Problem Set 1*

Due: in class, February 13th

1 Prospect Theory and first order stochastic dominance

Let $X \subset \mathbb{R}$ be a finite set. Consider a decision maker whose preferences over lotteries over X (that is preferences defined over $\Delta(X)$) are represented by

$$U(p) := \sum_{x \in X} \nu(x)\pi(p(x)) \tag{1}$$

where ν is a real valued strictly increasing function and $\pi : [0,1] \to [0,1]$ is a function with the following properties:

(A) π is strictly increasing,

(B) $\pi(0) = 0$ and $\pi(1) = 1$,

(C) there exists $\bar{y} \in (0,1)$ such that $\pi(y) > y$ for every $y \in (0, \bar{y})$.

Property (C) is the key property of Prospect Theory's probability weighting function, that prescribes that small probabilities be overweighted.

1. Show that U can accommodate the version of Allais' Paradox proposed on slide 14 of Lecture slides. You can choose a specific functional form if you wish.

Consider the following definition: a preference relation, represented by U, is monotone to first order stochastic dominance if $U(p) > U(q)$ for every p and q in $\Delta(X)$ such that p first order stochastically dominates q ($p >_{FOSD} q$).

It has been pointed out that every preference represented by equation (1) violates monotonicity to first order stochastic dominance whenever π is such that $\pi(y) \neq y$ for some $y \in [0,1]$.

2. Consider now a lottery d_k such that $d_k(x) = \mathbb{I}(x = k)$. Find a lottery q such that $d_k >_{FOSD} q$ but $U(d_k) \leq U(q)$

Consider now a modification of preferences represented by (1) as follows:

$$\tilde{U}(p) := \sum_{x \in X} \nu(x) \frac{\pi(p(x))}{\sum_{z \in X} \pi(p(z))}.$$

*I thank Sagar Saxena and Elia Sartori for useful comments.

3. Show that $\tilde{U}$ can accommodate the version of Allais' Paradox on slide 14 of Lecture slides.

4. Show that, when preferences are represented by $\tilde{U}$, $d_k >_{FOSD} q$ for any q implies d_k is preferred to q. Moreover, show that preferences are FOSD monotone for lotteries that attach positive probability to at most two values.

5. Are preferences represented by $\tilde{U}$ FOSD monotone? Prove it or provide a counterexample.

2 Mickey Mouse Asset pricing under loss aversion

An agent allocates consumption between dates 0 and 1. At date 0 there is no randomness, while at date 1 there are J states each occurring with probability π_j, $j = 1, ..., J$. The agent has wealth W and allocates it between present consumption c_0 and consumption at each state j. To secure consumption at state j he can buy, at price p_j, $x_j$units of a security that pays 1 in state j and 0 otherwise (Arrow-Debreu security). The agent has utility $u(c_0)$ from present consumption and, from future consumption, he has utility

$$\nu(c_j \mid \bar{c}) := u(c_j) + \eta \left[u(c_j) - u(\bar{c})\right] \mathbb{I}_{(c_j < \bar{c})} + \eta\lambda \left[u(c_j) - u(\bar{c})\right] \mathbb{I}_{(c_j \geq \bar{c})}$$

where

(A) $u : \mathbb{R}_+ \to \mathbb{R}$ is, strictly increasing, continuously differentiable and concave

(B) $\bar{c} > 0$ is the reference point for consumption

(C) $\eta > 0$ and $\lambda > 1$.

Hence the agent chooses $(c_0, c_1, ..., c_J)$ to maximize $\{u(c_0) + \sum_{j=1}^{J} \pi_j \nu(c_j \mid \bar{c})\}$ subject to the budget constraint.

1. Explain why ν captures loss aversion.

2. Find the prices of the Arrow-Debreu securities p_j, by using the first order conditions of the problem, as a function of $(c_i)_{i=0}^{J}$.

3. Let us assume now that the resource constraint of the economy forces $c_j = y_j$ for $j = 1, ..., J$. Let us assume that the status quo consumption $\bar{c}$ is equal to $\sum_{j=1}^{J} \pi_j y_j$. For simplicity, assume $u(c) = \log(c)$. Compare the prices of the Arrow-Debreu securities with the standard expected utility case ($\eta = 0$): when is a security overpriced compared to the standard case? When is it underpriced?

4. **(extra credit)** Find an expression for the risk premium of a generic asset. (hint: use the fact that every asset is a combination of Arrow-Debreu securities, and that the price of a linear combination of securities is the linear combination of the prices of the securities).

3 True/False/Uncertain

1. Loss aversion implies that, when choosing between lotteries, a decision maker is risk averse if the lotteries' realizations are framed in the gain domain and risk loving if they are framed in the loss domain.

2. Consider a decision maker with Prospect Theory preferences that chooses between lotteries. Loss aversion makes her evaluate a risky lottery less than one that pays its expected value with probability one.

3. Suppose an expected utility maximizer, with utility over consumption, and endowed with consumption $C > 0$ can choose between a lottery q_0 that gives 0 with probability 1 and a lottery

$$q_x = \begin{cases} kx & with\, probability\ \frac{1}{2} \\ -x & with\, probability\ \frac{1}{2} \end{cases}$$

with $k > 1$, and $C > x > 0$. If she is sufficiently risk averse, she will prefer q_0 to q_x no matter the value that x can take.