To adjust or not to adjust: The optimal inflation target in the face of a lower $r^*$

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Harvard macroeconomic policy seminar – November 5, 2019
The views expressed here are mine and do not necessarily reflect those of the Federal Reserve Bank of Boston or the Federal Reserve System.
Presentation based on Andrade-Galí-Le Bihan-Matheron

✓ “The optimal inflation target and the natural rate of interest”, BPEA Fall 2019

✓ VoxEU column, Oct. 2019
Outline

Introduction

Presentation of results in Andrade-Galí-Le Bihan-Matheron

✓ Quantitative analysis of the optimal inflation target ($\pi^*$) as a function of steady-state real interest rate ($r^*$)

✓ Based on an estimated NK model of the US economy

Discussion

✓ Costs & benefits of trend inflation compared to NK setup

✓ Why raising the target is currently not on the agenda

✓ Alternatives to increasing the target

Conclusion
Introduction

Presentation of results in Andrade-Galí-Le Bihan-Matheron (2019)

Discussion: Costs & benefits of trend inflation

Discussion: Why not on the current agenda

Discussion: What alternatives

Conclusion
Decline in the natural rate of interest $r^*$
Shift in the beliefs of FOMC members

**Figure:** FOMC members’ FFR long-run forecasts (SEP, median)
Decline in the natural rate of interest $r^*$
Downward trend in long-run forecasts of professionals & market participants

Figure: Short-term interest rate long-run forecasts (SEP, SPF, SPD)
Decline in the natural rate of interest $r^*$

Shift in expected real rates

**Figure:** Real interest rates long-run forecasts (SEP, SPF)
“The Federal Reserve announced in November 2018 that it would conduct a broad review of the strategy, tools, and communication practices it uses to pursue the monetary policy goals established by the Congress: maximum employment and price stability. […]

The U.S. economy appears to have changed in ways that matter for the conduct of monetary policy, particularly during economic downturns. For example, the neutral level of the policy interest rate, the level that keeps the economy on an even keel when employment and inflation are close to their objectives, appears to have fallen in the United States and abroad. This decline increases the risk that a central bank’s policy rate will fall to its effective lower bound, constraining the central bank’s ability to counter future downturns. [...]

Q: Please allow me to raise personal questions on this particular day. Your colleague, Peter Praet once told me that by the time he and you are leaving, you both would prefer to have monetary policy back to a kind of normality. We know that this unfortunately hasn’t happened. Can you give us some insight? […]

MD: […] It is true that during 2017 we gradually changed our monetary policy stance and we were preparing to exit that stance of monetary policy. But then conditions changed and what prevails over everything else is the determination to pursue the mandate for which this institution was created and for which we work. Therefore we had to change course and get back into the present stance. Let me also add one thing: that if there is one take from the recent IMF meetings, it’s that the paradigm of reference has changed. Until not long ago, the IMF and all the observers would say that, yes, interest rates are low and they may stay low for some time, but then they will go up. Now, the sense of many discussions at the IMF is actually that they will stay low for a long time because the real rate of interest has also declined. This implies that the exit from unconventional monetary policies has shifted forward in time. The way I feel? I feel like someone who tried to comply with his mandate in the best possible way.
Consequences for monetary policy

“The paradigm of reference has changed”

✓ More frequent Lower Bond on the nominal interest rate (difficulties to stabilize in a recession)

✓ More frequent recourse to unconventional monetary policies

✓ Low for long & search for yield: risks to financial stability

✓ Interest rate levels that used to be considered as very low / very accommodative may not be anymore
What can monetary policy do?

\[ i = r^* + \pi \]

✓ Drop in \( r^* \) can be compensated by increasing \( \pi \)
  \[ \rightarrow \] Blanchard et al. (2010), Ball (2014), ...

✓ However, trade-off: higher inflation also goes with costs
  \[ \rightarrow \] Increasing \( \pi \) to keep \( \text{pr}(ZLB) \) constant may not be optimal
  \[ \rightarrow \] Costs paid every day while ZLB is a rare event (Bernanke, 2016)

✓ Moreover, increasing \( \pi \) to compensate 1-for-1 for the decline in \( r^* \) does not ensure that \( \text{pr}(ZLB) \) stays constant
  \[ \rightarrow \] \( r^* \) determined by parameters that affect the whole dynamics of the economy (hence proba of hitting the ZLB)

✓ Is an increase in the inflation target warranted? By how much?
  \[ \rightarrow \] Quantify relation btw \( r^* \) and optimal inflation target \( \pi^* \)
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Discussion: Why not on the current agenda

Discussion: What alternatives

Conclusion
Should CB adjust their inflation target in the face of a lower $r^*$?

