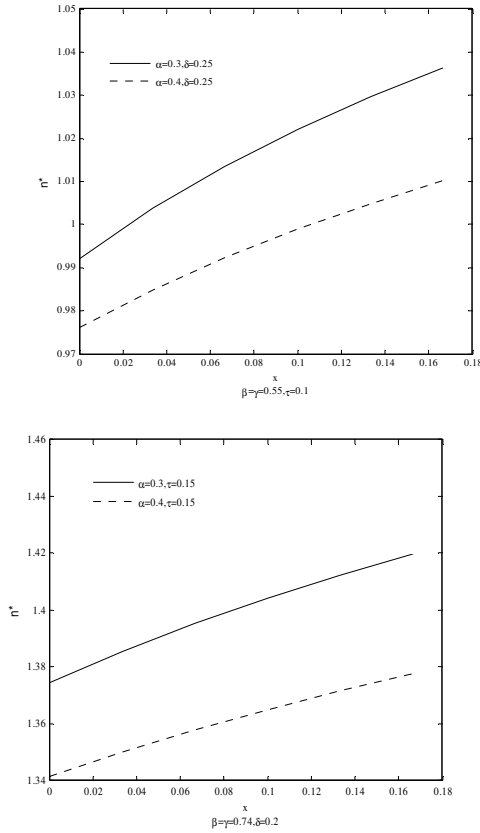


Fig.2. Effects of postponing the retirement age on the fertility rate

Fig.2 only gives several results. It shows that the long-run fertility rate increases as x increases. Of course, the conclusion still holds with other parameter settings. The effects of postponing the retirement age on fertility can be explained by life-cycle consumption model. When the retirement age is raised, the agent can get extra wage income in the second period, so she or he does not need more saving to support the life after retirement in the first period. The reducing saving can be used to rear children. Therefore, people would like to raise more children, the fertility rate increases as the retirement age increases. Furthermore, due to the postponement of the retirement age, the increase of the future income may reduce the credit constraints faced by the parents in the young, which can also contribute to increasing the fertility rate. From Fig.2, note that the expansion of pay-as-you-go social security decreases the fertility rate in the long run, but the effect is limited. However, the physical cost of child-rearing greatly affects the fertility rate. The results may have important implications for population policy.

3.2 PAYG social security benefit

The second effect is the agent's social security benefit. We consider how the postponement of the mandatory retirement age affects the social security benefit, in the long run, that is the P^* .

Proposition 3.2 (1) If $0 < \alpha < \frac{-\beta + \sqrt{\beta^2 + \beta(1+\gamma)}}{1+\gamma}$, then the social security benefit P^* increases in a small amount of x for every $\tau \in [0,1)$; (2) If $\frac{-\beta + \sqrt{\beta^2 + \beta(1+\gamma)}}{1+\gamma} < \alpha < \frac{1}{2}$, then there exists a unique $\bar{\tau} \in (0,1)$, such that for every $\tau \in (0, \bar{\tau})$, P^* decreases in a small amount of x and increases in a small amount of x for every $\tau \in (\bar{\tau}, 1)$; (3) If $\alpha \geq \frac{1}{2}$, P^* decreases in a small amount of x for every $\tau \in (0,1)$.

Proof Taking the derivative of P^* with respect to x and rearranging it, we have

$$\frac{\partial P^*}{\partial x} = \tau(1-\alpha)k^{*\alpha} \left[\frac{\partial n^*}{\partial x} + 1 + (n^* + x)\alpha k^{*-1} \frac{\partial k^*}{\partial x} \right].$$

At $x=0$, we have

$$n^*|_{x=0} = \frac{\gamma(1-\tau)(\tau - \tau\alpha + \alpha)}{\delta[\alpha(1+\beta+\gamma) + (1-\alpha)(1+\gamma)\tau]},$$

$$k^{*-1}|_{x=0} = \left[\frac{\alpha\beta(1-\alpha)(1-\tau)}{\alpha(1+\beta+\gamma) + (1-\alpha)(1+\gamma)\tau} \right]^{\frac{1}{1-\alpha}} \left[n^*|_{x=0} \right]^{\frac{1}{1-\alpha}},$$

