# INDEBTED DEMAND* 

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#### Abstract

We propose a theory of indebted demand, capturing the idea that large debt burdens lower aggregate demand, and thus the natural rate of interest. At the core of the theory is the simple yet under-appreciated observation that borrowers and savers differ in their marginal propensities to save out of permanent income. Embedding this insight in a two-agent perpetualyouth model, we find that recent trends in income inequality and financial deregulation lead to indebted household demand, pushing down the natural rate of interest. Moreover, popular expansionary policies-such as accommodative monetary policy-generate a debt-financed short-run boom at the expense of indebted demand in the future. When demand is sufficiently indebted, the economy gets stuck in a debt-driven liquidity trap, or debt trap. Escaping a debt trap requires consideration of less conventional macroeconomic policies, such as those focused on redistribution or those reducing the structural sources of high inequality. JEL Codes: E21, E43, G51, E52, E62.


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## I Introduction

Rising debt and falling rates of return have characterized advanced economies over the past 40 years. As shown in Figure 1, debt owed by households and the government in the United States has increased almost 100 percentage points of GDP since 1980, and real rates of return on financial assets have fallen by 3 to 5 percentage points for different securities. How did the twin phenomena of high debt levels and low rates of return come to be? What are the implications of high debt levels and low rates of return for the evolution of the economy and macroeconomic policymaking?

Figure 1: Debt and Returns.


The left panel shows the household plus government debt to GDP ratio for the United States. The right panel shows expected real returns on a variety of assets for the United States. Please see Online Appendix C for more information.

This study develops a new framework to tackle these difficult questions. The framework shows how rising income inequality and the deregulation of the financial sector can push economies into a low rate-high debt environment. Traditional macroeconomic policies such as monetary and fiscal policy turn out to be less effective over the long term in such an environment. On the other hand, less standard policies such as macro-prudential regulation, redistribution policy, and policies addressing the structural sources of high inequality are more powerful and long-lasting.

The model introduces non-homothetic consumption-saving behavior (e.g. Carroll 2000, De Nardi 2004, Straub 2019) into an otherwise conventional, deterministic two-agent endowment economy. The assumption of non-homotheticity implies that the saver in the model saves a larger fraction of lifetime income than the borrower. This is not a new idea in economics. In fact, it is pervasive in the work of luminaries such as John Atkinson Hobson, Eugen von Böhm-Bawerk, Irving Fisher, and John Maynard Keynes, and empirically supported by recent work (e.g. Dynan, Skinner and

Zeldes 2004, Straub 2019, and Fagereng, Holm, Moll and Natvik 2019). In the model, the wealthy lend to the rest of the population, which makes household debt an important financial asset in the portfolio of the wealthy. This implication of the model fits the data, as shown in Mian, Straub and Sufi (2020): a substantial fraction of household debt in the United States reflects the top $1 \%$ of the wealth distribution lending to the bottom $90 \%{ }^{1}$

The assumption of non-homotheticity in our model generates the crucial property that large debt levels weigh negatively on aggregate demand: as borrowers reduce their spending to make debt payments to savers, the latter, having greater saving rates, only imperfectly offset the shortfall in borrowers' spending. We refer to a situation in which demand is depressed due to elevated debt levels as indebted demand.

In general equilibrium, indebted demand thus implies that greater levels of debt go hand in hand with reduced natural interest rates. From the perspective of savers, reduced interest rates are necessary to balance the greater desire to save in response to greater debt service payments. In an interest rate - debt diagram, the savers' indifference condition is therefore represented by a downward-sloping saving supply schedule. We use the equivalence between indebted demand and the downward-sloping saving supply schedule extensively in our analysis.

The concept of indebted demand has broad implications for understanding what has led to the current high debt and low interest rate environment, and for evaluating what policies can potentially help advanced economies escape this equilibrium. An overarching theme of the model is that shifts or policies that boost demand today through debt accumulation necessarily reduce demand going forward by shifting resources from borrowers to savers; therefore, such shifts or policies actually contribute to persistently low interest rates.

The indebted demand framework predicts a number of patterns found in the datawhich models without non-homotheticity in the consumption-saving behavior of agents have a difficult time explaining. For example, since the 1980s, many advanced economies have experienced a large rise in top income shares (Katz and Murphy 1992, Piketty and Saez 2003, Piketty 2014, Piketty, Saez and Zucman 2017), in conjunction with a substantial decline in interest rates and increases in household and government debt. The model predicts exactly such an outcome: a rise in top income shares in the model shifts resources from borrowers to savers, pushing down interest rates due to savers' greater desire to save. Lower interest rates stimulate more debt, causing indebted demand-as debt is nothing other than an additional shift of resources in the form of debt service payments from borrowers to savers.

The framework also predicts that financial deregulation, which has been a prominent feature of advanced economies since the 1980s, leads to a decline in interest rates, a result that is difficult to generate in most macroeconomic models (e.g. Justiniano, Primiceri and Tambalotti 2017). In the indebted demand model, financial deregulation increases the amount of debt taken on by borrowers, which redistributes resources to savers. For the goods markets to clear, such a redistribution

[^1]requires interest rates to fall given that savers have a lower marginal propensity to consume out of these larger debt payments.

The concept of indebted demand also provides insight into discussions of monetary and fiscal policy. For example, deficit-financed fiscal policy in the model is associated with a short run rise in natural interest rates, which reverses into a reduction in interest rates in the long-run, as the government needs to raise taxes or cut spending in order to finance the greater government debt burden. ${ }^{2}$ As long as some of the taxes are ultimately imposed on borrowers, deficit-financed government spending is similar to any policy which attempts to boost demand through debt accumulation. Ultimately, such a policy shifts resources from borrowers to savers, depressing aggregate demand and therefore interest rates in the long run.

A similar argument applies to monetary policy, for which we extend our model to include nominal rigidities. Empirical evidence suggests that an important channel of accommodative monetary policy operates through an increase in debt accumulation (e.g. Bhutta and Keys 2016, Beraja, Fuster, Hurst and Vavra 2018, Di Maggio, Kermani and Palmer 2020, Cloyne, Ferreira and Surico 2019). This channel is also active in our model, boosting demand in the short-run. However, this boost reverses as monetary stimulus fades and debt needs to be serviced, beginning to drag on demand. Due to the presence of indebted demand, this drag can cause a persistent shift in natural interest rates after temporary monetary policy interventions. It is for this reason that monetary policy has limited ammunition in the model: successive monetary policy interventions build up debt levels, thereby lowering natural rates. This forces policy rates to keep falling with them to avoid a recession, thus approaching the effective lower bound.

When savers command sufficient resources in our economy, for instance due to high income inequality and large debt levels, the natural rate in our economy can be persistently below its effective lower bound. At that point, our economy is in a debt-driven liquidity trap, or debt trap, which is a well-defined stable steady state of our economy.

Once inside the debt trap, conventional policies that are based on debt accumulation only work in the short run. Eventually, the economy is "pulled back" into the debt trap. Certain unconventional policies, however, can facilitate an escape from the debt trap. For example, redistributive tax policies, such as wealth taxes, or structural policies that are geared towards reducing income inequality generate a sustainable increase in demand, persistently raising natural interest rates away from their effective lower bound. One-time debt forgiveness policies can also lift the economy out of the debt trap, but need to be combined with other policies, such as macroprudential ones, to prevent a return to the debt trap over time.

The idea of indebted demand helps explain the predicament faced by the world's leading central bankers, especially the absence of interest rate normalization. For example, a recent Wall Street Journal article cites monetary authorities worldwide in asserting that "borrowing helped pull countries out of recession but made it harder for policy makers to raise rates." Mark Carney,

[^2]Governor of the Bank of England observed that "the sustainability of debt burdens depends on interest rates remaining low." Philip Lowe, Governor of the Reserve Bank of Australia has warned that "if interest rates were to rise . . . many consumers might have to severely curtail their spending to keep up their repayments. ${ }^{13}$ This paper formalizes these intuitions.

Literature. Our paper is part of a burgeoning literature on the causes of the recent fall in natural interest rates, referred to as "secular stagnation" by Summers (2014). Among the existing explanations are population aging (Eggertsson, Mehrotra and Robbins 2019), income risk and income inequality (Auclert and Rognlie 2018, Straub 2019), the global saving glut (Bernanke 2005, Coeurdacier, Guibaud and Jin 2015) and a shortage of safe assets (Caballero and Farhi 2017, Caballero, Farhi and Gourinchas 2017). ${ }^{4}$ Our theory suggests a new force for reduced natural interest rates, namely indebted demand. It can act both as an amplifier of existing explanations-as we demonstrate for rising income inequality-or give rise to new explanations, as we demonstrate for financial deregulation, which is commonly thought to be a force against low interest rates. ${ }^{5}$

The central element of our theory is the assumption of non-homothetic preferences, generating heterogeneous saving rates out of permanent income transfers. ${ }^{6}$ As we mentioned above, such heterogeneity was an important aspect of many early studies of (non-optimizing) consumption behavior. Among the more recent papers in this tradition are Stiglitz (1969), Von Schlicht (1975), and Bourguignon (1981), who study the implications of such behavior on inequality. The earliest models of optimal consumption behavior that we know of and that allow for such preferences are Strotz (1956), Koopmans (1960) and Uzawa (1968). More recently, Carroll (2000), De Nardi (2004), and Benhabib, Bisin and Luo (2019) argue that non-homothetic preferences are important to understand wealth inequality, and Straub (2019) studies their implications for a rise in income inequality.

Our implications for monetary policy are related to the debate around "leaning vs. cleaning" (Bernanke and Gertler 2001, Stein 2013, Svensson 2018) and to the nascent academic literature surrounding the idea that monetary policy might have limited ammunition. McKay and Wieland (2019) explore this idea in a model of durables spending, Caballero and Simsek (2019) in a model with asset price crashes.

The closest antecedents to our paper are Kumhof, Rancière and Winant (2015), Cairó and Sim (2018) and Rannenberg (2019). Kumhof, Rancière and Winant (2015) study a two-agent endowment economy, where savers are more patient than borrowers and savers have non-homothetic preferences. They find that a rise in income inequality leads to greater debt levels and a greater likelihood of a financial crisis due to endogenous default, but no change in long-run interest rates.

[^3]The driving force behind this result is the specific structure and heterogeneity of preferences. It generates a higher saving rate of savers out of labor income, compared to borrowers, but a lower saving rate out of financial income. This is why the model does not feature indebted demand: in fact, an increase in debt raises aggregate demand in the model and thus dampens the effects of income inequality. The model in Cairó and Sim (2018) builds on Kumhof, Rancière and Winant (2015) and studies implications for a richer set of shocks and for the conduct of monetary policy. The recent paper by Rannenberg (2019) also builds on Kumhof, Rancière and Winant (2015) but shows that income inequality can also reduce natural interest rates in addition to generating greater debt.

Finally, as a paper about household and government debt, it relates to a vast empirical and theoretical literature on the origins and consequences of high debt levels. Among the empirical papers, Schularick and Taylor (2012) document the well known "financial hockey stick" behavior of private debt; Mian and Sufi (2015), Jordà, Schularick and Taylor (2016), Mian, Sufi and Verner (2017) document that expansions in household debt predict weak future economic growth; Reinhart and Rogoff (2010) assess the consequences of large government debt. Among the theoretical papers, Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017) study the effects of debt deleveraging on the economy. Our model emphasizes that even without deleveraging, debt reduces aggregate demand. This aspect is shared with Illing, Ono and Schlegl (2018), who show that debt can lead to persistent stagnation in the context of insatiable preferences for money (Ono 1994).

Layout. Section II introduces the model, and Section III studies equilibrium in the model, introducing the concept of indebted demand. Section IV provides evidence to support the key feature of the model that long-run saving supply schedule for the rich are downward sloping. Section V examines how income inequality and financial deregulation affect debt levels and interest rates in the economy. Next, we study the implications of fiscal and monetary policy (Section VI), and what indebted demand means for an economy in a liquidity trap (Section VII). Section VIII provides two extensions, and Section IX offers the perspective of a richer model. Section X concludes.

## II Model

The model is a deterministic, infinite-horizon endowment economy, populated by two separate dynasties of agents trading debt contracts. Endowments can be thought of as dividends of real assets, or "Lucas trees", owned by the two dynasties. Each such asset produces one unit of the consumption good each instant. There are $Y$ real assets in total, where we normalize $Y=1$ for now.

The agents in the two dynasties share the same preferences and only differ by their endowments of the real asset. For reasons that will become clear below, we refer to the poorer ("nonrich") dynasty as borrowers $i=b$ and wealthier ("rich") dynasty as the savers $i=s$. At any point in
time, there is a mass $\mu^{b}=1-\mu$ of borrowers and a mass $\mu^{s}=\mu$ of savers. We sometimes simply refer to all dynasties of type $i$ as "agent" $i$.

The model is intentionally kept simple and tractable for now; several extensions can be found in Section VIII and Online Appendix B.

## II.A. Preferences

We begin by setting up the agents' common preferences. An agent in dynasty $i \in\{b, s\}$ dies at rate $\delta>0$ and discounts future utility at rate $\rho>0$. At any date $t$, total consumption by dynasty $i$ is $c_{t}^{i}$ and total wealth by dynasty $i$ is $a_{t}^{i}$. The average type- $i$ agent therefore consumes $c_{t}^{i} / \mu^{i}$ and owns wealth $a_{t}^{i} / \mu^{i}$, with a utility function given by ${ }^{7}$

$$
\begin{equation*}
\int_{0}^{\infty} e^{-(\rho+\delta) t}\left\{\log \left(c_{t}^{i} / \mu^{i}\right)+\frac{\delta}{\rho} v\left(a_{t}^{i} / \mu^{i}\right)\right\} d t \tag{1}
\end{equation*}
$$

Utility is derived from two components: each instant, utility over flow consumption per capita $c_{t}^{i} / \mu^{i}$; and, arriving at rate $\delta$, a warm-glow bequest motive captured by the function $v(a) / \rho$. We assume for now that upon death, the entire asset position of an agent is bequeathed to a single newborn offspring, ruling out any cross-dynasty mobility. ${ }^{8}$ The consolidated budget constraint of all agents of type $i$ is therefore simply given by

$$
\begin{equation*}
c_{t}^{i}+\dot{a}_{t}^{i} \leq r_{t} a_{t}^{i} \tag{2}
\end{equation*}
$$

where $r_{t}$ is the endogenous flow interest rate at date $t$.
The function $v(a)$ represents a crucial aspect of this model. It characterizes the relationship between wealth of a dynasty and its saving rate. To see this, consider the special case where $v(a)=\log a$. This choice of $v(a)$ makes the preferences in (1) homothetic: the borrower and saver dynasties would exhibit the exact same saving behavior, just scaled by their current wealth positions. ${ }^{9}$

This is no longer true as $v(a)$ deviates from $\log a$. To capture such deviations, we define $\eta^{i}(a)$ to be the marginal utility of $v$ relative to the marginal utility of log, that is,

$$
\begin{equation*}
\eta^{i}(a) \equiv a / \mu^{i} \cdot v^{\prime}\left(a / \mu^{i}\right) . \tag{3}
\end{equation*}
$$

$\eta^{i}(a)$ is defined in per-capita terms and therefore depends on $i . \eta^{i}(a)$ plays an important role in the analysis, especially $\eta^{s}(a)$ which henceforth we also denote by $\eta(a)$. When $\eta^{i}(a)$ is constant, for instance $\eta^{i}(a)=1$ when $v(a)=\log a$, utility is homothetic as marginal utility of bequests and marginal utility of consumption are proportional. When $\eta^{i}(a)$ is decreasing, the marginal utility of

[^4]bequeathing assets decreases relatively more quickly than the marginal utility of consumption; in this case, wealthier agents save relatively less. When $\eta^{i}(a)$ is increasing, marginal bequest utility decays more slowly than that of consumption, implying that wealthier agents have a stronger desire to save. As shown in Section IV below, the empirical evidence supports the non-homothetic case in which $\eta^{i}(a)$ is increasing. As a result, the development of the model in this section and in Section III emphasizes this non-homothetic case.