✓ Yes: increase.

By how much?

✓ A 1pp drop in $r^*$ calls for an increase in $\pi^*$ of 0.9pp to 1pp.

Does anything modify this result within our modeling setup?

✓ Yes: changes in the reaction function / strategy of the CB
Modelling setup

✓ Medium-scale NK model
✓ Staggered price and wage setting à la Calvo
✓ Imperfect indexation of prices to lagged price inflation; and of wages to lagged price inflation and productivity
✓ Shocks: demand (risk premium, marginal U of cons., MP) & supply (technology, price and wage markups)
✓ Trend growth: \( r^* = \rho + \mu z \)
✓ Monetary policy rule:

\[
i_t = \max\{i^n_t, 0\}
\]

where

\[
i^n_t = (1 - \rho_i) i + \rho_i i^n_{t-1} + (1 - \rho_i) [a_\pi (\pi_t - \pi) + a_y (y_t - y^n_t)] + \zeta_{r,t}
\]

with \( i = r^* + \pi \) and where \( \pi \) defines the inflation target
✓ Parameters estimated using Bayesian approach

✓ Sample period: 1985Q1-2008Q3 (pre-ZLB; Great Moderation)

✓ Estimated parameters are in the ball park of existing results
  → Calvo: prices updated every 9 month; wages every 6 month
  → Indexation: 20% on past inflation for prices; 45% for wages
  → Taylor rule: strong “smoothing” parameters; 85% of current stance pre-determined by lagged interest rate
  → $\Pr(ZLB) \approx 5\%$
✓ $W(\pi; \theta)$: 2d-order approximation to HH expected utility

✓ NK setup: nominal rigidities induce price distortions & suboptimal reaction (employment/output...) to macro shocks

→ ZLB costly: larger price distortions because central bank lacks its usual stabilization instrument

✓ Positive inflation target in NK setup

→ Beneficial because it limits pr(ZLB)

→ Costly because imperfect indexation distort relative prices / real wage hence output permanently below its efficient level
Computing the Optimal Inflation Target

✓ Simulate model for a fixed parameter values $\theta$ and an inflation target $\pi$ and compute $W(\pi; \theta)$

✓ Do that for various values of $\pi$

✓ Optimal inflation target is:

$$\pi^*(\theta) \equiv \arg \max_\pi W(\pi; \theta)$$

✓ Do that for various values of $r^*$

Pre-crisis benchmark

$$r^* \approx 2.4\% \Rightarrow \pi^*_{US} \in [1.85\%, 2.20\%]$$
The \((r^*, \pi^*)\) relation

Figure: \((r^*, \pi^*)\) locus (at the posterior mean)
Relation between $Pr[\text{ZLB}|\text{optimal inflation}]$ and $r^*$
Relation between $Pr[ZLB]$ and $r^*$ at fixed $\pi$
What if shocks are larger?

Set standard deviation of demand shocks to 1.3 their baseline value

Figure: \((r^*, \pi^*)\) relation with larger demand shocks

Note: blue dots \(\equiv\) baseline scenario: all the structural parameters set at posterior mean \(\bar{\theta}\).
The red dots counterfactual simulation with \(\sigma_q\) & \(\sigma_g\) set to 30% higher than their baseline value.
What if markups are lower (or larger)?

Vary elasticity of substitution accross products:
baseline $\theta_p$ ($\mu = 1.2$) / low ($\mu = 1.5$) / high ($\mu = 1.11$)

Figure: ($r^*, \pi^*$)
What if there is more or (less) indexation?

Vary indexation degree:

Figure: \((r^*, \pi^*)\)
✓ Draw $\theta$ from posterior distribution, simulate model and compute $\mathcal{W}(\pi; \theta)$

✓ Repeat these $N$ times so as to get a distribution of $\mathcal{W}(\pi; \theta)$

✓ Optimal inflation

$$\pi^{**} \equiv \arg\max_{\pi} \int_{\theta} \mathcal{W}(\pi; \theta)p(\theta|X_T)d\theta$$

Pre-crisis benchmark

$$\Rightarrow \pi^{**}_{US} = 2.40\%$$
Accounting for parameter uncertainty
Counterfactual posterior distribution of $r^*$

Figure: Posterior Distributions of $r^*$ and counterfactual $r^*$

Plain curve: PDF of $r^*$; Dashed vertical line: Mean value, i.e. $E_{\theta}(\pi^*(\theta))$. Remark: distribution of $r^*$ roughly symmetric; does not explain the asymmetry in distribution of $\pi^*$. 
Accounting for parameter uncertainty
Change in the optimal inflation target