$$\frac{\partial k^*}{\partial x}|_{x=0} = -\frac{1}{1-\alpha} \frac{[\alpha\beta(1-\alpha)(1-\tau)]^{\frac{1}{1-\alpha}}}{[\alpha(1+\beta+\gamma) + (1-\alpha)(1+\gamma)\tau]^{\frac{1}{1-\alpha}+1}} \left[n^*|_{x=0} \right]^{-\frac{1}{1-\alpha}} \left[1 + \alpha\beta + \gamma + \frac{\alpha\beta(1-\alpha)(1-\tau)}{\tau - \tau\alpha + \alpha} \right],$$

$$\begin{aligned} \frac{\partial p^*}{\partial x} \Big|_{x=0} &= \tau(1-\alpha)k^{\alpha} \Big|_{x=0} \left[\frac{\partial n^*}{\partial x} \Big|_{x=0} + 1 + n^* \Big|_{x=0} \alpha k^{\alpha-1} \Big|_{x=0} \frac{\partial k^*}{\partial x} \Big|_{x=0} \right] \\ &= \tau(1-\alpha)k^{\alpha} \Big|_{x=0} \frac{(1-\alpha)^3(1+\gamma)\tau^2 + \alpha(1-\alpha)(1-2\alpha)(1+\gamma)\tau + \alpha\beta(1-2\alpha) - \alpha^3(1+\gamma)}{(1-\alpha)[\alpha(1+\beta+\gamma) + (1-\alpha)(1+\gamma)\tau](\tau - \tau\alpha + \beta)} \end{aligned}$$

Therefore, the sign of $\frac{\partial p^*}{\partial x} \Big|_{x=0}$ depends on the sign of the function $g(\tau)$, $g(\tau)$ is the numerator of the second term of the above equation on the right-hand side. Thus,

$$g(\tau) := (1-\alpha)^3(1+\gamma)\tau^2 + \alpha(1-\alpha)(1-2\alpha)(1+\gamma)\tau + \alpha\beta(1-2\alpha) - \alpha^3(1+\gamma)$$

If $0 < \alpha < 1/2$,

$$\begin{aligned} g(1) &= (1-\alpha)^3(1+\gamma) + \alpha(1-\alpha)(1-2\alpha)(1+\gamma) + \alpha\beta(1-2\alpha) - \alpha^3(1+\gamma) \\ &= (1-\alpha)^3(1+\gamma) - \alpha^3(1+\gamma) + \alpha(1-\alpha)(1-2\alpha)(1+\gamma) + \alpha\beta(1-2\alpha) > 0 \end{aligned}$$

If $g(0) = \alpha\beta(1-2\alpha) - \alpha^3(1+\gamma) > 0 \Leftrightarrow 0 < \alpha < \frac{-\beta + \sqrt{\beta^2 + \beta(1+\gamma)}}{1+\gamma} < \frac{1}{2}$,

in this case, according to the shape of this function, $g(\tau) > 0$ for

every $\tau \in (0, 1)$. Therefore, $\frac{\partial p^*}{\partial x} \Big|_{x=0} > 0$ for every $\tau \in (0, 1)$. Which implies a small amount of postponement of the mandatory retirement age can improve the social security benefit in the long run.

If $g(0) = \alpha\beta(1-2\alpha) - \alpha^3(1+\gamma) < 0 \Leftrightarrow \frac{-\beta + \sqrt{\beta^2 + \beta(1+\gamma)}}{1+\gamma} < \alpha < \frac{1}{2}$, then there is

a unique $\bar{\tau} \in (0, 1)$ such that $g(\bar{\tau}) = 0$ and $g(\tau) < 0$ for every $\tau \in (0, \bar{\tau})$. Otherwise, $g(\tau) > 0$ for every $\tau \in (\bar{\tau}, 1)$. Therefore, under the

condition $\frac{-\beta + \sqrt{\beta^2 + \beta(1+\gamma)}}{1+\gamma} < \alpha < \frac{1}{2}$, we have $\frac{\partial p^*}{\partial x} \Big|_{x=0} < 0$ for every $\tau \in (0, \bar{\tau})$ and

$\frac{\partial p^*}{\partial x} \Big|_{x=0} > 0$ for every $\tau \in (\bar{\tau}, 1)$. The former implies that a small amount of raising the mandatory retirement age can decrease the social security benefit, and the latter can

improve it in the long run.