## II.B. Borrowing constraint

The two types of agents in the model maximize utility (1) subject to the budget constraint (2) and a borrowing constraint. To formulate the borrowing constraint, we separate type-i agents' wealth positions into two components: their real assets $h_{t}^{i}$ and their financial assets, which if negative, we refer to as debt $d_{t}^{i}$, that is,

$$
\begin{equation*}
a_{t}^{i}=h_{t}^{i}-d_{t}^{i} \tag{4}
\end{equation*}
$$

We assume for now that the agents' debt is adjustable-rate long-term debt which decays at some rate $\lambda>0$.

Agents of type $i$ own a fixed total endowment of $\omega^{i} \in(0,1)$ of real assets (trees), where $\omega^{s}+$ $\omega^{b}=1$. Within the endowment, we assume that $\ell^{i}<\omega^{i}$ are pledgeable real assets (e.g. land, houses, businesses, etc) and $\omega^{i}-\ell^{i}$ are non-pledgeable real assets (e.g. human capital). Denoting

$$
\begin{equation*}
p_{t} \equiv \int_{t}^{\infty} e^{-\int_{t}^{s} r_{u} d u} Y d s \tag{5}
\end{equation*}
$$

the price of a single real asset (tree), type-i agents' total wealth in real assets is

$$
\begin{equation*}
h_{t}^{i}=p_{t} \omega^{i} \tag{6}
\end{equation*}
$$

and type-i agents' pledgeable wealth is $p_{t} \ell^{i}$. Henceforth we assume that pledgeable wealth (per capita) is equal across agents, $\ell^{b} /(1-\mu)=\ell^{s} / \mu$, and denote $\ell \equiv \ell^{b}$, so that the only source of heterogeneity between the two agents are the endowments $\omega^{i}$, or equivalently, the agents' real-asset earning shares. We assume that savers' per capita earnings exceed those of borrowers, $\omega^{s} / \mu^{s}>\omega^{b} / \mu^{b}$.

We impose the borrowing constraint

$$
\begin{equation*}
\dot{d}_{t}^{i}+\lambda d_{t}^{i} \leq \lambda p_{t} \ell \tag{7}
\end{equation*}
$$

where, due to asset market clearing, $d_{t}^{s}+d_{t}^{b}=0 .{ }^{10}$ We henceforth focus exclusively on the borrowers' total debt position $d_{t} \equiv d_{t}^{b}$, the key state variable for our analysis. $d_{t}$ essentially captures how much borrowers have spent beyond earnings $\omega^{b} Y$ in the past, and how much of a debt burden borrowers need to service in the future.

[^5]According to borrowing constraint (7), new debt issuance $d_{t}^{i}+\lambda d_{t}^{i}$ is bounded above by the value of pledgeable assets. As we emphasize below, most of our results do not rely on the specific constraint (7). ${ }^{11}$

## II.C. Homothetic benchmark

Throughout the analysis, we compare the model to a homothetic benchmark model. This model is characterized by $\eta(a)=1$, so that agents' preferences are indeed homothetic. Moreover, to avoid a continuum of steady state equilibria in the homothetic model, we allow the saver's discount factor to be different from, and smaller than, the borrower's discount factor, $\rho^{s}<\rho$. Heterogeneity of discount factors is not assumed in the non-homothetic model.

## II.D. Equilibrium

We formally define equilibrium next.
Definition 1. Given initial debt $d_{0}=d_{0}^{b}$ a (competitive) equilibrium of the model are sequences $\left\{c_{t}^{i}, a_{t}^{i}, d_{t}^{i}, h_{t}^{i}, p_{t}, r_{t}\right\}$ such that both agents choose $\left\{c_{t}^{i}, a_{t}^{i}\right\}$ to maximize utility (1) subject to the budget constraint (2) and the borrowing constraint (7); $d_{t}^{i}$ is determined by (4); $h_{t}^{i}$ is determined by (6); $p_{t}$ is determined by (5); and financial markets clear at all times, that is, $d_{t}^{s}+d_{t}^{b}=0$. The goods market clears by Walras' law.

A steady state (equilibrium) is an equilibrium in which $c_{t}^{i}, a_{t}^{i}, d_{t}^{i}, h_{t}^{i}$ and $r_{t}$ are all constant.
A steady state with debt $d$ is stable if there exists an $\epsilon>0$ such that any equilibrium with initial $\operatorname{debt} d_{0} \in(d-\epsilon, d+\epsilon)$ has debt converge back to $d, d_{t} \rightarrow d$. All other steady states are unstable.

## II.E. Discussion

What does $\eta(a)$ capture? The literature has pointed out numerous examples of why agents might care about their wealth beyond its value for financing their own consumption behavior. This includes bequests (De Nardi 2004), out-of-pocket medical expenses in old age (De Nardi, French, Jones and Gooptu 2011), utility over status (Cole, Mailath and Postlewaite 1992, Corneo and Jeanne 1997), inter-vivos transfers (Straub 2019), and numerous other reasons that are documented in other papers in the literature (e.g. Carroll 2000, Dynan, Skinner and Zeldes 2004, Saez and Stantcheva 2018). Many of these examples are more salient or applicable to wealthier agents and can be captured in reduced form by assuming a specific shape $\eta(a)$. In addition to these examples, $\eta(a)$ could also capture the idea that assets other than a given stock of liquid assets or human capital are illiquid and therefore being saved "by holding" (Fagereng, Holm, Moll and Natvik 2019).

[^6]Observe that a standard altruistic saving motive would correspond to $\eta(a) \propto \log a$ and thus be equivalent to our homothetic benchmark model (cf footnote 9). ${ }^{12}$

Aggregate scale invariance. Our baseline non-homothetic model, with increasing $\eta(a)$, is not scaleinvariant in aggregate. If aggregate output $Y$ doubles, all agents are wealthier and thus, in line with a rising $\eta(a)$, would raise their savings by more than double. Taken at face value, this would generate rising saving rates in all growing economies, which seems counterfactual.

We believe that the key to understanding why a non-homothetic model, which breaks individual scale invariance, need not necessarily break aggregate scale invariance is that many of the motives for non-homothetic saving are relative to some economy-wide aggregates. For example, bequests are likely especially valued among the rich if they are large relative to the average wage or income in the economy, relative to the price of land, or relative to the average bequest. This suggests that $\eta(a)$ should really be thought of as a function of $a$ relative to $Y$ or aggregate wealth, i.e. $\eta(a / Y)$ or $\eta\left(a /\left(a^{b}+a^{s}\right)\right)$. To incorporate this idea and reduce clutter in the formulas, we henceforth assume that $\eta$ is of the form $\eta(a / Y)$ but output $Y$ is normalized to $1, Y=1$. We demonstrate in Online Appendix B. 5 that our results carry over to the case where $\eta$ is of the form $\eta\left(a /\left(a^{b}+a^{s}\right)\right)$, as in Corneo and Jeanne (1997).

Trading debt vs. trading assets. In the model, households trade debt contracts, rather than real assets. There are two simple reasons behind this assumption. First, in a deterministic model like ours, debt contracts and real assets are priced with the same rate of return, so trading one versus the other does not matter except for one-time revaluation effects. Second, debt contracts have been and continue to be a very important vehicle for saving and dissaving across the U.S. wealth distribution. This fact is shown in Mian, Straub and Sufi (2020), where saving and borrowing across the income and wealth distribution are explored for the United States between 1963 and 2016. The analysis shows that a substantial amount of borrowing by households in the bottom $90 \%$ of the wealth distribution was financed through the accumulation of financial assets by the top $1 \%$. Even though much of this debt was collateralized by housing, the bottom $90 \%$ did not actually accumulate additional housing assets while borrowing. ${ }^{13}$ In fact, Mian, Straub and Sufi (2020) show that the bottom $90 \%$ actually decumulated other assets, aggravating their dissaving.

Savers and borrowers. In the model, the rich agents are savers, and the non-rich agents are borrowers. While this is a simplifying assumption, it also fits with empirical evidence in the United States, as shown in Mian, Straub and Sufi (2020). In particular, from 1998 to 2016, individuals in the top $1 \%$ of the income or wealth distribution saved on average between 6 and 8 percentage points of national income annually depending on the methodology used. Individuals in the next $9 \%$ saved between 2 and 4 percentage points of national income. Estimates of savings by individuals in the bottom $90 \%$ range from $-4 \%$ to $0 \%$ of national income per year. So while the model

[^7]provides a simplified view of who in the economy saves and who borrows, this simplified view is not too far from reality.

Rate of return. The precise rate of return $r_{t}$ in the model is the expected return on the loans extended by savers to borrowers, which can be thought of as the expected return on consumer or home mortgage debt. More broadly, the rate of return should include both the expected return on household debt and the expected return on other financial assets that savers have been accumulating relative to non-savers since the 1980s. Mian, Straub and Sufi (2020) show that the non-rich in the United States have indeed boosted their borrowing from the rich significantly since the early 1980s. However, the non-rich have also been decumulating financial asset holdings relative to the rich, who have boosted their holdings of both household debt and other financial assets. As a result, $r_{t}$ in the model can also be thought of as the general return on wealth for households. As shown in Figure 1, real rates of return across asset classes have fallen substantially since the early 1980s.

## III Indebted Demand

We next characterize the equilibria in our model. We focus exclusively on equilibria in which debt is positive $d_{t}>0$, that is, the borrower actually borrows and the saver actually saves. ${ }^{14}$ Such equilibria always exist in our economy.

## III.A. Saving supply schedules

The saver's Euler equation is given by

$$
\begin{equation*}
\frac{\dot{c}_{t}^{s}}{c_{t}^{s}}=r_{t}-\rho-\delta+\delta \frac{c_{t}^{s}}{\rho a_{t}^{s}} \eta\left(a_{t}^{s}\right) . \tag{8}
\end{equation*}
$$

In a steady state, quantities and prices are constant, so that the budget constraint reads $c^{s}=$ $r a^{s}$. Substituting this into the Euler equation (8), we find our first key steady state equilibrium condition

$$
\begin{equation*}
r=\rho \cdot \frac{1+\delta / \rho}{1+\delta / \rho \cdot \eta\left(a^{s}\right)} \tag{9}
\end{equation*}
$$

This equation can be understood as a long-run saving supply schedule, describing the saving behavior of a possibly non-homothetic saver. Specifically, for each wealth position $a^{s}$, it describes the interest rate $r$ that is necessary for a saver to find it optimal to keep his wealth constant at $a^{s}$. Equation (9) can thus be thought of as an indifference condition. It is defined as the unique interest rate at which borrowers are indifferent between saving and dissaving. ${ }^{15}$

The crucial object that determines the shape of the saving supply schedule is the function $\eta(a)$, as illustrated in Figure 2. In the homothetic benchmark economy, where $\eta(a)$ is equal to 1 (or

[^8]Figure 2: Long-run saving supply schedules.

another constant), we recover the standard infinitely elastic long-run supply schedule, $r=\rho$. When $\eta(a)$ falls in $a$, in which case saving is treated as a necessity by agents, the saving supply schedule slopes up. Finally, and most importantly, when $\eta(a)$ rises in $a$ and thus saving is treated as a luxury, the saving supply schedule slopes down. This is the key property of our non-homothetic model. We summarize it in the following proposition.

Proposition 1. The long-run saving supply schedule (9) is downward sloping if and only if wealthier agents have a greater marginal propensity to save, that is, when $\eta(a)$ is increasing in a.

What is the intuition behind the negative slope? In a model in which wealthier agents save at higher rates, the higher an agent's wealth is, the lower must be the the return on wealth for the agent to be indifferent between saving and dissaving. ${ }^{16}$

## III.B. Steady state equilibria

Steady states are the intersections of saving supply schedules with debt demand curves, as we characterize in the following proposition.

Proposition 2. Any steady state with positive debt $d>0$ corresponds to an intersection of a long-run saving supply schedule

$$
\begin{equation*}
r=\rho \cdot \frac{1+\delta / \rho}{1+\delta / \rho \cdot \eta\left(\omega^{s} / r+d\right)} \tag{10}
\end{equation*}
$$

with a long-run debt demand curve

$$
\begin{equation*}
d=\frac{\ell}{r} . \tag{11}
\end{equation*}
$$

exist one in which savers borrow and borrowers save. With a realistic gap in $\omega^{i} / \mu^{i}$, this possibility vanishes.
${ }^{15}$ It is not necessarily like a conventional "supply curve", which typically describes the level of wealth $a^{s}$ savers tend towards for a fixed interest rate $r$.
${ }^{16}$ One may think that the individual saving dynamics displayed by the arrows in Figure 2 imply our economy is unstable. This is not the case, as we show below. Our model gives rise to a unique stable steady state. Moreover, note that the aggregate saving supply schedule can slope down in even as individual saving dynamics are stationary, as we illustrate in an extension in Online Appendix B.10.

Figure 3: Steady state equilibrium.


There is at most one such steady state.
Proposition 2 shows that the relevant saving supply schedule is that of the saver, and that the relevant debt demand curve is given by the borrowing constraint of the borrower. We write both conditions in terms of the interest rate (return on wealth) $r$ and debt $d$. Similar to models with discount rate heterogeneity, the borrower is up against the borrowing constraint in the steady state. In this formulation, the debt demand curve slopes down in $r$. The slope of the saving supply schedule depends on the slope of $\eta(a)$. If $\eta(a)$ is strictly increasing in $a$, then the saving supply schedule slopes downward.