Figure: $E_{\theta}(\mathcal{W}(\pi, \theta))$

Blue curve: $E_{\theta}(\mathcal{W}(\pi, \theta))$; Red curve: $E_{\theta}(\mathcal{W}(\pi, \theta))$ with lower $r^*$
ELB: the nominal rate $i_t$, such that $i_t \geq ELB$
Here set $ELB$ for US to $-40$ basis points instead of zero.
Matches the ECB Deposit Facility Rate level attained in March 2016.
Baseline $\rho_i = 0.85$ / low $\rho_i = 0.80$ / high $\rho_i = 0.95$
Alternative Monetary Policy Rules and Environments

Price level targeting rule

\[ i_{t}^{plt} = (1 - \rho_i)i + \rho_i i_{t-1}^{plt} + (1 - \rho_i) \left( a_p (p_t - \bar{p}_t) + a_y (y_t - \bar{y}_t^n) \right) + \zeta R, t \]

with \( \bar{p}_t = p_0 + \pi . t \)
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Conclusion
Does the NK setup understate the costs of trend inflation

Effect on $\pi^*$: understate (−) / overstate (+)

Model delivers $\pi^* \simeq 2\%$ pre-crisis (consistent with FOMC choice)

Some specificities of NK setup

✓ Welfare puts little weight on output gap (+)
✓ Nominal rigidities less costly under idiosyncratic shocks and menu costs (+)
✓ Nominal rigidities could decline & indexation increase with average inflation (+)
✓ No money in the utility function (taxation of real balances) (−)
✓ No “other costs” of inflation (uncertainty) (−)
Does the NK setup overerstate the benefits of trend infl.

Effect on $\pi^*$: overstate ($-$) / understate ($+$)

✓ Other policies can step in ($-$)
✓ Expectations are more anchored in reality than in model
  $\rightarrow$ less deflation spiral at the ZLB ($-$)
  $\rightarrow$ non-conventional MP less potent ($+$)
✓ ZLB and financial stability ($+$)
✓ Downward wage rigidity ($+$)
✓ Effect on public finance (non indexation of the tax code) ($+$)
✓ Tendency to overstate inflation in statistics ($+$)
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“The Federal Reserve announced in November 2018 that it would conduct a broad review of the strategy, tools, and communication practices it uses to pursue the monetary policy goals established by the Congress: maximum employment and price stability. [...] The review will take the Federal Reserve’s statutory mandate as given, and will also take as given that an inflation objective of 2 percent is the most consistent, over the longer run, with the assigned mandate of price stability. Within these parameters, the review will be wide-ranging. It will consider whether the Federal Reserve can best meet its dual-mandate objectives with its existing monetary policy strategy, whether the existing monetary policy tools are adequate to achieve and maintain the dual mandate, and whether the communications about the policy strategy and tools can be improved.”
Reasons central bankers give for not considering it

✓ Alternatives policies are better than increasing $\pi^*$
   - More frequent unconventional MP
   - More aggressive “lower for longer” MP strategies

✓ Increasing the target raises credibility issues
   - Hard won anchoring of inflation expectations will be put at risk
   - How increasing the target can be credible in a world where CBs cannot even meet the current one?

✓ However
   - More frequent ZLB also raises credibility issues as inflation will tend to be on average below target
   - If current toolkit is enough to control inflation, it should allow for meeting / increasing the inflation target
   - Alternative polices may also be costly (see below)
Other potential reasons

✓ High subjective cost of inflation (higher than in NK models?)

✓ Change in target involves redistribution during transition

✓ So difficult to reach a consensus in favor of a higher target

✓ Move to “make-up” strat. easier / closer to current framework

→ But what if it involves inflation $\geq 3\%$ for some times (years)?

→ Once agreed, credibility of commitment to a new target would be much stronger
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What are the alternatives

✓ Stick to inflation targeting and tolerate somewhat lower inflation at the ZLB (and on average)
  → Rational is to limit financial stab. risks (too low for too long)
  → Risk of deanchoring hence even lower IR and more frequent ZLB

✓ More asset purchases
  → Costs as redistribution effects / political pressures
  → Financial stability risks (low for long)

✓ More forward guidance
  → Commitment is time inconsistent
  → Raises communication challenges (Delphic FG)
  → Markets expect rates to be low for long; lowering them further require communicating about the long-run (credibility)
  → Very powerful in theory but expectations might not be as a powerful channel in reality
What are the alternatives