If $\alpha=1/2$, $g(\tau)=1/8(1+\gamma)(\tau^2-1)$, then $g(\tau)<0$ for every $\tau \in (0,1)$. Hence, the agent's social security decreases in the long run.

$$\text{If } \alpha > 1/2, \text{ then } g(0) = \underbrace{\alpha\beta(1-2\alpha)}_{<0} - \underbrace{\alpha^3(1+\gamma)}_{<0} < 0.$$

Notice that

$$\begin{aligned} g(1) &= (1-\alpha)^3(1+\gamma) + \alpha(1-\alpha)(1-2\alpha)(1+\gamma) + \alpha\beta(1-2\alpha) - \alpha^3(1+\gamma) \\ &= \underbrace{(1-\alpha)^3(1+\gamma)}_{<0} + \underbrace{\alpha(1-\alpha)(1-2\alpha)(1+\gamma)}_{<0} + \alpha\beta(1-2\alpha) < 0 \end{aligned}$$

Hence, according to the shape of $g(\tau)$, we know that $g(\tau)<0$ for every $\tau \in (0,1)$. Therefore, under the assumption $\alpha \geq 1/2$, we have $\frac{\partial p^*}{\partial x}|_{x=0} < 0$ for every $\tau \in (0,1)$. It implies that a small amount of postponement of the mandatory retirement age can decrease the social security benefit in the long run.

The intuition behind this result is that the postponement of the mandatory retirement age has two effects on the agent's social security benefit. One is that a small amount of the postponement of the mandatory retirement age can decrease the precautionary savings, and thus increases disposable income, which encourages the young to have more children. Therefore, it can improve the fertility rate in the long, but the magnitude is very limited. Hence, it causes more workers to support the PAYG social security system. In addition, the postponement of retirement age has prolonged the pension contribution period, which also can increase the social security benefit. As a consequence, it can increase the social security benefit. The other is that the rising fertility rate and a small amount of the postponement of the mandatory retirement age can decrease the capital of per worker, and then decrease the real wage. As a consequence, it can decrease the social security benefit in the long run.

When the output elasticity of capital is small enough, the first effect dominates the second effect, so P^* increases. However, when the output elasticity of capital is more

than 0.5, the second effect dominates the first effect, so P^* decreases. In addition, when the output elasticity of capital is in a small range, the dominance of the effect depends on the contribution rate of the social security.

3.3 Welfare

A small amount of the postponement of the mandatory retirement age can decrease the saving rate, and thus decrease the real wage level in the equilibrium. However, it increases the long-run fertility rate. The decreasing wage will reduce the agent's well being, but it will increase welfare by improving the fertility rate. Therefore, we want to investigate how it affects the agent's welfare. In the steady state, the agents' welfare is defined by

$$U(x) := Lnc^y + \beta \ln c^o + \gamma \ln n^*,$$

(15)

where $c^y = (1 - \tau - \delta n^*)(1 - \alpha)k^{*\alpha} - (n^* + x)k^*$, and $c^o = [(\alpha + \tau - \alpha\tau)n^* + x]k^{*\alpha}$. First, we want to know the effects of raising the mandatory retirement age on an adult agent's consumption and on an old agent's consumption in the steady state. We have the following proposition:

proposition 3.3 (1)The young agent's consumption c^y decreases in a small amount of x for every $\tau \in (0,1)$; (2)If $0 < \alpha \leq 1/2$, the old agent's consumption c^o increases in a small amount of x for every $\tau \in (0,1)$; (3)If $\frac{1+2\beta+\gamma-\sqrt{(1+2\beta+\gamma)(1+\beta+\gamma)}}{\beta} < \alpha < 1$, the old agent's consumption c^o decreases in a small amount of x for every $\tau \in (0,1)$; If $1/2 < \alpha < \frac{1+2\beta+\gamma-\sqrt{(1+2\beta+\gamma)(1+\beta+\gamma)}}{\beta}$, then there exist a unique $\hat{\tau} \in (0,1)$, c^o increases in a small amount of x for every $\tau \in (0, \hat{\tau})$, and decreases in a small

amount of x for every $\tau \in (\hat{\tau}, 1)$.