We illustrate the saving supply schedule of the saver and the debt demand curve and their intersection in Figure 3. We prove in Online Appendix A there can only be a single intersection, and hence a single steady state, in the model. ${ }^{17}$

Steady state in the homothetic economy. In the homothetic economy, the interest rate in the unique steady state is necessarily pinned down by the saver's discount rate, $r=\rho^{s}$. The associated debt level is then $d=\ell / \rho^{s}$.

Analytical example. The steady state conditions in Proposition 2 can be solved analytically in a simple special case, where $\eta(a)$ is a linear function in the relevant region of the state space. For example, assuming $\eta(a)=a$, there is a unique stable steady state in this region, with interest rate

$$
r=\rho+\delta-\delta / \rho\left(\omega^{s}+\ell\right)
$$

and associated debt level

$$
d=\frac{\ell}{\rho+\delta-\delta / \rho\left(\omega^{s}+\ell\right)} .
$$

## III.C. Indebted demand

At the core of many of the results in this paper is the idea that an increase in debt service costs by some $d x$, e.g. caused by a greater level of debt $d a$ so that $d x=r d a$, may lower aggregate

[^9]demand. We next explore this idea starting at the steady state and in partial equilibrium, holding the interest rate $r$ fixed.

Proposition 3 (Indebted demand). Assume the economy is in its steady state and hold $r$ fixed. A permanent increase dx in debt service costs, or equivalently a permanent transfer from borrowers to savers, moves aggregate spending on impact by

$$
\begin{equation*}
d C=d c^{b}+d c^{s}=-\frac{\rho+\delta}{r} \frac{1}{2}\left(1-\sqrt{1-4\left(1-\frac{r}{\rho+\delta}\right) \frac{r}{\rho+\delta} \epsilon_{\eta}}\right) d x \tag{12}
\end{equation*}
$$

Here, $\epsilon_{\eta} \equiv \frac{\eta^{\prime}(a) a}{\eta(a)}$ is a measure of the degree of non-homotheticity in preferences. In particular, aggregate spending falls, $d C<0$, if and only if $\epsilon_{\eta}>0$.

Proposition 3 highlights that any increase in debt service costs weighs down on aggregate demand, $d C<0$, precisely if and only if $\epsilon_{\eta}>0$, a phenomenon we henceforth call indebted demand.

Why is demand indebted in this case? The increase in debt service costs $d x$ passes through to the borrower's spending one-for-one, $d c^{b}=-d x$. However, since savers have a greater saving propensity, their spending initially rises by less than the transfer, $d c^{s}<d x$. Thus, aggregate spending falls, $d C<0$. For the goods market to clear, the equilibrium interest rate must therefore fall. As this mechanism only relies on heterogeneity in saving propensities out of a small permanent transfer $d x$, any model that generates such heterogeneity along the wealth distribution exhibits the property of indebted demand. The model studied in this paper can be regarded as an example of such an economy.

The sign of $d C$ in Proposition 3 is directly related to the slope of the saving supply schedule in Figure 2. The indebted demand property holds, i.e. $d C$ is negative, precisely when savers are situated on a downward-sloping saving supply schedule. This is because, holding $r$ fixed, a marginal increase in wealth $d a$ corresponds to a permanent transfer of $d x=r d a$. When savers' consumption $d c^{s}$ responds to this transfer less than one for one, $d c^{s}<d x$, their saving must become positive, $d \dot{a}^{s}>0$. But this implies that the shift in wealth $d a$, without an offsetting shift in interest rates, must have moved savers above their saving supply schedule, into the region where wealth increases.

Therefore, a downward-sloping saving supply schedule is isomorphic to a marginal propensity to consume out of a permanent transfer of less than one. Indebted demand emerges if and only if the saving supply schedule slopes down. Given the critical role of the slope of the saving supply schedule in the model, Section IV below provides both microeconomic and macroeconomic evidence to support the plausibility of the idea that the the saving supply schedule is in fact downward sloping.

The homothetic model, despite its discount rate heterogeneity, has $\epsilon_{\eta}=0$ and thus does not generate indebted demand. The reason for this is that there is no heterogeneity in saving propensities out of a small permanent transfer $d x$ : borrowers do not save out of a small transfer as they
are hand-to-mouth; savers do not either as they smooth their consumption perfectly, with $r=\rho^{s}$.
As a side remark, observe that our non-homothetic model predicts a positive consumption response, $d C>0$, to a reduction in debt service payments, $d x<0$. Such a reduction could occur in reality when households refinance their mortgages to bring down the interest rate ("rate refi"). In homothetic models, as $\epsilon_{\eta}=0$, there is no effect of "rate refis" on aggregate consumption (Greenwald 2018), which quantitatively limits their macroeconomic relevance (Berger, Milbradt, Tourre and Vavra 2018). In non-homothetic models, such as ours, "rate refis" could instead have sizable consequences for aggregate consumption.

## III.D. Transitions

Having discussed the set of steady state equilibria in this economy, we now explore the entire set of equilibria, including the transitions along which the economy approaches the steady state.

The transitions follow along a system of ordinary differential equations (ODEs), with a single backward-looking state variable, debt $d_{t}$, and a single endogenous equilibrium price, the interest rate $r_{t}$. One can show that borrowers are always up against their borrowing constraint along the transition unless debt is below some threshold $\underline{d}$, which lies below the steady state debt position. Figure 4 illustrates the transitional dynamics in the interest rate - debt space. We next describe the equations characterizing these transitions, for simplicity for the case of a binding borrowing constraint (that is for $d_{t} \geq \underline{d}$ ).

Due to the binding borrowing constraint, debt evolves as in (7), that is

$$
\begin{equation*}
\dot{d}_{t}+\lambda d_{t}=\lambda p_{t} \ell . \tag{13a}
\end{equation*}
$$

Here, the price of real assets $p_{t}$, defined in (5), is the first forward-looking state variable which follows the ODE

$$
\begin{equation*}
\dot{p}_{t}=r_{t} p_{t}-1 \tag{13b}
\end{equation*}
$$

The second forward-looking state variable is the consumption of savers, which is determined by the Euler equation (8)

$$
\begin{equation*}
\frac{\dot{c}_{t}^{s}}{c_{t}^{s}}=r_{t}-\rho-\delta+\delta \frac{c_{t}^{s}}{\rho a_{t}^{s}} \eta\left(a_{t}^{s}\right) \tag{13c}
\end{equation*}
$$

where wealth of savers can be expressed as $a_{t}^{s}=\omega^{s} p_{t}+d_{t}$. Finally, the interest rate is pinned down by the budget constraint of savers (2), which can be cast as

$$
\begin{equation*}
c_{t}^{s}+\dot{d}_{t}=r_{t} d_{t}+\omega^{s} \tag{13d}
\end{equation*}
$$

Together, the four equations (13a)-(13d) jointly determine the evolution of the three state variables $\left(d_{t}, p_{t}, c_{t}^{s}\right)$ as well as the interest rate $r_{t} .{ }^{18}$ It turns out that this evolution is unique for any given

[^10]Figure 4: Equilibrium transitions in the baseline model.


Note. Red: saving supply schedule. Black: debt demand curve. Green: transitional dynamics.
initial level of debt $d_{0}>0$. We verified this using phase diagrams, confirmed it in our numerical simulations, and provide an analytical local uniqueness \& existence result in Online Appendix A.

If $d_{0}$ is to the left of the steady state (region I), the borrower levers up, hitting the borrowing constraint as soon as $d_{t}$ crosses $\underline{d}$, and ultimately converging to the steady state $d$. If $d_{0}$ is to the right of the steady state (region II), interest rates are pushed down relative to steady state, and the borrower has a desire to deleverage. The magnitude of the decline in interest rates in response to the accumulation of debt depends on the degree of non-homotheticity, as when there is more non-homotheticity, the saver spends less of the additional debt payments.

Observe that the black line in Figure 4 only corresponds to the borrowing constraint in steady state, $d=\ell / r$. Along the transition from the left, the expectation of lower interest rates in the future implies an asset price $p_{t}$ that lies above $\ell / r_{t}$. Thus, there can be points $\left(d_{t}, r_{t}\right)$ during the transition that lie to the right of the black line. The opposite happens during transitions from the right.

## III.E. Illustrative calibration of the basic model

We next provide an illustrative calibration of our model. The calibration is meant to capture the US economy in the 1980s, before the recent increase in income inequality. We interpret the saver as comprising the top $1 \%$ earning households of the economy, i.e. with a population share $\mu=0.01$, and the borrower as the bottom $99 \%$. We choose the saver's real (non-bond) earnings share $\omega^{s}$ to match the post-tax income share (excluding returns to household debt) to be consistent with the calibration of our richer model in Section IX, giving $\omega^{s}=0.06$. This ensures that the steady state distribution of income is the same as in our richer model.

We assume an initial interest rate of $5.5 \%$, consistent with an expected real return on wealth of $7.5 \%$ (see Figure 1 and discussion in Section II.E.) net of $2 \%$ productivity growth. We calibrate $\ell$ to match the US household debt to GDP level in 1980 of $45 \%$, giving $\ell=0.0248$. We choose $\delta=0.025$ corresponding to an expected duration of a generation of 40 years. ${ }^{19}$ The discount rate $\rho$, which approximately corresponds to the discount rate of borrowers as the bequest motive is

[^11]less relevant to them, is chosen at $10 \%$. Whenever we refer to the homothetic benchmark model, we use a discount rate of savers of $\rho^{s}=r=0.055$.

We directly calibrate $\eta(a)=v^{\prime}(a / \mu) a / \mu$, letting it take a flexible functional form,

$$
\begin{equation*}
\eta(a)=1+\frac{1}{\tilde{\eta} \tilde{a}} \log \left(1+e^{\tilde{\eta}(a-\tilde{a})}\right) \tag{14}
\end{equation*}
$$

where $\tilde{\eta}, \tilde{a}>0$. This form is arguably the simplest "activation function", with the following desirable properties: ${ }^{20}$ it is positive and strictly increasing everywhere; it is flat at 1 for low levels of assets $a$, implying near-homothetic behavior then; it rises linearly for large asset levels $a$, with slope $\tilde{a}^{-1}$; when $\tilde{a} \rightarrow \infty, \eta(a)$ remains flat for all $a$; the speed at which $\eta(a)$ moves from flat to linear is parametrized by $\tilde{\eta}$; its elasticity $\epsilon_{\eta}(a)=\frac{\eta^{\prime}(a) a}{\eta(a)}=1-\frac{-v^{\prime \prime}(a) a}{v^{\prime}(a)}$ always lies in ( 0,1$)$, consistent with $v(a)$ being a concave function. We jointly calibrate $\tilde{\eta}$ and $\tilde{a}$ to ensure that the steady state Euler equation (10) is satisfied and that savers have an MPC out of wealth of 0.01 in line with our discussion in the next section. ${ }^{21}$

The remaining parameter to be determined is $\lambda$, which is less important for our results as it only matters for the transitional dynamics. It governs the speed of the debt response. To calibrate it, we compare the impulse response of household debt over GDP to a monetary policy shock implied by our model to that commonly found to identified monetary policy shocks. In particular, we feed a 100 basis point interest rate cut with a half-life of 2 years (similar to (20) below) into the Section VI.B. variant of our model. We compare the household debt / GDP response at its peak (approximately 0.75 percentage points after 2 years) to the response of US household debt / GDP to a Romer and Romer (2004) shock. This procedure implies a $\lambda$ approximately equal to 0.5 .

## IV Evidence for a Downward Sloping Saving Supply Schedule

The slope of the long-run saving supply schedule is a crucial aspect of the model. This section provides both microeconomic and macroeconomic evidence supporting the plausibility of a downward saving supply schedule among individuals at the top of the income or wealth distribution.

## IV.A. Saving rates out of lifetime income

As shown in Figure 2, the saving supply schedule slopes downward in our model if and only if $\eta(a)$, which is the marginal utility of wealth relative to the homothetic benchmark, is strictly increasing in $a$ for savers in the economy. In the model, savers are those in the top of the permanent income distribution, which we interpret as the top $1 \%$. As mentioned in Section II.E., Mian, Straub and Sufi (2020) show that most of the savings in the U.S. economy come from those in the top $1 \%$ of the wealth or income distribution. The critical empirical question is whether $\eta(a)$ is strictly increasing in $a$ for those at the top of the distribution.

[^12]Empirical research measuring saving rates out of lifetime income can inform us on the slope of $\eta(a)$ for the rich. In the homothetic benchmark, saving rates are constant across the lifetime income distribution. In contrast, saving rates are rising in lifetime income if $\eta(a)$ is increasing in a. There is a long line of influential work that supports the view that saving rates out of lifetime income are higher for wealthy individuals. For example, this idea features prominently in the writings of John Atkinson Hobson, Eugen Bohm von Bawerk, Irving Fisher, and John Maynard Keynes, among others. More recently, formal empirical work has validated the notion that saving rates are highest at the top end of the lifetime income distribution.

Dynan, Skinner and Zeldes (2004) use panel data from the Survey of Consumer Finances (SCF) to show that individuals in the top $20 \%$ of the income distribution have saving rates out of lifetime income that are substantially larger than the rest of the population. The saving rates for the top $1 \%$ and top 5\% out of income are estimated to be particularly large, almost four times larger than at the median of the distribution ( 0.51 compared to 0.13 out of a dollar of permanent income, Table 4 , column (2)). ${ }^{22}$

Straub (2019) uses the Panel Study of Income Dynamics (PSID) to estimate an elasticity of consumption to lifetime income. If preferences were homothetic, then the elasticity of consumption with respect to lifetime income should be one, implying that changes in permanent income inequality do not impact aggregate consumption. However, the paper estimates that the elasticity of consumption with respect to permanent income is around 0.7 , which is evidence in favor of nonhomothetic preferences and a concave relationship between consumption and lifetime income.

The advantage of these two studies is that they seek to estimate the saving rate out of lifetime income, which is the main object of interest in determining the shape of $\eta(a)$. In addition, there also exists recent evidence from studies estimating saving rates out of income more generally.

Fagereng, Holm, Moll and Natvik (2019) use administrative panel data from Norway to estimate saving rates out of income across the wealth distribution. The study finds substantially higher saving rates for wealthier households, with saving rates for the top $1 \%$ estimated to be almost double the saving rates for the median of wealth distribution.

