Complexity and the Reform Process*

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Abstract

Decision makers called to evaluate and approve a reform, proposed by an interest group, a politician, or a bureaucracy, suffer from a double asymmetric information problem: about the competence of the proposer and the consequences of the proposal. Moreover, the ability of decision makers to evaluate proposals depends on the complexity of the legislative environment, itself a product of past reforms. We model the strategic interaction between reformers and decision makers as a function of legislative complexity, and study the dynamics of endogenous complexity and stability of reforms. Complexification-simplification cycles can occur on the equilibrium path, and expected long-run complexity may be higher when competence of reform proposers is lower. The results apply to regulatory reforms, legislative politics, and institutional design.

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1 Introduction

Many recent technological innovations have led to the creation of new products or markets, or have fundamentally changed existing ones. Closely linked to these changes is the need for regulatory reforms capable of addressing these developments. For instance, the increased capabilities of artificial intelligence have led to calls for rules regarding privacy and data harvesting. Similarly, scientific advances on the effects of climate change spurred proposals for a wide range of reforms including to energy production, emission standards, and construction codes. As these examples suggest, many of these reforms must address complex policy problems. This requires a regulatory system capable of understanding and implementing complex policies. Yet the political decision making process and legislative incrementalism, where new rules build upon existing ones, may limit the regulatory system's ability to efficiently regulate complex policy domains.

In this paper, we model the reform process and examine how political incentives along with economic conditions determine what types of rules are adopted, when complex rules are implemented even though they do not aptly address the corresponding policy problems, and when reforms are likely to be blocked. We trace the endogenous evolution of regulation, and characterize its dynamics. The joint role of political and economic factors in determining complexity of reforms and complexity of the legislative environment explains patterns in recent empirical evidence on the relationship between increases in regulatory complexity, efficiency and economic growth.

Most legislative or regulatory reform processes typically consist of three stages: An entry stage (stage zero) at which interest groups are formed and reform proposers emerge; An *internal* evaluation and approval process (stage one), which takes place within the political or bureaucratic institution; and an implementation and *external* evaluation by market participants or voters (stage two). The expected rewards in stage two, for example a politician's increased re-election prospects once a reform is implemented, determine who decides to propose reforms in stage zero. The variation across domains and polities in terms of stage two rewards may generate differences in the types of proposers expected to enter and the

expected quality of proposals.[1] Since our goal is to explain the interplay of reform incentives and complexity, we take the differences as given and focus on stage one, that is, on the mapping from the expected quality of the proposals to the evaluation and approval process internal to any political or bureaucratic decision body.

We model the "stage one" process in which a *proposer* (an interest group, a specialized agency, or a politician, depending on the context) offers a reform proposal for approval by a *decision maker* (politician, regulator, or institution, depending on the context). The reform proposal comes in response to a change in the state of the world due for example to technological innovations or economic shocks. As is common to delegation models, the proposer has better information about the state of the world than the decision maker, that is, there is information asymmetry over the state of the world. For example, the proposer has more expertise on the topic or access to better private data. We augment this common setup by differentiating between simple versus complex reform proposals. A simple reform is one which contains few or no contingencies, a "one-size-fits-all" type of policy. A complex reform, on the other hand, specifies many contingencies, tailored to the specifics of the policy problem. This makes the reform better suited for addressing a more complicated policy problem, in the same way that more contingencies make a contract more complete. Yet, as with more complete contracts, writing down more contingencies increases the cost of drafting and bureaucratic implementation. If done badly, a complex reform imposes inefficiently high costs relative to benefits. Thus, a second dimension of information asymmetry becomes relevant for complex reforms: the proposer's ability (high or low) to produce a good complex reform.

To illustrate this key distinction between simple and complex reforms, consider the example of possible governmental responses after an economic crisis: one is implementing a simple blanket stimulus policy, for instance a universal tax rebate; the other is a set of complex targeted subsidies for specific sectors. Typically a government agency or dedicated task force proposes one of the two, and the executive (or the parliamentary majority) faces the

[1] See, for instance, De Mesquita and Stephenson (2007) for a model where the stage two rewards affect the stage zero investment in quality.

information problem we postulate: first, the executive faces uncertainty about the nature of the crisis, e.g., whether it is concentrated primarily in a few sectors or generalized; second, if the executive were to pursue the tailored policy, it matters whether the agency is able to distinguish and target the sectors which benefit the most from subsidies. A blanket policy is likely to be effective if the crisis is likely to be generalized. A complex reform is effective if the crisis is concentrated *and* the implementing agency is capable of carrying it out.

After the proposer offers a simple or a complex reform, the decision maker evaluates it and chooses to adopt or reject it. The evaluation produces a noisy public signal about the state of the world. For instance, the decision maker may evaluate technical data, in order to understand what type of reform it calls for. The proposer's objective is to have the reform proposal adopted. The decision maker would like to adopt a reform called for given the state of the world, and to maintain the status quo if the reform is bad or if it does not fit the state of the world. For example, the proposer may be an interest group who drafts a reform, and the decision maker is a politician who may adopt or reject that reform to the legislation. Alternatively, the proposer may be a politician who makes a reform bill proposal, and the decision maker is the legislature's majority leader, who can support or reject the bill.