- More fiscal expansions
  - Debt sustainability issue

- Move from inflation targeting strategy to “make-up’ strategy
  - Commitment is time inconsistent: (how much inflation $\geq 2\%$ needed to compensate for infl. below target over last decade?)
  - Very powerful in theory but expectations might not be as a powerful channel in reality

- Negative interest rates (& cashless economy)

- Implement policies that will increase $r^*$
  - Increase supply of safe assets (By how much? Public debt?)
  - Increase potential growth (How?)
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✓ 2% is not a universal constant
   → Chosen at a time when $r^*$ was (thought to be) higher
✓ Solutions to fix low $r^*$ difficult to implement and will take time
✓ In the meantime CB will face the consequences of a more frequent ZLB and its associated drawbacks
   → More important limits to stabilize macro outcomes / meet the dual mandate
   → Risk of deanchoring of inflation expectation and permanently depressed aggregate demand
   → Risk to financial stability in a low interest rate world
   → Risk to credibility & independence if CB is deemed to be too interventionist / to miss on its mandate
✓ Be sure that the costs of increasing the target exceed the ones above before dismissing this option
Appendix
### Estimation Results

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<th>Parameter</th>
<th>Description</th>
<th>Post. mean</th>
<th>Post. s.d</th>
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See detailed US results
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Computing the Optimal Inflation Target
The case of no parameter uncertainty

✓ Freeze parameters $\theta$ to a fixed value (posterior mean)
✓ 2d-order approximation to HH expected utility $\mathcal{W}(\pi; \theta)$
✓ Simulate large sample ($T = 100,000$) and compute $\mathcal{W}(\pi; \theta)$
✓ Solution under ZLB: Bodenstein et al. (2009) algo.
✓ Optimal (welfare-maximizing) inflation target is:

$$\pi^*(\theta) \equiv \arg \max_\pi \mathcal{W}(\pi; \theta)$$

Pre-crisis benchmark

$$\Rightarrow \pi^*_US \in [1.85\%, 2.20\%]$$
A second–order approximation to welfare:

\[ U_0 = \frac{1}{2} \frac{1}{1 - \eta} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \lambda_y [x_t - \delta x_{t-1} + (1 - \delta)\bar{x}]^2 \right. \]

\[ + \lambda_p [(1 - \gamma_p)\pi + \hat{\pi}_t - \gamma_p \hat{\pi}_{t-1}]^2 \]

\[ + \lambda_w [(1 - \gamma_z)\mu_z + (1 - \gamma_w)\pi + \hat{\pi}_{w,t} - \gamma_w \hat{\pi}_{t-1}]^2 \}

+ t.i.p. + O(||\zeta, \pi||^3)

where gaps are defined as: \( x_t \equiv \hat{y}_t - \hat{y}_t^n \), \( \bar{x} \equiv \log \left( \frac{Y_z}{\hat{Y}_z^n} \right) \)

Strictly positive inflation
\[ \rightarrow \] is harmful due to induced dispersion in prices and quantities
\[ \rightarrow \] but limits variability of \( \hat{\pi}_t \) that results from ZLB
The Model: Solution under ZLB

Log-linearized version of the model

Simulation under ZLB via a “OccBin” algorithm following Bodenstein et al. (2009) or Guerrieri-Iacoviello (2015)

General idea
At each date $t$, given shocks $\epsilon_t$

✓ Postulate ZLB entry date $T_e$ and ZLB exit date $T_x$

✓ Solve by backward induction for time-varying state-space representation

✓ Check whether postulated dates are correct; else shift leftward or rightward as appropriate

✓ Iterate upon convergence
Example of (Normalized) Welfare Functions

Blue: parameters set at posterior mean; light blue: parameters set at the posterior median; Lighter blue: parameters set at posterior mode.
The \((r^*, \pi^*)\) relation

Figure: \((r^*, \pi^*)\) locus when \(\mu_z\) varies

Blue: parameters set at the posterior mean; light blue: parameters set at the posterior median; Lighter blue: parameters set at the posterior mode
Memo: \(r^* = \rho + \mu_z\). Range for \(\mu_z\): 0.4% to 10% (annualized)
The \((r^*, \pi^*)\) relation — 

**Figure:** \((r^*, \pi^*)\) locus when \(\rho\) varies

**Memo:**  
\(r^* = \rho + \mu_z\). Range for \(\rho\): 0.4\% to 10\% (annualized)
The probability of ZLB under large shocks

Figure: Relation between probability of ZLB at optimal inflation and $r^*$ (at the posterior mean)
The distribution of ZLB spells duration

**Figure:** Distribution of ZLB spells duration at the posterior mean
The welfare cost of inflation

Figure: Welfare cost of inflation at the posterior mean