The empirical strategy of Fisher, Johnson, Smeeding and Thompson (2018) estimates a consumption share and after-tax income share of the top $1 \%$ of the income distribution of $s_{c}=0.066$ and $s_{y}=0.171$, respectively, for the 2004 to 2016 period. Together with an estimate of the average propensity to consume out of income APC in the aggregate, one can estimate the saving rate of the top $1 \%$ as $1-A P C \cdot s_{c} / s_{y}=0.649$. Here, the APC is measured as personal consumption expenditures divided by disposable personal income from the National Accounts. This same calculation implies a saving rate of -0.025 for the bottom $99 \%$ as a whole. The top $1 \%$ have a much higher saving rate than the bottom $99 \%$. In addition to showing a higher saving rate of the top $1 \%$, the SCF evidence also provides further support to the idea that most of the saving in the economy is done by the top $1 \%$.

[^13]IV.B. MPCs and the return on wealth

The slope of the saving supply schedule can also be discerned through a comparison of the observed marginal propensity to consume out of a change in wealth versus the expected return on wealth for the rich. More specifically, let $\mathcal{C}(r, a)$ be the steady state consumption of rich households in an economy. The definition of the saving supply schedule $r(a)$ as a function of rich households' wealth requires that

$$
\mathcal{C}(r(a), a)=r(a) a .
$$

Total differentiation of this equation with respect to $a$ allows us to isolate the local slope of the saving supply schedule

$$
\begin{equation*}
\frac{d r}{d \log a}=\frac{M P C^{\text {wealth }}-r}{1-\epsilon_{r}} \tag{15}
\end{equation*}
$$

where $\epsilon_{r} \equiv \frac{\partial \log \mathcal{C}}{\partial \log r}$ is the elasticity of consumption with respect to a permanent shift in interest rates, and MPC wealth is the marginal propensity to consume out of wealth for the rich. Given the preponderance of illiquid wealth among the rich, this ought to be interpreted as the MPC out of illiquid wealth or capital gains. The denominator of the right hand side is necessarily positive, and so the sign of $M P C^{\text {wealth }}-r$ for the rich gives us the local slope of the saving supply schedule. ${ }^{23}$

Recent studies using data from a number of European countries suggest that the MPC wealth of the rich is about $1.0 \%$. More specifically, Arrondel, Lamarche and Savignac (2019) estimate $M P C^{\text {wealth }}$ across the wealth distribution in France and find that the top $10 \%$ has an MPC wealth of $0.6 \%$. Garbinti, Lamarche, Lecanu and Savignac (2020) estimate MPC wealth for the top $10 \%$ of the wealth distribution across five European countries, and they find estimates of $0.3 \%$ for Cyprus, $0.6 \%$ for Germany, $0.8 \%$ for Spain, $1.2 \%$ for Belgium, and $2.3 \%$ for Italy. Using administrative data from Sweden, Di Maggio, Kermani and Palmer (2020) estimate an MPC wealth of $2.8 \%$ for the top $5 \%$ of the wealth distribution. The median and mean of these estimates across countries suggests an MPC wealth of about $1.0 \%$ for those in the top of the wealth distribution. ${ }^{24}$

How does this compare to $r$, or the expected return on wealth for the rich? As shown in Figure 1 , the expected real return on wealth for the U.S. economy as a whole has averaged about $5 \%$ from 1982 to 2016. It started out at around $7.5 \%$ and has fallen to about $3 \%$ in recent years. In order to apply it in (15), we need to subtract real GDP growth in the United States, which is about 2\%, as (15) was derived in a model without growth. Given these facts on expected returns in conjunction with the estimates of the $M P C^{\text {wealth }}$ for the top $10 \%$ above, the numerator $M P C^{\text {wealth }}-r$ is almost assuredly negative, thereby indicating a downward sloping saving supply schedule.

[^14]We can also use formula (15) to get a sense of magnitudes. To do so, we assume $\epsilon_{r}=0$, which would be implied by $\log$ preferences. ${ }^{25}$ Using an estimate of $1 \%$ for $M P C^{\text {wealth }}$, and $3 \%$ for $r$ net of GDP growth, the average for the United States from 1982 to 2016, we obtain:

$$
\frac{d r}{d \log a} \approx-2 \% .
$$

In words, this implies that if the richest households' wealth rises by $10 \%$, the interest rate has to come down by 20 basis points. While this is not a precise calculation, it gives a rough sense of the magnitudes that are at play in the model.
IV.C. Evidence using wealth to income ratios

Another implication of higher saving rates of the rich is a positive correlation between top income shares and wealth to income ratios. ${ }^{26}$ This implication is robustly supported by time series data in the United States, as shown in the left panel of Figure 5. The share of income earned by the top $1 \%$ of the income distribution is strongly positively correlated with the aggregate wealth to income ratio across years from 1913 to 2019.

One may be concerned, however, that other time series factors could have influenced both inequality and wealth to income ratios in the aggregate. To try to identify the effect of top income shares on wealth to income ratios more cleanly, the right panel of Figure 5 uses cross-sectional variation across states in the rise in the top $1 \%$ share from 1982 to $2007 .{ }^{27}$ As it shows, there is a strong positive correlation. States in which the top $1 \%$ earned a larger share of the state's total income over time also experienced larger wealth accumulation. While the figure only displays a correlation, Mian, Straub and Sufi (2020) show that this result is robust to variety of controls. ${ }^{28}$

[^15]Figure 5: Top Income Shares and Wealth to Income Ratios.


The left panel plots the aggregate wealth to income ratio against the share of income going to the top $1 \%$ for the United States from 1913 to 2019. Every third year is labeled. The data are from Saez and Zucman (2020). The left panel plots the change from 1982 to 2007 in the ratio of total household wealth to household income at the state level against the change in the top $1 \%$ share of income over the same period. For full details on data in the right panel, see Mian, Straub and Sufi (2020).

## IV.D. Comparing saving supply schedules across models

The evidence is supportive of the view that the saving supply schedule slopes downward for rich households. However, our model does not imply a downward sloping saving supply across the entire distribution. The borrowers in our model are on the upward part of their saving supply schedule as in other models used in the literature.

Perhaps the most prominent of models with an upward sloping saving supply schedule is Aiyagari (1994). In this model, there is a precautionary saving motive for households given the potential for hitting a borrowing constraint after negative idiosyncratic productivity shocks. In the Aiyagari (1994) model, a permanent transfer to households acts as additional insurance, cushioning the household in states of low realizations of idiosyncratic productivity. A household therefore responds to the transfer by raising consumption more than one for one, decumulating wealth. As a result, a permanent transfer leads to a lower saving rate.

While this logic is sound for households near a borrowing constraint, it is unlikely to be relevant for those near the top of the lifetime income distribution. Such households already have ample resources to buffer negative idiosyncratic shocks, and therefore it is unlikely that they will
have lower saving rates if they become richer. ${ }^{29}$
However, it is important to note that borrowers in our model, which we calibrate to correspond to the bottom $99 \%$ of the income distribution, are all on the upward-sloping part of the saving supply schedule, as in Aiyagari (1994). In fact, since debt is negative saving, their downward-sloping debt demand curve is the exact mirror image of their upward-sloping saving supply schedule. In our model, the upward slope stems from a borrowing constraint, but as we emphasize in Online Appendix B.6, many other formulations are possible, including a precautionary savings motive.

Borrowers are on the upward sloping part of the saving supply schedule while savers are on the downward sloping part. This is possible despite the fact that all agents share the same preferences. Each agent's saving supply schedule is first upward-sloping for low levels of wealth near the borrowing constraint, and then flattens out as wealth increases. Only if wealth is sufficiently high does it turn down again. Recall from Section II.E. that most of the saving in the U.S. economy over the past 20 years has come from those in the top $1 \%$, which supports the view that most saving in the United States is done by rich households that are likely to be on the downward sloping part of the saving supply schedule.

## V Inequality, Financial Deregulation, and Indebted Demand

The framework developed in the previous sections may help understand the underlying factors that contributed to the simultaneous increase in debt and decline in interest rates that many advanced economies have experienced in the past 40 years. We explore this next.

## V.A. Inequality

As is well understood by now, many advanced economies have experienced a significant rise in income inequality (Atkinson, Piketty and Saez 2011). In the model, a rise in income inequality can be captured as an increasing share $\omega^{s}$ of real earnings going to savers, and a corresponding fall in $\omega^{b}=1-\omega^{s} \cdot{ }^{30}$ The following proposition characterizes the long-run implications of rising income inequality.

Proposition 4. An increase in income inequality (greater $\omega^{s}$ ) unambiguously reduces long-run equilibrium interest rates and raises household debt. In the homothetic model, long-run interest rates and household debt are unaffected by rising income inequality.

The long-run implications of rising inequality are best understood in the context of our model's saving supply schedule and debt demand curve. Figure 6 shows supply and demand diagrams

[^16]Figure 6: The effects of rising income inequality for long-run saving supply and debt demand.

for the homothetic economy in panel (a), and the non-homothetic economy in panel (b). In the homothetic case, the supply schedule is pinned down by the discount factor and thus independent of inequality. The demand curve is also independent of inequality, and therefore the old and new steady states coincide.

In the non-homothetic economy, savers have a greater propensity to save. Thus, if they earn a greater share of income, total saving increases. This manifests itself in a shift of the saving supply schedule (10) to the left: for a given level of debt $d$, savers earn more resources and are willing to save more. As Proposition 4 shows, and as is illustrated in Figure 6, the equilibrium interest rate falls and the amount of debt in the economy rises in response to the rise in inequality. The non-homothetic model thus helps rationalize the close empirical association between the rise in inequality and the simultaneous increase in debt and decline in interest rates across advanced economies.

Transition. This is confirmed numerically in Figure 7, which simulates the responses of a homothetic and a non-homothetic economy to a permanent increase in income inequality. Since this is a perfect-foresight transition, borrowers begin raising their debt levels already early on, in anticipation of lower interest rates in the future, which raises demand and thus interest rates initially. ${ }^{31}$

Interestingly, the transition shows a hump-shaped profile in the debt service ratio, which ultimately falls back to its pre-transition value. This demonstrates that the debt service ratio is a highly endogenous object, which can be low either when there is little debt (early in the transition), or, when there is high debt but interest rates are low (late in the transition).

One reaction to the strong increase in debt in Figure 7 may be to point out that in the data, borrowers typically use debt to acquire assets (houses) and that their net worth actually remained more or less constant (Bartscher, Kuhn, Schularick and Steins 2020). Shouldn't this be reflected in the model?

It turns out that it already is. Clearly, most of the run-up in debt over the last few decades

[^17]Figure 7: Rising income inequality and debt.


Note. Plots show transitions from our calibrated steady state with saver income share $\omega^{s}=0.06$ (dotted gray line) to a steady state with $\omega^{s}=0.10$. The dashed blue line corresponds to the homothetic model with $\rho^{s}=0.055$.
is mortgage debt, and thus ultimately collateralized by housing. As we show in our companion paper, Mian, Straub and Sufi (2020), however, when taken together, the bottom $90 \%$ of the wealth distribution did not use the increase in debt to accumulate more housing. Instead, housing was bought and sold within the bottom $90 \%$, likely from old homeowners to young homebuyers, and thus ultimately financed consumption expenditure by old homeowners (Bartscher, Kuhn, Schularick and Steins 2020). Net worth only remained stable because house prices were rising.

At a stylized level, this is precisely the mechanism in our model. A natural measure of borrowers' financial net worth is their pledgeable wealth net of debt, $p_{t} \ell-d_{t}$, where $\ell$ can be interpreted as land or housing owned by borrowers. Figure 7 shows how borrowers' net worth evolves, and splits it up into its components, $p_{t} \ell$ and $d_{t}$. Similar to the data, net worth remains stable in the transition. Underlying the stability, however, are two opposing trends. On the one hand, pledgeable wealth increased tremendously, as asset prices $p_{t}$ rise; on the other, greater pledgeable wealth relaxes the borrowing constraint and thus leads to greater debt accumulation. ${ }^{32}$

[^18]Figure 8: Decomposing borrowers' net worth.


If net worth of borrowers did not change, why then is there indebted demand? Couldn't borrowers sell their assets, annihilate their debt and finance the same level of consumption as before? The answer is no. What matters for borrowers' consumption stream—and hence their contribution to aggregate demand-is not their net worth; instead it is their income stream after making debt payments. Valuation effects from lower discount rates and greater asset prices do not alter the income stream. Thus, indebted demand occurs when rich households save and non-rich households dissave; this may or may not coincide with a reduction in borrowers' net worth.

## V.B. Financial deregulation

Another widespread recent trend in advanced economies has been financial liberalization and deregulation. Especially the "mortgage finance revolution" of the 1970s and 1980s allowed new institutions to enter mortgage markets, led to securitization of mortgages and to a general loosening of borrowing constraints (Ball, 1990). For example, Bokhari, Torous and Wheaton (2013) document large increases in the fractions of mortgages originated with an LTV ratio above $90 \%$ and a debt-to-income ratio above $40 \%$ from 1986 to 1995. One tension in the literature noted by Justiniano, Primiceri and Tambalotti (2017) and Favilukis, Ludvigson and Van Nieuwerburgh (2017) is that in most standard models, a loosening of such borrowing constraints should be associated with an increase in interest rates. We next explore the effects of financial deregulation on debt and interest rates in the model developed here.

To do so, financial deregulation is modeled as an increase in the pledgability $\ell$ of real assets. ${ }^{33}$ We find the following result.

Proposition 5. Financial deregulation (greater $\ell$ ) unambiguously reduces long-run equilibrium interest rates and increases household debt. By contrast, in the homothetic model, long-run interest rates are unaffected by financial deregulation and household debt rises by less.

Kaplan, Mitman and Violante 2020) and relaxed collateral constraints. Both can be captured to some extent by shifts in $\ell$. We analyze such shifts in the next section.
${ }^{33}$ In our housing application in Section B.6.1 we show that a rising LTV ratio amounts to an increase in $\ell$.

Figure 9: The effects of financial deregulation for long-run saving supply and debt demand.


Figure 9 plots the implied shifts in the debt demand curve, as well as the qualitative transitional dynamics from the old steady state to the new one (green arrows). As can be seen, in both homothetic and non-homothetic models, the short-run saving supply schedule is upwardsloping: the loosening of borrowing constraints initially increases interest rates, as household demand grows in response. In the long run, the saving supply schedule is flat in the homothetic benchmark model, so that there is no long run effect of deregulation on interest rates.