Proof it is easy to show that

$$\begin{aligned} \frac{\partial c^y}{\partial x} \Big|_{x=0} &= \frac{1}{c^y} \left[-\delta(1-\alpha) \frac{\partial n^*}{\partial x} \Big|_{x=0} k^{*\alpha} \Big|_{x=0} + (1-\tau - \delta n^* \Big|_{x=0})(1-\alpha) \alpha k^{*\alpha-1} \Big|_{x=0} \frac{\partial k^*}{\partial x} \Big|_{x=0} \right. \\ &\quad \left. - \left(\frac{\partial n^*}{\partial x} \Big|_{x=0} + 1 \right) k^* \Big|_{x=0} - n^* \Big|_{x=0} \frac{\partial k^*}{\partial x} \Big|_{x=0} \right] \\ &= \frac{1}{c^y} k^* \Big|_{x=0} \frac{1}{\left[\alpha(1+\beta+\gamma) + (1-\alpha)(1+\gamma)\tau \right] \left[\tau - \tau\alpha + \alpha(1-\alpha) \right]} \times \\ &\quad \frac{-\left(\alpha(1-\tau) + \tau \right) \left(\alpha^2 \beta^2 + \alpha(\beta(2+\gamma) + (1+\gamma)(1-\tau)) + (1+\gamma)\tau \right)}{\beta} < 0. \end{aligned}$$

Therefore, The young agent's consumption c^y decreases in a small amount of x for every $\tau \in (0, 1)$.

In order to investigate how the old agent's consumption changes as the mandatory retirement age increases, I take the derivative of c^o with respect to x . Then, at $x=0$, we get

$$\frac{\partial c^o}{\partial x} \Big|_{x=0} = \frac{(1-\alpha)[(1-2\alpha)(1+\gamma) - \alpha\beta(1-\alpha)]\tau + \alpha[(1-2\alpha)(1+2\beta+\gamma) + \alpha^2\beta]}{(1-\alpha)[\alpha(1+\beta+\gamma) + (1-\alpha)(1+\gamma)\tau]} k^{*\alpha} \Big|_{x=0}.$$

From the above equation, it shows that the sign of $\frac{\partial c^o}{\partial x} \Big|_{x=0}$ depends on the numerator of the first term on the right-hand side of this equation. Let

$$h(\tau) := (1-\alpha)[(1-2\alpha)(1+\gamma) - \alpha\beta(1-\alpha)]\tau + \alpha[(1-2\alpha)(1+2\beta+\gamma) + \alpha^2\beta].$$

If $0 < \alpha < 1/2$, then $h(0) = \alpha[(1-2\alpha)(1+2\beta+\gamma) + \alpha^2\beta] > 0$, and

$h(1) = (1-2\alpha)(1+\alpha\beta+\gamma) > 0$. This function is linear in τ . This implies $h(\tau) > 0$ for every $\tau \in (0, 1)$. Therefore, in this case, the old agent's consumption increases in a small amount of x for every $\tau \in [0, 1)$.

If $\alpha = 1/2$, then $h(\tau) = \frac{1}{8}\beta(1-\tau) > 0$ for all $\tau \in (0, 1)$. Therefore, the old agent's

consumption c^o increases in a small amount of x for every $\tau \in (0,1)$.

If $\alpha > 1/2$, the first term of $h(\tau)$ is negative. According to the shape of the function, its sign depends on the sign of the term $l(a) = (1-2\alpha)(1+2\beta+\gamma) + \alpha^2\beta$. When $l(a) < 0$, the solution of the inequality

$$\frac{1+2\beta+\gamma - \sqrt{(1+2\beta+\gamma)(1+\beta+\gamma)}}{\beta} < \alpha < 1$$

is .Furthermore,

$h(1) = (1-2\alpha)(1+\alpha\beta+\gamma) < 0$. Hence, $h(\tau)$ is negative for all $\tau \in (0,1)$. In this case,

the old agent's consumption c^o decreases in a small amount of x for every $\tau \in (0,1)$.

When $l(a) > 0$, the solution is $1/2 < \alpha < \frac{1+2\beta+\gamma - \sqrt{(1+2\beta+\gamma)(1+\beta+\gamma)}}{\beta}$. In this

case, there is a unique $\hat{\tau} \in (0,1)$ such that $h(\tau) > 0$ for every $\tau \in (0, \hat{\tau})$, and

$h(\tau) < 0$ for every $\tau \in (\hat{\tau}, 1)$. Therefore, at a low payroll tax rate, the old agent's

consumption increases in a small amount of x , while at a high rate, the old agent's consumption decreases in a small amount of x .