In the non-homothetic model, by contrast, the increased debt burden ultimately leads to a fall in equilibrium interest rates, as the long-run saving supply schedule is downward-sloping. ${ }^{34}$ This then again contributes to increasing debt further. Interestingly, this resolves the puzzle faced in the literature: the model shows that financial deregulation might only put upward pressure on interest rates in the short run, and it actually contributes to a declining interest rate in the longrun. This is in line with a common narrative for the early and mid 2000s in the US (Summers 2015): despite sharp credit growth, the economy did not experience a booming economy.

One common argument in favor of financial deregulation is that it enables households to better smooth consumption over the life cycle and insure themselves against financial shocks. Our results in this section imply that this benefit has to be weighed against the potential cost of pushing overall debt levels higher and interest rates towards the zero lower bound. The cost of hitting the zero lower bound is an aggregate demand externality (see Section VII) and thus not internalized by individual borrowers (e.g. Korinek and Simsek 2016). A deeper discussion of the implications of this trade-off for optimal financial regulation is left for future research.

[^19]
## VI Implications for Fiscal and Monetary Policy

## VI.A. Fiscal policy: Deficits and redistribution

The previous section showed how private deficits lead to the accumulation of household debt, and thus indebted demand. A considerable portion of the recent increase in debt, however, has been public debt. According to conventional wisdom, a rise in government debt exerts upward pressure on interest rates (e.g. Blanchard 1985, Aiyagari and McGrattan 1998).

What are the implications of a rise in government debt in our non-homothetic model? This section focuses on this question in the context of the equilibrium introduced in Section II.D., in which output is fixed at $Y=1$, and therefore interest rates endogenously adjust to clear the goods market. Section VII revisits fiscal policy in the presence of nominal rigidities and a binding zerolower bound.

We consider fiscal policy in this section, as well as other policies in subsequent sections, mainly from a positive perspective, documenting its effects in our model without any notion of welfare. The reason for this choice is that there are several real-world considerations that are first-order for welfare but outside our model. For example, high debt levels and low interest rates are often associated with instability and risk-taking in the financial sector, and thus raise the likelihood of a financial crisis (e.g. Reinhart and Rogoff 2009, Schularick and Taylor 2012, Stein 2012). Low interest rates may also reduce growth (Liu, Mian and Sufi 2019). Behavioral aspects, such as timeinconsistent preferences, would lead borrowers to accumulate too much debt. One important dimension of welfare an extension of our model can speak to is the potential for a liquidity trap when the (natural) interest rate is sufficiently depressed. We discuss the welfare implications of our model in this context in Section VII.

Government. We introduce a standard government sector into the economy. Specifically, the government is assumed to choose a debt position $B_{t}$, government spending $G_{t}$, and proportional income taxes $\tau_{t}^{i}$ on agent $i$ such that its flow budget constraint

$$
\begin{equation*}
G_{t}+r_{t} B_{t} \leq \dot{B}_{t}+\tau_{t}^{s} \omega^{s}+\tau_{t}^{b} \omega^{b} \tag{16}
\end{equation*}
$$

is satisfied at all times $t$. Ponzi schemes are ruled out by assuming that $B_{t}$ is bounded above, uniformly in $t$. In the baseline model, government bonds pay the same interest rate as other assets. We discuss the implications for when this is not the case below.

For simplicity, government spending is treated here as purchases of goods that are either wasted, or-which is equivalent for the purposes of this current positive exercise-enter agents' utilities in an additively-separable form. Taxes are assumed to enter agents' real wealth in the natural way, $r_{t} h_{t}^{i}=\left(1-\tau_{t}^{i}\right) \omega^{i}+\dot{h}^{i}$. Taking fiscal policy as given, the definition of a competitive equilibrium is unchanged from before, with the exception that the bond market clearing condition is now given by $d_{t}^{b}+d_{t}^{s}+B_{t}=0$.

Figure 10: Long-run effect of an increase in public debt $B$.


Long-run effects of fiscal policy. We begin by studying the long-run effects of fiscal policy, focusing on constant policies $\left(G, B, \tau^{s}, \tau^{b}\right)$. In this case, the equilibrium conditions for steady state equilibria are given by

$$
\begin{gather*}
r=\rho \frac{1+\delta / \rho}{1+\delta / \rho \cdot \eta(a)}  \tag{17}\\
a=\left(1-\tau^{s}\right) \frac{\omega^{s}}{r}+\frac{\ell}{r}+B \tag{18}
\end{gather*}
$$

Equations (17) and (18) characterize the long-run implications of fiscal policy. We are specifically interested in increases in $B$, financed by raising taxes $\tau^{i}$ on both agents or cutting expenditure $G$; as well as tax-financed increases in $G$. This yields the following result.

Proposition 6 (Long-run effects of fiscal policy on interest rates and debt.). In the long run,
a) larger government debt ( $B \uparrow$ ) depresses the interest rate ( $r \downarrow$ ) and crowds in household debt ( $d \uparrow$ ).
b) tax-financed government spending ( $G \uparrow$ ) increases the interest rate ( $r \uparrow$ ) and crowds out household $\operatorname{debt}(d \downarrow$ ).
c) fiscal redistribution ( $\tau^{s} \uparrow, \tau^{b} \downarrow$ ) increases the interest rate ( $r \uparrow$ ) and crowds out household debt (d $\downarrow$ ).

With a homothetic saver, none of these policies have any effect on the long-run interest rate and on household debt.

An intuition for these results can be explained with the help of Figure 10. Consider the first policy in Proposition 6, and assume the greater debt level $B$ is entirely paid for by a reduction in government expenditure $G$. As savers do not raise their consumption one-for-one with the increase in debt service payments by the government, aggregate demand would fall were it not for a reduction in interest rates. Graphically, the policy corresponds to an increase in the economy's total demand for debt, $d+B$, which shifts out to the right. Notably, the reduction in interest rates will crowd in household debt.

When the greater level of debt is not paid for by government spending cuts, but instead by greater taxation, the result in Proposition 6 is qualitatively the same. However, the exact magnitude of the interest rate decline now depends on the distribution of taxation: in the corner case
where borrowers pay all additional taxes, the interest rate decline is as large as when government spending is cut; in the corner case where savers pay all additional taxes, interest rates do not respond. ${ }^{35}$

Tax-financed government spending and fiscal redistribution reallocate resources from the saver to a "spender", which is either the government-in the case of government spending-or the borrower-in the case of redistribution. Such resource reallocation would raise aggregate demand were it not for an increase in interest rates.

Proposition 6 and Figure 10 prescribe a very different role for fiscal policy in influencing interest rates than is typically assumed. What helps in the long-run is first and foremost redistribution between spenders and savers, not redistribution of taxes over time in the form of public deficits, which, paradoxically, lowers long-run interest rates even further as government demand becomes indebted.

One caveat to Proposition 6 is that the interest rate on government debt $B, r^{B}$, might differ form the interest rate $r$ on other debt $d$. Here, the result depends on why $r^{B}$ is different from $r$ and, crucially, whether $r^{B}$ is above or below the growth rate, which is zero in the baseline model. For example, it is conceivably that the first result in Proposition 6 breaks when the interest rate on government debt $r^{B}$ lies sufficiently below zero. We leave a characterization of the interaction of convenience yields on government debt and indebted demand for future research.

Fiscal policy in the analytical example. We can illustrate the effects of fiscal policy in the analytical example in Section III.B.. It is straightforward to obtain the steady state given a set of tax policies ( $G, B, \tau^{s}, \tau^{b}$ )

$$
r=\frac{\rho+\delta-\frac{\delta}{\rho}\left(\left(1-\tau^{s}\right) \omega^{s}+\ell\right)}{1+\frac{\delta}{\rho} B} \quad \text { and } \quad d=\frac{\left(1+\frac{\delta}{\rho} B\right) \ell}{\rho+\delta-\frac{\delta}{\rho}\left(\left(1-\tau^{s}\right) \omega^{s}+\ell\right)} .
$$

$G$ and $\tau^{b}$ do not enter the expressions, as they are implicitly used to balance the government budget for any values of $B, \tau^{s}$; it does not matter which of the two is used. We see that greater redistribution and greater spending (both financed through greater $\tau^{s}$ ) raises $r$ and lowers $d$. Greater public debt $B$ lowers $r$ and crowds in $d$. Finally, greater $B$ reduces the sensitivity of $r$ to changes in $\tau^{s}, \omega^{s}$ or $\ell$.

Short run effects. Despite its novel long-run effects of government debt, the model predicts conventional short-run effects of debt-financed fiscal stimulus programs (whether through government spending or tax cuts). As before, in terms of saving supply schedule and debt demand curve, this is due to an upward-sloping short-run saving supply schedule. We illustrate this in

[^20]Figure 11: Deficit spending.


Note. Plot shows response of non-homothetic economy to temporary government spending shock (AR(1) spending path with $g_{0}=5.5 \%$ of GDP and a half-life of 2 years). Fiscal rule: $\tau^{i}=r_{\infty} B$ where $r_{\infty}$ is eventual steady state interest rate.

Figure 11, which plots the dynamic response of the economy to temporary deficit-financed government spending. There is a short-run rise in the natural interest rate, lasting about as long as the fiscal stimulus itself. During this time, household debt is crowded out by higher interest rates. Afterward, however, the interest rate declines, falling below its original level and allowing debt to increase. The opposite of the dynamics in Figure 11 would materialize in response to an austerity program, causing a short-term reduction in the natural rate but raising natural rates in the longer term.

In practice, this suggests a dilemma for economies that are currently stuck in a steady state with low interest rates and high public debt, but, for some reason outside our model, wish to raise rates going forward. If they expanded public debt even further, rates would rise in the short run, but subsequently fall again, even below their already undesirable previous levels. If they contracted public debt, rates will fall in the short-run-possibly below the effective lower bound, causing a recession-despite the prospect of greater rates in the long run.

## VI.B. Monetary policy: Limited ammunition

In the previous section, we saw that deficit-financed fiscal stimulus reduces natural interest rates in our model in the long run. We next argue that monetary stimulus also moves natural interest rates, possibly persistently so.

Extending the model. To do this, it is necessary to move away from an endowment economy, where output $Y$ is fixed at 1 and interest rates endogenously adjust to clear goods markets. Instead, we now let actual output, henceforth denoted by $\hat{Y}_{t}$, adjust endogenously in response to monetary policy and differ from potential output $Y_{t}$. In the interest of space, we only explain here the main ingredients of the model, as they closely follow the existing literature, e.g. Werning (2015) and

Auclert, Rognlie and Straub (2018). We describe the full model extension in great detail in Online Appendix B.7.

To allow output $\hat{Y}_{t}$ to be endogenous, we assume it is produced using efficiency units of labor $N_{t}, \hat{Y}_{t}=N_{t}$, which are supplied by both types of agents. We assume that dynasty $i$ has labor productivity $\omega^{i}$ and supplies hours $n_{t}^{i}$, such that total labor units are $N_{t}=\omega^{b} n_{t}^{b}+\omega^{s} n_{t}^{s}$. We modify dynasty $i$ 's utility function to include a standard additively separable disutility from labor supply. Dynasty $i^{\prime}$ s total real asset wealth $h_{t}^{i}$ is now given by its human capital, $h_{t}^{i}=\int_{t}^{\infty} e^{-\int_{t}^{s} r_{u} d u} \omega^{i} n_{s}^{i} d s$, so that dynasty $i$ 's budget constraint simply becomes

$$
c_{t}^{i}+r_{t} d_{t}^{i} \leq d_{t}^{i}+\omega^{i} n_{t}^{i} .
$$

We follow Werning (2015) and Auclert, Rognlie and Straub (2018) in assuming that prices are flexible, nominal wages are perfectly rigid, and that aggregate labor demand $N_{t}$ is allocated across agents using a simple uniform allocation rule, namely $n_{t}^{i}=N_{t} .{ }^{36}$ This implies that real earnings by type $i$ are $\omega^{i} \hat{Y}_{t}$, and thus that the income distribution is unaffected by the level of aggregate output $\hat{Y}_{t}$. Moreover, as we show in Online Appendix B.7, the disutility can be specified such that the allocation with $\hat{Y}_{t}=N_{t}=n_{t}^{i}=1$ is the natural allocation. We continue to denote potential output by $Y_{t}=1$.

To ensure continued tractability of the model, we treat monetary policy as controlling the real rate directly, as in Werning (2015), McKay, Nakamura and Steinsson (2016) and Auclert, Rognlie and Straub (2020). ${ }^{37}$ Transmission of monetary policy then works in the standard way: monetary policy changes the interest rate, which steers aggregate demand and thus output (and labor) in the economy. With exogenous interest rates $\left\{r_{t}\right\}$, the goods market now clears because $\left\{\hat{Y}_{t}\right\}$ is endogenous. We define as natural interest rates the sequence of real interest rates $\left\{r_{t}^{n}\right\}$ that achieves the natural allocation, that is, it implements a path of aggregate demand at potential, $\hat{Y}_{t}=Y=1$. Given the assumptions above, this model is "backwards compatible", in that its natural allocation precisely corresponds to our baseline model from Section II.

Monetary policy shocks. We consider two types of monetary policy shocks, which hit the economy at a stable steady state $\left(c^{b}, c^{s}, r, d\right)$. The first type is a $T$-period long interest rate reduction, before a reversal back to the original interest rate

$$
r_{t}=\left\{\begin{array}{ll}
\hat{r} & t \leq T  \tag{19}\\
r & t>T
\end{array} .\right.
$$

[^21]The second type also starts with a $T$-period long interest rate reduction, but then reverses back to the path of natural interest rates

$$
r_{t}= \begin{cases}\hat{r} & t \leq T  \tag{20}\\ r_{t}^{n} & t>T\end{cases}
$$

ensuring that for any $t>T$ after the intervention $\hat{Y}_{t}=Y=1$ in this case.

Monetary policy and debt. We begin by studying monetary policy shocks of the first kind. In our model, they stimulate the economy via two separate channels. First, they relax borrowing constraints and encourage borrowers to use additional household debt for spending (debt channel). Second, through income and substitution effects, they provide incentives for savers to spend more (saver channel). To study the role of these channels for monetary transmission, we define the following present values

$$
\begin{aligned}
& P V^{\tau}\left(\left\{c_{t}^{i}\right\}\right)=\int_{0}^{\tau} e^{-\int_{0}^{t} r_{s} d s} c_{t}^{i} d t-\int_{0}^{\tau} e^{-r t} c^{i} d t \\
& P V^{\tau}\left(\left\{\hat{Y}_{t}\right\}\right)=\int_{0}^{\tau} e^{-\int_{0}^{t} r_{s} d s} \hat{Y}_{t} d t-\int_{0}^{\tau} e^{-r t} Y d t
\end{aligned}
$$

The first is the increase in the present value of agent $i^{\prime}$ s spending until period $\tau$; the second is the increase in the present value of output until period $\tau$. The next proposition shows that the two channels have asymmetric implications for the path of aggregate demand.

Proposition 7. The $\tau$-period present value of the output response to the monetary policy shock (19) is given by

$$
\begin{equation*}
P V^{\tau}\left(\left\{\hat{Y}_{t}\right\}\right)=\underbrace{\frac{1}{\omega^{s}} P V^{\tau}\left(\left\{c_{t}^{s}\right\}\right)}_{\text {saver channel }}+\underbrace{\frac{1}{\omega^{s}} e^{-\int_{0}^{\tau} r_{s} d s}\left(d_{\tau}-d\right)}_{\text {debt channel }} . \tag{21}
\end{equation*}
$$

In the long run $(\tau=\infty)$, the present value of output is entirely determined by the saver channel,

$$
\begin{equation*}
P V^{\infty}\left(\left\{\hat{Y}_{t}\right\}\right)=\frac{1}{\omega^{s}} P V^{\infty}\left(\left\{c_{t}^{s}\right\}\right) \tag{22}
\end{equation*}
$$

This implies that any output stimulus generated by debt accumulation necessarily weighs negatively on output going forward.

Proposition 7 shows that the two channels of monetary transmission have vastly different implications for the path of output. While the saver's consumption response to the interest rate change affects output permanently, the debt channel only has a temporary effect. In fact, as any additional debt taken out by borrowers eventually has to be serviced or even repaid, future demand is reduced by an active debt channel. Put differently, when monetary policy is used to stimulate the economy, any resulting increase in demand that is debt-financed does not sustainably raise demand and will contribute to reduced demand in the future.

One implication of the result in Proposition 7 is that if monetary policy is accommodative now,

Figure 12: Debt limits the ammunition of monetary policy.

it endogenously limits its room to be accommodative in the future, as it also needs to ensure that the accumulated debt burden from past interventions does not cause a shortfall in demand. The accurate object summarizing the "room to be accommodative in the future" is the natural rate of interest $r_{t}^{n}$. Our next proposition studies the effect that monetary policy has on $r_{t}^{n}$.

Proposition 8. To first order, a monetary policy shock as in (19) or (20) causes debt to rise and the natural rate to fall, $r_{t}^{n}<r$ for any $t$. For a given increase in debt, the natural rate falls by more (as measured by $\int_{s}^{\infty} e^{-r(t-s)} r_{t}^{n} d t$ for any s) if there is more non-homotheticity (as measured by the elasticity $\epsilon_{\eta}$ of $\eta$ ); and if interest rates are lower (lower $\hat{r}$ ) for longer (larger $T$ ).

Accommodative monetary policy systematically reduces natural interest rates in our model, and thus endogenously limits the "ammunition" that is available to monetary policy in the future, before economy approaches the effective lower bound. This happens because in the presence of a greater debt burden, natural rates $r_{t}^{n}$ cannot possibly be equal to $r$ after $t=T$ as this would tighten borrowing constraints, and lead to the borrower severely contracting demand. Therefore, natural rates $r_{t}^{n}$ are below $r$ at least for some time after $t=T$ while the borrower deleverages (see Figure 12).

This logic operates even absent non-homotheticity. However, in a homothetic model, the convergence process $r_{t}^{n} \rightarrow r$ is sped up significantly by the fact that the saver's consumption rises significantly due to the increase in the saver's permanent income, pushing the natural rate up, closer to $r$. In a non-homothetic model such as ours, an additional reason for a decline in $r_{t}^{n}$ emerges-indebted demand-which leads to lower natural rates and a significantly reduced convergence rate back to $r$. In other words, non-homotheticity and indebted demand significantly aggravate the "limited ammunition" property of monetary policy (see Figure 12). This can be sufficiently strong to permanently lower natural rates. Such behavior occurs when the economy exhibits multiple steady states (see our discussion in Section III.B.), and the monetary intervention is "too low for too long", that is, $\hat{r}$ is sufficiently low and $T$ sufficiently long.

Relationship to literature. A number of economists have recently emphasized how the effectiveness of monetary policy interventions can be reduced by past interventions (e.g. Eichenbaum, Rebelo and Wong 2019, Berger, Milbradt, Tourre and Vavra 2018). In this paper, we do not consider consecutive interventions. Instead, we focus on how much "ammunition" in terms of the natural interest rate, a single intervention costs. This aspect of our paper is closest to McKay and Wieland (2019). To give an analogy with the IS curve, we focus on the effect of monetary policy on the future level of the IS curve as opposed to its slope. ${ }^{38}$

Practical implications for the conduct of monetary policy. Monetary policy can have long-lasting effects on natural rates through the accumulation of debt. This should be taken into account when contemplating the force with which to respond to different kinds macroeconomic shocks. Temporary shocks to borrowers' ability to borrow for instance-e.g. during a financial crisis-can be met with aggressive monetary easing as debt is unlikely to rise in this context. However, when reacting to shocks that do not directly affect borrowers' demand for debt-e.g. negative shocks to business investment as during the 2001 recession-aggressive monetary policy could lead to significant and persistent increases in household debt, and therefore reduce monetary policy ammunition going forward.

When used in conjunction with macroprudential policies that are designed to keep debt in check, thereby dampening the debt channel, monetary policy can be used more aggressively. That way, the economy does not merely "pull forward" demand through debt, demand which it then lacks in the future.

## VII Debt Trap and Policies to Escape It

The most serious implications of indebted demand occur when it tips the economy into a liquidity trap. We next discuss ways in which an economy can slide into a liquidity trap, and evaluate policy options that may help the economy recover.

To do so, we focus on the case of our model where the interest rate associated with our model's steady state is possibly below its effective lower bound (ELB). Let $\underline{r}>0$ denote that (real) effective lower bound. It needs to be positive as we take $r$ to be the real return on wealth in our model. To get a number for $\underline{r}$, we propose to take an estimate of the real return on wealth during the ELB period-e.g. around $3.5 \%$ in Figure (1)—which then needs to be de-trended by productivity growth during the ZLB episode-e.g. $1.5 \%$-to obtain $\underline{r}$. In this example, $\underline{r}=2 \% .{ }^{39}$

We next study a situation where the steady state natural interest rate $r$ lies below $\underline{r}$, so that monetary policy cannot achieve full employment in steady state.

[^22]Figure 13: Falling into the liquidity trap in response to greater income inequality.


Note. These plots simulate an increase in income inequality from $\omega^{s}=0.10$ to $\omega^{s}=0.11$ in the non-homothetic economy. The black line assumes a lower bound of $\underline{r}=2 \%$, in line with the real return on wealth during the US ELB period ( $3.5 \%$ in Figure 1 net of $1.5 \%$ growth).

## VII.A. The debt trap steady state

We focus on a steady state $(r, d)$ with a natural interest rate $r$ below the effective lower bound, $r<\underline{r}$, that is,

$$
\begin{equation*}
\underline{r}<\rho \frac{1+\delta / \rho}{1+\delta / \rho \eta\left(\omega^{s} / \underline{r}+d\right)} \tag{23}
\end{equation*}
$$

In this case, the economy gives rise to a stable liquidity trap steady state, which we henceforth also call debt trap due to its association with high levels of debt.

Proposition 9. In the presence of an effective lower bound with (23), there exists a stable liquidity trap steady state ("debt trap"), in which output is reduced to

$$
\begin{equation*}
\hat{Y}=Y \frac{\underline{r}}{\omega^{s}+\ell} \cdot \eta^{-1}\left(\frac{\rho}{\underline{r}}(1+\rho / \delta)-\rho / \delta\right)<Y \tag{24}
\end{equation*}
$$

In the debt trap, household debt is high and output is permanently reduced due to indebted demand. The reduction in output in (24) is larger the greater income inequality is (greater $\omega^{s}$ ) and the higher the effective lower bound $\underline{r}$ is. Moreover, in our model, household debt is the key endogenous state variable that determines whether or not an economy is able to generate sufficient demand to avoid a liquidity trap. This implies that more household debt (greater $\ell$ ) makes the trap more likely, and the output reduction greater. Any force or policy that boosts household debt in the debt trap will push output even lower in the long run.

It also means that the economy can slide into the trap over time, as debt levels increase (see Figure 13). Interestingly, the prospect of falling into debt trap itself accelerates its arrival. The reason for this is that both agents anticipate a recession in the debt trap, and thus, in an attempt to smooth consumption, cut back on their spending already in advance. This, however, only pushes down the natural rate further, closer to the effective lower bound. We illustrate this in Figure 13
by plotting the transition path without imposing an ELB, which crosses the ELB later (blue dashed line).

The presence of debt as endogenous state variable sets our model apart from several prominent recent papers modeling secular stagnation, e.g. Benigno and Fornaro (2018), Caballero and Farhi (2017), Eggertsson, Mehrotra and Robbins (2019), and Ravn and Sterk (2020). Moreover, the liquidity trap here is indeed a trap, meaning associated with a stable steady state, rather than a relatively brief episode driven by household deleveraging, as in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017).

## VII.B. Potential policies to escape the trap

Fiscal policy in the debt trap. Once an economy finds itself in the debt trap, how does it get back out again? We can formally introduce the government exactly as in Section VI.A. and obtain the following result. ${ }^{40}$

Proposition 10. With proportional income taxes $\tau^{s}, \tau^{b}$ on the saver and borrower, output in the debt trap steady state is given by

$$
\begin{equation*}
\hat{Y}=Y \frac{\underline{r}}{\left(1-\tau^{s}\right) \omega^{s}+\ell} \cdot \eta^{-1}\left(\frac{\rho}{\underline{r}}(1+\rho / \delta)-\frac{\rho}{\delta}\right) \tag{25}
\end{equation*}
$$

In particular, greater redistribution through taxes raises output and greater tax-financed government spending raises output.

This result is the mirror image of Proposition 6, except that at the effective lower bound, adjustments in the natural rate correspond to adjustments in aggregate demand and output. In particular, Proposition 10 suggests that redistributive tax policies-greater $\tau^{s}$ —can raise output $\hat{Y}$, reducing the severity of the liquidity trap. What would be the consequences of such a policy for the two agents? As long as the effective lower bound still binds, steady state consumption and wealth are given by

$$
\begin{equation*}
c^{s}=\underline{r} \eta^{-1}\left(\frac{\rho}{\underline{r}}(1+\rho / \delta)-\frac{\rho}{\delta}\right) Y, \quad a^{s}=\frac{c^{s}}{\underline{r}}, \quad c^{b}=\hat{Y}-c^{s} . \tag{26}
\end{equation*}
$$

The only object that is endogenous to the tax choice is $\hat{Y}$. Remarkably, greater redistribution therefore leaves the steady state consumption and wealth of savers entirely unaffected, while boosting the consumption of borrowers. The reason for this result is that the income loss of greater taxation of savers' incomes is exactly offset by rising overall incomes. This can happen in the liquidity trap due to aggregate demand externalities, as in Korinek and Simsek (2016) and Farhi and Werning (2016).

[^23]What are the implications for welfare? While (26) only holds across steady states, observe that any policy change that raises $\hat{Y}$ also relaxes the borrowing constraint and thus generates additional consumption for borrowers during the transition period. Moreover, one can show that consumption and wealth of savers are constant at the levels in (26) throughout the transition. Thus, our model implies that, in the liquidity trap, greater redistribution is Pareto-improving.

Wealth taxes. Our framework provides an interesting and novel perspective to the current debate (as of January 2021) surrounding wealth taxes (e.g. Saez and Zucman 2019, Guvenen, Kambourov, Kuruscu, Ocampo-Diaz and Chen 2019, Sarin, Summers and Kupferberg 2020). A (progressive) wealth tax $\tau^{a}$ in our model taxes the saver's wealth $a^{s}=d+\frac{\omega^{s}}{r}+B$ each period, that is, the savers' consolidated budget constraint (2) becomes

$$
\begin{equation*}
c_{t}^{s}+\dot{a}_{t}^{s}=\left(r_{t}-\tau^{a}\right) a_{t}^{s} \tag{27}
\end{equation*}
$$

The returns to this policy are rebated to borrowers. As can be seen in (27), the wealth tax effectively reduces the after-tax return on wealth realized by savers. This changes their steady state saving supply schedule (9) to

$$
r-\tau^{a}=\rho \cdot \frac{1+\delta / \rho}{1+\delta / \rho \cdot \eta\left(a^{s}\right)}
$$

The relevant interest rate for savers' saving behavior is the after-tax return on wealth $r-\tau^{a}$. Thus, the presence of $\tau^{a}$ effectively relaxes the effective lower bound, raising output (25) during the liquidity trap according to

$$
\hat{Y}=Y \frac{\underline{r}-\tau^{a}}{\left(1-\tau^{s}\right) \omega^{s}+\ell(\underline{r})} \cdot \eta^{-1}\left(\frac{\rho}{\underline{r}-\tau^{a}}(1+\rho / \delta)-\frac{\rho}{\delta}\right) .
$$

Thus, a progressive wealth tax is successful in mitigating the impact of secular stagnation in our framework. ${ }^{41}$

Macroprudential policies in the debt trap. A commonly prescribed remedy for economies with high debt burdens is macroprudential policy designed to bring down debt. Being the opposite of financial deregulation, we think of such a policy as a reduction in $\ell$. Equation (24) immediately implies that such a policy raises demand and output $\hat{Y}$ in the long run, mitigating the recession. However, during the period of deleveraging, the economy goes through a significant short-run bust. This emphasizes that debt is best reduced by reducing saving supply rather than the demand for debt.

Debt jubilees. An alternative way to deal with a liquidity trap caused by high levels of debt is a debt jubilee. We define a debt jubilee of size $\Delta>0$ as an immediate reduction in both private debt $d$ and saver's assets $a^{s}$ by $\Delta$. The following result lays out the long-run implications of a debt jubilee

[^24]policy:
Proposition 11 (Bailouts and debt jubilee.). A debt jubilee raises output in the short run, but unless combined with structural changes to inequality, redistribution or debt limits, there is no change to the economy in the long-run.

A debt jubilee amounts to a sudden reduction in the state variable $d_{t}$. With a unique stable steady state, this cannot have a long-run effect. In other words, borrowers would get themselves into debt again. However, a debt jubilee has positive short run effects, as it lifts the economy temporarily out of the debt trap. To prevent it from falling into it again, it can be paired with other policies, e.g. redistributive or macroprudential policies (see above). Such a combination can jointly address short and long-run issues.