Now, we want to investigate how the postponement of the statutory retirement age affects the agent's welfare. Since it is difficult to use methods of comparative statics to determine the sign, we provide a numerical illustration to analyze the effect of raising the mandatory retirement age on the agent's welfare. Let us set the output elasticity of capital $\alpha = 0.3, 1/3, 0.4, 0.5$. A generation is 30 years (Feldstein, 1985).

Hence, for the values of β and γ , we set $\beta = \gamma = 0.99^{30} = 0.74$. For the cost of childrearing, Apps and Rees (2001) argue that each child's cost of rearing accounts for 20-30% of the family's income, which is also suggested by Deaton and Muellbauer (1986), while it is 0.25 in Miyazaki (2013). We set it as 0.2 and 0.25. For the value of the payroll tax rate, we set it as 0.15, which is used by Fanti (2014). We note that the mandatory retirement age will be increased by 1 to 5 years. Therefore, table 1 below displays the effects of raising the mandatory retirement age on welfare.

Table 1. Effects of raising the statutory retirement age upon welfare

Parameter settings	Delaying retirement age				
	1	2	3	4	5
$\alpha = 0.3, \beta = \gamma = 0.74, \tau = 0.15, \delta = 0.2$	0.60	1.14	1.62	2.06	2.45
	%	%	%	%	%
$\alpha = 1/3, \beta = \gamma = 0.74, \tau = 0.15, \delta = 0.2$	0.32	0.60	0.84	1.06	1.24
	%	%	%	%	%
$\alpha = 0.4, \beta = \gamma = 0.74, \tau = 0.15, \delta = 0.2$	-0.1	-0.2	-0.3	-0.5	-0.6
	2%	4%	7%	0%	3%
$\alpha = 0.5, \beta = \gamma = 0.74, \tau = 0.15, \delta = 0.2$	-0.5	-1.0	-1.5	-1.9	-2.4
	2%	1%	0%	6%	1%
$\alpha = 0.3, \beta = \gamma = 0.74, \tau = 0.15, \delta = 0.25$	0.71	1.32	1.86	2.33	2.76
	%	%	%	%	%
$\alpha = 1/3, \beta = \gamma = 0.74, \tau = 0.15, \delta = 0.25$	0.38	0.70	0.97	1.20	1.40
	%	%	%	%	%
$\alpha = 0.4, \beta = \gamma = 0.74, \tau = 0.15, \delta = 0.25$	-0.1	-0.3	-0.4	-0.6	-0.7
	4%	0%	6%	2%	9%
$\alpha = 0.5, \beta = \gamma = 0.74, \tau = 0.15, \delta = 0.25$	-0.6	-1.2	-1.8	-2.4	-2.9
	5%	7%	6%	3%	8%

From the table 1, when the output elasticity of capital is less than 0.4, the welfare increases as retirement age increases. Otherwise, the welfare decreases as retirement age increases. Of course, the magnitude is very small. This has an important policy implication for some developing countries. In order to reduce the burden of pension payouts, many countries want to implement the policy of raising the mandatory retirement age, such as China. However, it shows that, in some developing countries with higher output elasticity of capital, the delaying retirement age has a negative

effect on the individual welfare and also a negative effect on pension benefits. Hence, these countries should reduce the output elasticity of capital by improving labor productivity for the individual well being and pension benefits.

4 Conclusion

This paper uses a neoclassical growth model with endogenous fertility and studies how the postponement of the mandatory retirement age affects the fertility rate, social security benefit, and welfare in the long run. It is shown that a small amount of raising the retirement age can increase the fertility rate in the long run. Data analysis indicates that the fertility rate is a limited improvement as the retirement age increases, but the reduction of cost of child-rearing can greatly improve the fertility. However, it will not necessarily lead to the improvement of the social security benefit in the long run. When the elasticity of capital is no less than 0.5, the agent's social security benefit decreases in a small amount of the postponement of the retirement age in the long run. Only when the elasticity is small enough, the social security benefit increases in the retirement age in the long run. Otherwise, it depends on the contribution rate. As to the welfare, when the output elasticity of capital is small, the welfare increases as retirement age increases. Otherwise, the welfare decreases as retirement age increases. But the magnitude is very small.

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