## VIII Two Extensions

Our baseline model was intentionally kept simple. We now present two extensions to the baseline model in Section II, beginning with investment. ${ }^{42}$

## VIII.A. Role of Investment

To introduce investment, we allow output $Y$ to be produced from three factors, capital $K$ and both types of agents' labor supply $n^{i}$, and we write the net-of-depreciation production function as ${ }^{43}$

$$
Y=F\left(K, n^{b}, n^{s}\right) .
$$

Dynasty $i$ is assumed to have a labor endowment of $n^{i}=1$ whose factor income is capitalized in its real asset wealth, or human capital, $h_{t}^{i} \cdot{ }^{44}$ We assume without loss of generality that $Y$ has constant returns to scale; otherwise we include a fixed factor owned by savers and/or borrowers. Thus, total income $Y$ can be split up into income going to savers and income going to borrowers. The income shares only depend on the level of capital $K$, which itself is pinned down by the interest rate,

$$
F_{K}=r .
$$

As only savers hold capital in our economy, the agents' income shares are then described by the following functions of the interest rate

$$
\omega^{s}(r)=\frac{F_{K} K}{F}+\frac{F_{n^{s}} n^{s}}{F} \quad \text { and } \quad \omega^{b}(r)=\frac{F_{n^{b}} n^{b}}{F}=1-\omega^{s}(r) .
$$

[^25]Figure 14: Capital and indebted demand.


With these income shares, we can now characterize our economy's steady states as

$$
\begin{equation*}
r=\rho \cdot \frac{1+\delta / \rho}{1+\delta / \rho \cdot \eta\left(\omega^{s}(r) / r+d\right)} \quad \text { and } \quad d=\frac{\ell}{r} . \tag{28}
\end{equation*}
$$

Crucially, $\omega^{s}(r)$ is now possibly a function of $r$. As we demonstrate in the following proposition, the shape of $\omega^{s}(r)$ depends on the (Allen) elasticity of substitution $\sigma \equiv F_{n^{b}} F_{K} /\left(F_{K n^{b}} F\right)$ between capital and borrowers' labor supply.

Proposition 12. Let $\sigma$ be the elasticity of substitution between capital and borrowers' labor supply in the economy with investment. Denote by $\omega^{s}(r)$ the savers' income share.
a) If $\sigma=1: \omega^{s}(r)$ is independent of the interest rate $r$. The steady state is identical to the one in the economy without investment.
b) If $\sigma<1$ : $\omega^{s}(r)$ falls with lower $r$. This flattens the saving supply schedule.
c) If $\sigma>1$ : $\omega^{s}(r)$ rises with lower $r$. This steepens in the saving supply schedule.

Proposition 12 precisely characterizes the role of investment for the long-run economy. When an increase in capital leaves the income distribution unchanged, investment will have no effect in the long-run (case (a)). The extent to which capital matters depends on whether it crowds the share of income going to borrowers in or out. If capital is complementary to borrowers' labor supply, it reduces the extent of indebted demand (even if it can never fully undo it) as lower interest rates go hand in hand with greater capital and a more equitable income distribution (case (b)). If capital is substitutable with borrowers' labor supply-one may think of capital-skill complementarity and automation as in Krusell, Ohanian, Rios-Rull and Violante (2000) and Autor, Levy and Murnane (n.d.)-the opposite is the case. Lower interest rates endogenously lead to a more unequal income distribution, effectively steepening the saving supply schedule and amplifying the problem of indebted demand (case (c)). ${ }^{45}$ We illustrate the three cases in Figure 14.

This discussion focused on the long run. Investment contributes to demand in the short run as interest rates fall, irrespective of the structure of the production function $F$. For example, if $F$

[^26]is Cobb-Douglas, and the economy sees a shift in income inequality, investment picks up initially, temporarily slowing the decline in $r$. As the investment boom recedes, however, the fall in $r$ accelerates again, eventually falling to the exact same steady state level as would have occurred without investment (as in Section V).

Observe that the recent US experience does not align well with this description of investment. It did not seem that investment (as a fraction of GDP) rose as interest rates fell. This is subject of a recent literature (e.g. Gutiérrez and Philippon 2016, Liu, Mian and Sufi 2019, Farhi and Gourio 2018, Eggertsson, Robbins and Wold 2018).

Which kind of debt causes indebted demand? Productive vs. unproductive debt. Investment may be funded with (corporate) debt, raising the question, which kind of debt actually causes indebted demand? In Section III, we argued that, starting in some steady state, a one-time exogenous increase in debt by some $d D$, holding $r$ fixed, causes a response of aggregate spending of

$$
\begin{equation*}
d C=-\frac{\rho+\delta}{2}\left(1-\sqrt{1-4\left(1-\frac{r}{\rho+\delta}\right) \frac{r}{\rho+\delta} \epsilon_{\eta}}\right) d D \tag{29}
\end{equation*}
$$

(see Proposition 12). The debt in this experiment is unproductive, that is, it is being used for consumption.

We now repeat this exercise with productive debt, that is, debt that is being used to raise the capital stock of the economy, $d K=d D$. Holding $r$ and household debt $d$ fixed, what are the implications for aggregate spending?

Proposition 13 (Indebted demand when debt is productive). Starting from a steady state and holding $r$ and $d$, fixed, an exogenous increase in debt $d D$ that raises the capital stock by the same amount affects aggregate spending by

$$
\begin{equation*}
d C=-\frac{\rho+\delta}{2}\left(1-\sqrt{1-4\left(1-\frac{r}{\rho+\delta}\right) \frac{r}{\rho+\delta} \epsilon_{\eta} \chi}\right) d D . \tag{30}
\end{equation*}
$$

where $\chi \equiv\left(1-\sigma^{-1}\right) \omega^{b}-r \frac{r d}{Y}<1$.
A first observation about Proposition 13 is that productive debt, (30), always causes strictly less indebted demand than unproductive debt, (29). To see this, note that $\chi=\left(1-\sigma^{-1}\right) \omega^{b}-$ $r \frac{r d}{Y}<1$. Thus, even if capital is perfectly substitutable with borrowers' labor supply, $\sigma=\infty$, productive debt does not cause as much indebted demand. The reason for this is that even if $\sigma=\infty$, capital raises aggregate output. A second observation is that the negative effect of $d D$ on aggregate spending $d C$ falls for lower values of $\sigma$, as one would expect given Proposition 12. In the case $\sigma=1$, the effect is positive at first, $d C>0$. Recall that (30) is the contemporary effect on spending-but once we allow household debt to increase with $Y$, the positive effect fades.

The distinction between productive and unproductive debt is not always obvious. Consider, for example, investment in infrastructure (e.g. airports, buildings, public transport) in remote
locations, as some argue can be found in China today. Debt that is financing such investments ought to be thought of as unproductive.

A special case where debt is used productively but $\chi$ is still very high is residential investment into borrowers' owner-occupied housing. For example, imagine borrowers remodel or extend their houses. Clearly this is productive, as a greater housing stock produces more housing services. However, these additional housing services are consumed by borrowers themselves and do not increase borrowers' marginal product of labor. Thus, in this case, $\sigma=\infty$ and $\chi=\omega^{b}-r \frac{r d}{Y}$, the largest possible value for productive debt.

In sum, our results suggest that productive debt always causes weaker indebted demand than unproductive debt. The degree to which it is weaker depends on the elasticity of substitution $\sigma$ between the capital that is accumulated using debt and the borrowers' labor supply. This suggests that debt-financed productive investments made by firms or governments need not necessarily contribute to indebted demand.

Does student debt contribute to indebted demand? Since the end of the Great Recession, the US has witnessed a significant increase in student debt, begging the question whether student debt is a likely contributor to indebted demand. At the face of it, it seems that the answer is no. After all, student debt finances investment in human capital, and thus can be analyzed like investment in physical capital.

There is, however, an important difference. The rise in student debt partly reflects the rising cost of college tuition. To the extent that this is the case, student debt is indeed a source of indebted demand, as borrowers have to service larger piles of student debt without having accumulated greater human capital.

## VIII.B. Longer duration debt

A recent literature highlighted that responses of economies to interest rate changes differ according to the type of debt contract agents hold (e.g. adjustable-rate vs. fixed-rate contracts) as well as the debt's maturity (e.g. Campbell 2013, Calza, Monacelli and Stracca 2013, Di Maggio et al. 2017). In this extension, we briefly investigate the conceptual role of debt duration for indebted demand.

Consider a version of our baseline model in Section II. Assume debt has an entirely fixed rate, equal to the steady state debt payment $\ell$ (which is equal across steady states). How does this change the economy?

While fixed rate (FR) debt does not change the steady state, it does affect the transitional dynamics. To show this, consider first the experiment of rising income inequality in Figure 7. In that figure, due to the anticipated fall in interest rates, the present value of pledgeable wealth increases, while the present value of debt remains unchanged, as debt is adjustable rate. Thus, borrowers have more room to spend, pushing interest rates up in the short run.

With fixed rate debt, the present value of debt jumps on impact, in lockstep with pledgeable wealth. In fact, given our assumption of completely fixed debt, the value of debt exactly equals
pledgeable wealth. This implies that borrowers have no additional room to spend, and the economy adjusts immediately on impact.

Proposition 14 (Transition with completely fixed rate debt.). When debt carries a completely fixed rate $\ell$, an unexpected permanent change in income inequality (greater $\omega^{s}$ ) lets the economy jump immediately to its new steady state.

Thus, the presence of fixed rate debt implies weaker aggregate demand during transitions with rising debt levels and falling interest rates, which speeds up the transition.

Now consider a shock that pushes real interest rates up, such as a reduction in inequality or an increase in progressive taxation. The present value of adjustable-rate (AR) debt is again unchanged, but the present value of FR debt falls, again speeding up the transition. Thus, in this sense, AR debt makes it harder to leave a steady state with high levels of debt since any increase in interest rates leads to an immediate sharp fall in demand without favorable revaluation effect.

These discussions highlight that AR debt contracts slow down transitions into states with low $r$ and high debt, and FR debt contracts speed up transitions away from such states. Fixed-rate contracts with automatic refinancing achieve both of these arguably favorable outcomes. ${ }^{46}$ Policies that raise the share of refinancing among US fixed-rate mortgage owners are therefore beneficial from this perspective.

## IX Quantitative Exploration

Thus far, we illustrated several important properties in various extensions of the baseline model. In this section, we combine those extensions into a single richer model. Relative to the baseline model in Section II, the model in this section includes additional forces: taxation and government bonds in the steady state; land; investment subject to Hayashi (1982) type capital adjustment cost; and the possibility of a transition without perfect foresight. We use the model to revisit the effect of inequality on interest rates and debt.

Extending the model. Households - Dynasties continue to maximizes preferences (1), subject to their budget constraints (2). Their wealth $a_{t}^{i}$ is still given by $h_{t}^{i}-d_{t}^{i}$. Real asset wealth is now given by the discounted stream of after-tax human wealth as well as the dynasty's share $\kappa^{i}$ of capital holdings

$$
h_{t}^{i}=\int_{t}^{\infty} e^{-\int_{t}^{s} r_{u} d u}\left(1-\tau_{s}^{i}\right) w_{s} \mu^{i} n_{s}^{i} d s+\kappa^{i} J_{t}
$$

where $\kappa^{s}+\kappa^{b}=1 ; J_{t}$ is the value of land and capital combined. We continue to assume that agents can pledge part of their real asset wealth, in this case part of their land holdings, giving rise to the same borrowing constraint (7), in which $p_{t}$ is the price of land.

[^27]Government - The government finances a fixed amount of government spending $G$ and a fixed amount of government debt $B$ using proportional taxes $\tau_{t}^{i}=\tau_{t}$ that are set to satisfy the government budget constraint (16).

Investment - There is a representative firm making investment decisions subject to quadratic capital adjustment costs. The firm uses a nested CES production function of the form

$$
Y=F\left(\mathcal{L}, K, n^{b}, n^{s}\right)=\mathcal{L}^{\psi}\left(\frac{\alpha}{\alpha+\omega^{b}}\left(\frac{K}{K^{*}}\right)^{\frac{\sigma-1}{\sigma}}+\frac{\omega^{b}}{\alpha+\omega^{b}}\left(n^{b}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\left(\alpha+\omega^{b}\right)}\left(n^{s}\right)^{\omega^{s}}
$$

where $\psi$ is the share of income going to owners of land $\mathcal{L}$ and, as before, $\sigma$ is the Allen elasticity between $K$ and $n^{b} .{ }^{47}$ The normalization parameter $K^{*}$ will be chosen to be equal to the initial steady state capital stock, in order for there not to be a direct effect of changes in $\sigma$ on output. When $\sigma=1$, the production function is Cobb-Douglas, with capital share $\alpha$ and labor income shares $\omega^{i}$. We assume $\psi+\alpha+\omega^{b}+\omega^{s}=1$. The firm maximizes its value

$$
\begin{equation*}
r_{t} J_{t}\left(K_{t}\right)=\max _{n_{t, n}^{b}, n_{t}^{s}, I_{t}} F\left(K_{t}, n_{t}^{b}, n_{t}^{s}\right)-w_{t}^{b} n_{t}^{b}-w_{t}^{s} n_{t}^{s}-I_{t}-\zeta\left(\frac{I_{t}-\delta_{K} K_{t}}{K_{t}}\right) K_{t}+\frac{d}{d t} J_{t}\left(K_{t}\right) \tag{31}
\end{equation*}
$$

subject to the law of motion of capital $\dot{K}_{t}=I_{t}-\delta_{K} K_{t}$. The adjustment cost function is quadratic $\zeta(x)=\frac{1}{2 \delta_{K} \epsilon_{K}} x^{2}$. The price of capital is given by $p_{t} \equiv \partial J_{t}\left(K_{t}\right) / \partial K_{t}$.

Equilibrium - The definition of equilibrium is like in Section II.D.. Households maximize their utility (1), firms maximize their value (31), and markets clear:

$$
c_{t}^{s}+c_{t}^{b}+I_{t}+G=Y_{t} \quad d_{t}^{s}+d_{t}^{b}+B=0 .
$$

Calibration - We pursue a similar calibration strategy to the one in Section III.E.. We continue to use the same values for $\mu, \rho, \delta, \ell, r, \lambda$. The land share is set to $\psi=0.05$, depreciation $\delta_{K}$ is set to 0.06 , and the capital share $\alpha$ is set to 0.20 , so that the value of capital to output is approximately 2.5 in the steady state, a standard value. Government spending $G$ is $15 \%$ of GDP, government debt is $40 \%$ of GDP. Parameters $\tilde{\eta}, \tilde{a}$ are still chosen to match a steady state interest rate of $5.5 \%$ and an MPC out of wealth of savers of 0.01 . Here, this gives $\tilde{\eta}=1.14, \tilde{a}=0.51 . \omega^{s}$ and $\kappa^{s}$, which for simplicity we choose to be the same, are picked to match a top $1 \%$ income share of $10 \%$ (Smith, Yagan, Zidar and Zwick 2019) in 1980. This gives $\omega^{s}=\kappa^{s}=4 \%{ }^{48}$ The adjustment cost parameter $\epsilon_{K}$ is chosen to match a semi-elasticity of investment to Tobin's $\mathrm{Q}, p_{t}$, of 1 as in Auclert and Rognlie (2018), so that $\epsilon_{K}=1$. Our baseline assumes an Allen elasticity of $\sigma=1$. We allow both $\epsilon_{K}$ and $\sigma$ to vary below.

Effects of rising inequality. We revisit the experiment of rising income inequality from Section V.A., studying the implications of a number of assumptions. We increase inequality from $\omega^{s}=4 \%$ to

[^28]Figure 15: Rising income inequality-Robustness in richer models.


Note. Plots show transitions from our calibrated steady state with $\omega^{s}=0.04$ to one with $\omega^{s}=0.11$.
$\omega^{s}=11 \%$, in line with the findings of Smith, Yagan, Zidar and Zwick (2019). ${ }^{49}$
Figure 15 shows the resulting paths of interest rates and debt levels, for six different version of the quantitative model. The dashed gray line shows the homothetic economy, in which debt barely moves, and just like before, natural rates fall briefly but then converge back to their steady state level of $5.5 \%$. The solid black line shows the transition in our baseline non-homothetic model. It shows a continuous upward trend in debt levels, along with a strong decline in the natural interest rate. The trends in debt levels are amplified somewhat when the elasticity of substitution $\sigma$ is assumed to be 2 , and mitigated somewhat when $\sigma=0.5 .{ }^{50}$ Interestingly, both trends are similar when the capital stock is assumed to be constant at its initial level, with infinite adjustment cost, $\epsilon_{K}=0$.

The red densely dashed line shows the baseline transition, but without the assumption of perfect foresight. In particular, we assume that each instant, when presented with a greater $\omega^{s}$ level, agents simply expect $\omega^{s}$ to remain constant at the current level forever after. While this has no bearing on the eventual natural rate and debt levels, it no longer exhibits an increase of the natural interest rate on impact, in line with our discussion of Figure 7.

Quantitatively, the simulations predict an eventual interest rate decline of around $3 \%$, from one steady state to another, which is in the range of estimates for the decline of the natural interest rate over the past four decades (Laubach and Williams 2016). Debt levels are predicted to eventually increase by about 80 percentage points relative to GDP (steady state to steady state), which is greater than the actual increase that happened in the US so far ( $55 p p$ until the financial crisis of 2008/09, 40pp until Q2 2020). This is likely due to the fact that our analysis in this section worked with a simplified borrowing constraint (7) and did not include an effective lower bound, which-

[^29]as shown in Figure 13-mitigates the increase in debt levels.

## X Conclusion

In this paper, we proposed a new theory connecting several recent secular trends: the increase in income inequality, financial deregulation, the decline in natural interest rates, and the rise in debt by households and governments.

The central element in our theory are non-homothetic preferences, which lead richer households to have greater saving rates out of a permanent income transfer. This gives rise to the idea of indebted demand: greater debt levels mean a greater transfer of income in the form of debt service payments from borrowers to savers, and thus depress demand.

We identified three main implications of indebted demand. First, secular economic shifts that raise debt levels (e.g. income inequality or financial deregulation) also lower natural interest rates, which then itself has an amplified effect on debt. Second, monetary and fiscal policy, to the extent that they involve household or government debt creation, can persistently reduce future natural interest rates. This means that there is only a limited number of such policy interventions that can be used before economies approach the effective lower bound. Finally, when the lower bound is binding, the economy is in a liquidity trap with depressed output. In this "debt trap", debtfinanced stimulus deepens the recession in the future, whereas redistributive policies and policies addressing the structural sources of inequality mitigate it.

Our results suggest that economies face a sort of "budget constraint for aggregate demand". They can stimulate aggregate demand through debt creation, but that reduces future demand (and thus natural interest rates). This logic suggests a new trade-off for debt-based stimulus policies. We view an exploration of this trade-off in an optimal policy setting as a promising avenue for future research.

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[^1]:    ${ }^{1}$ For example, households in the top $1 \%$ of the wealth distribution financed $30 \%$ of the rise in the net household debt position of the bottom $90 \%$ of the wealth distribution from 1982 to 2007. See Figure 9 in Mian, Straub and Sufi (2020).

[^2]:    ${ }^{2}$ As we discuss below, we find that a similar result holds up in the presence of spreads between government bond yields and the returns on other assets.

[^3]:    ${ }^{3}$ See also Borio and White (2004), Koo (2008), Borio and Disyatat (2014), Lo and Rogoff (2015), Turner (2015), Dalio (2018) and Pettis (2019) for similar ideas.
    ${ }^{4}$ For an overview of multiple forces see Rachel and Summers (2019). For an alternative theory of secular stagnation based on preferences for wealth, see Michau (2018).
    ${ }^{5}$ For a notable exception, see Iachan, Nenov and Simsek (2015).
    ${ }^{6}$ This is not to be confused with heterogeneity in marginal propensities to consume out of transitory income transfers, which, as we explain below, are not sufficient to generate indebted demand.

[^4]:    ${ }^{7}$ Our results also hold with utility functions over consumption different from log, see Online Appendix B.5.
    ${ }^{8}$ We relax this assumption in Section B.3.
    ${ }^{9}$ In fact, given the normalization with $1 / \rho, v\left(a_{t}\right)=\log a_{t}$ exactly corresponds to an altruistic bequest motive in an equilibrium in which $r_{t}=\rho$.

[^5]:    ${ }^{10}$ We multiply the right hand side by $\lambda$ so that in a steady state, the constraint simplifies to $d^{i} \leq p \ell$. This is immaterial to our results.

[^6]:    ${ }^{11}$ In fact, we can allow for a more general constraint of the form $d_{t}^{i}+\lambda d_{t}^{i} \leq \lambda \mathcal{L}\left(\left\{r_{s}\right\}_{s \geq t}\right)$ where $\mathcal{L}$ is a general function of current and future interest rates. Denoting by $\mathcal{L}(r)$ the function $\mathcal{L}\left(\left\{r_{s}\right\}_{s \geq t}\right)$ in the case where rates are constant $r_{s}=r$ for all $s \geq t$, our results require that $\mathcal{L}$ is decreasing in $r$. We show in Online Appendix B. 6 that many alternative models of borrowing have this feature.

[^7]:    ${ }^{12}$ We discuss evidence in Section IV that is incompatible with the altruistic model.
    ${ }^{13}$ These findings are also in line with Bartscher, Kuhn, Schularick and Steins (2020).

[^8]:    ${ }^{14}$ If we assumed away heterogeneity in per-capita real earnings $\omega^{i} / \mu^{i}$, "borrowers" and "savers" become entirely symmetric, so that for each equilibrium in which borrowers borrow and savers save, strictly speaking there would also

[^9]:    ${ }^{17}$ Multiple steady states can occur under more general borrowing constraints (see footnote 11).

[^10]:    ${ }^{18}$ The three boundary conditions are (a) an initial level of debt $d_{0}$; (b) the terminal level of the asset price $\lim _{t \rightarrow \infty} p_{t}=$ $1 / r$; and (c) the terminal level of savers' consumption $\lim _{t \rightarrow \infty} c_{t}^{s}=c^{s}$. $r$ and $c^{s}$ are the steady state values.

[^11]:    ${ }^{19}$ This is conservative given the alternative reasons for non-homothetic saving, see e.g. the discussion in Section II.E..

[^12]:    ${ }^{20}$ The functional form in (14) is a transformation of a "SoftPlus" function commonly used in Machine Learning.
    ${ }^{21}$ The formula for the MPC is $r-\frac{\rho+\delta}{2}\left(1-\sqrt{1-4\left(1-\frac{r}{\rho+\delta}\right) \frac{r}{\rho+\delta} \epsilon_{\eta}}\right)$.

[^13]:    ${ }^{22}$ See also the influential article by Carroll (2000) which highlights some of the empirical work on the subject from the 1990s.

[^14]:    ${ }^{23} \epsilon_{r}<1$ holds for any function $\mathcal{C}(r, a)$ describing the response of initial consumption $c_{0}$ that is the solution to a standard utility maximization problem with monotone and concave preferences over paths $\left\{c_{t}\right\}$ with prices $e^{-r t}$ relative to the present and initial wealth $a$.
    ${ }^{24} \mathrm{To}$ the best of our knowledge, there are no estimates of how the $M P C^{\text {wealth }}$ varies across the wealth distribution in the United States. Chodorow-Reich, Nenov and Simsek (2019) estimate $2.8 \%$ in the aggregate. The estimates by Garbinti, Lamarche, Lecanu and Savignac (2020) in other countries suggest an MPC wealth for the bottom $90 \%$ that is on average five times larger than the top $10 \%$. Applying this pattern to the United States implies that the MPC wealth of the top of the wealth distribution in the United States is likely to be substantially lower than $2.8 \%$.

[^15]:    ${ }^{25}$ Despite log utility over consumption, $\epsilon_{r}$ is slightly negative in our model due to the bequest motive in the utility function (1).
    ${ }^{26}$ Wealth to income is independent of the lifetime income distribution when saving rates are constant in lifetime income (Straub, 2019). Since we do not have data on top lifetime income shares, we use top (current) income shares. We believe this is appropriate given that the rise in (current) income inequality was largely driven by rising inequality in lifetime income (Kopczuk, Saez and Song, 2010, Guvenen, Kaplan, Song and Weidner, 2017).
    ${ }^{27}$ These years are chosen as they are the years for which state-level information is available to construct the wealth to income ratio in a state. See Mian, Straub and Sufi (2020) for more details.
    ${ }^{28}$ The slope of the relationship in the time series is larger than the slope of the relationship across states. One reason for this is that the time series relationship includes the endogenous response of interest rates to a rise in inequality. If interest rates fall due to a rise in top income shares (as we argue below), then wealth to income ratios will rise even further. The cross-sectional specification holds fixed the interest rate which is why this endogenous response is absent in the cross-sectional specification. In this sense, the cross-sectional relationship is the more direct test of non-homotheticity without general equilibrium effects.

[^16]:    ${ }^{29}$ Even in the Aiyagari (1994) model, saving supply schedules go from upward sloping to flat at the highest wealth levels. The reason is that the precautionary motive ceases to materially influence saving rates for the wealthy. There is no force such as non-homotheticity in the Aiyagari (1994) model to generate a downward sloping saving supply schedule at higher wealth levels.
    ${ }^{30}$ The rise in income inequality in the model is a rise in the permanent income of the high endowment agents relative to the low endowment agents. This experiment matches the data. Both Kopczuk, Saez and Song (2010) and Guvenen, Kaplan, Song and Weidner (2017) show that lifetime income inequality has increased substantially in the United States since the early 1980s.

[^17]:    ${ }^{31}$ Similarly, the homothetic economy shows an on-impact drop in the interest rate, below its initial steady state value (dashed gray line) before converging back to it.

[^18]:    ${ }^{32} \mathrm{An}$ important caveat here is that this is a perfect foresight transition with rational expectations. In practice, especially in the early 2000s, house prices and borrowing partly increased (and later reversed) due to optimism (see, e.g.

[^19]:    ${ }^{34}$ We conjecture that the point at which interest rates fall below their prior steady state level is related to the point at which resources start flowing in net from borrowers to savers, similar to the evidence in Drehmann, Juselius and Korinek (2018).

[^20]:    ${ }^{35}$ Interest rates are constant in government debt when savers are taxed because savers are Ricardian. In a nonRicardian model with downward-sloping saving supply schedule, e.g. the one in Online Appendix B.9, interest rates would increase with greater government debt if it is financed by savers alone. Importantly, whether savers are Ricardian or not has no bearing on the cases in which borrowers are being taxed or the government cuts back its spending.

[^21]:    ${ }^{36}$ The precise formulation of wage stickiness is actually irrelevant for this section as we make the simplifying assumption below that monetary policy sets the real interest rate directly. See Auclert, Rognlie and Straub (2018) for a related argument.
    ${ }^{37}$ This corresponds to a Taylor rule with a Taylor coefficient of 1 . One can easily study other Taylor coefficients, after specifying the wage Phillips curve. The transmission from real rates to real economy outcomes remains unaffected. Note that, different from a textbook New-Keynesian model, our economy is locally determinate under a real-rate rule.

[^22]:    ${ }^{38}$ Aside from these implications, our model also finds that forward guidance is less powerful than in standard NewKeynesian models, for a reason similar to the one in Michaillat and Saez (2019).
    ${ }^{39} \underline{r}$ can be microfounded in the context of our model in Online Appendix B.2, where it would be equal to the convenience yield. In Online Appendix B.8, we show how allowing for wage deflation amplifies our findings.

[^23]:    ${ }^{40} \mathrm{We}$ leave out policies related to government debt here as those require a well-microfounded model of the interest rate spread between government and private debt that is beyond the scope of this paper.

[^24]:    ${ }^{41}$ Interestingly, our model also suggests that a consumption (or VAT) tax would reduce natural interest rates and hence worsen the liquidity trap.

[^25]:    ${ }^{42}$ Online Appendix B shows a number of other important extensions.
    ${ }^{43}$ We assume the typical: $F$ is strictly concave, satisfying Inada conditions.
    ${ }^{44} \mathrm{We}$ assume that the economy is not in the debt trap for this section. Investment would be negatively affected in the debt trap and thus amplify the output loss there, similar to the logic in Benigno and Fornaro (2018).

[^26]:    ${ }^{45}$ See Straub (2019) for a related point.

[^27]:    ${ }^{46}$ Lengthening the maturity of debt when debt is high, as in Campbell, Clara and Cocco (2020), would further speed up transitions back to lower debt states as they would lengthen the duration of debt. Automatic refinancing is reminiscent of the idea to convert FR into AR contracts (Guren, Krishnamurthy and McQuade 2021), albeit such a conversion would slow down transitions back to states with lower debt levels.

[^28]:    ${ }^{47}$ Here, it is more convenient to define $\sigma$ directly on the gross production function.
    ${ }^{48}$ The fact that we choose $\kappa^{s}$ to be the same as $\omega^{s}$ has almost no bearing on our results.

[^29]:    ${ }^{49}$ See Figure IX in Smith, Yagan, Zidar and Zwick (2019). The share of labor income plus $75 \%$ of the share of business income going to the top $1 \%$ rose by approximately 7 pp .
    ${ }^{50}$ These forces are harder to see for the interest rate as it tends to converge more slowly to its long-run level.