We obtain a full characterization of the best Perfect Bayesian Equilibrium for every pair of parameters defining the asymmetric information. Examining the types of reforms proposed in equilibrium, we find that bad complex reforms are proposed only when the decision maker expects the state of the world to require complexity. In that case, she is more likely to adopt a reform in line with her prior. This leads to complex reforms being proposed even when they are not desirable, as the proposer offers the reform most likely to be adopted. The likelihood of complex reforms being offered is, however, non-monotonic in the expected ability of the proposer, and by extension in the expected quality of these reforms. When ability is expected to be high, the decision maker is inclined to approve reform proposals, and thus the proposer can safely offer the reform that matches the state of the world. When ability is expected to be low, the decision maker expects complex reforms to be bad. She therefore favors simple reforms, the outcome of which does not depend on the proposer's

ability. In turn, the proposer offers a simple reform, to increase his chance of approval. Thus, expectations of low ability, for instance due to bureaucratic inefficiency, lead to simple rules or to gridlock, if the reform is rejected, even when the state of the world requires a complex reform. This widens the gap between what the economic state requires and what the regulatory system delivers. However, when the decision maker's expectation of ability is intermediate, the high-ability proposer offers a complex reform even when it is not needed, leading to over-complexification. The decision maker is willing to adopt complex reforms, because she expects them to be more likely to come from a high-ability proposer than a low-ability proposer. Thus, both unnecessarily complex reforms and needed but bad quality complex reforms are adopted.

So far we have described the determinants of simple vs complex reforms in a given legislative environment. However, equally important is to consider that reforms change the legislative environment in which future reforms may be undertaken. We consider these dynamics. We start by assuming that the decision maker and the proposer are both short-lived, and replaced each period by a new set of agents. If a simple reform is adopted, then the legislation also becomes simpler. This means that next period's policy problem becomes easier for a future decision maker: evaluating a new reform will deliver a more precise signal. Similarly, if a complex reform is adopted, then the legislation becomes more complex. This makes next period's evaluation problem more difficult: evaluating a new reform will deliver a noisier signal to a future decision maker. If the decision maker rejects the reform, then the status quo is maintained, and the next period's evaluation problem will be identical.

Dynamically, the evolution of the legislative environment leads to two main insights. First, there is the possibility of complexity cycles: faced with high complexity, the decision maker receives simple reform proposals that she adopts, and the legislative environment becomes less complex. But once the decision maker has better information about the state of the world, complex reform proposals materialize on the equilibrium path, as the proposer offers such policies in the state in which they are beneficial. This makes the system fluctuate around an intermediate level of legislative complexity. This is driven by cycling in the

proposer's equilibrium strategy as the legislative environment's complexity changes. We show that this dynamic has some important features that are unique to the "checks and balances" nature of our model: in our setting the level of legislative complexity around which cycling happens *decreases* in the probability that the proposer is high ability, while in a situation where a decision maker can freely pick reforms it would *increase* in the probability of high proposer ability. In the former case, the proposer responds to the decision maker's expectation of higher ability by strategically offering a simple reform. In the latter case, the decision maker simply responds to less uncertainty about the state of the world.

Second, the evolution of the legislative environment exhibits path dependence. This happens when there is a higher probability that the state of the world favors a complex reform and that the proposer is low-ability. In this case, starting from low complexity of the environment ensures continued low complexity. Starting from high complexity of the environment leads to a complexity trap in which complexity begets complexity.

Finally, we examine how changing the tenure of the decision maker or the proposer affects the dynamic inefficiencies. If the decision maker is allowed a longer tenure, the effect of complex reforms on the future legislative environment is partially internalized. The proposer in turn becomes more likely to avoid complex reforms. The downside is the rejection of good complex reforms. Stagnation also becomes more likely, as the decision maker is wary of the long-term effects of reforms. If, however, the proposer has a longer tenure and reputational concerns, the opposite effect is obtained. A high-ability proposer produces complex regulation in order to reveal his type, leading to excessive legislative complexity.

The predictions of the model help to reconcile different findings in the recent literature on the relationship between legislative complexity and efficiency. The negative effects of excessive legislative activism by politicians described in Gratton et al. (2020) are rationalized in our model by the combination of intermediate valence (mapped to ability in the model) of politicians in Italy and a large demand for reforms (mapped to high uncertainty about the state of the world) post-1992, leading to unwarranted complexification; on the other hand, the positive effects of complexification on efficiency shown in Ash et al. (2019) are

obtained when the expected quality of proposals (mapped to proposer ability) increases due to improved transmission of information from other jurisdictions on what reforms are good. Put together, these empirical findings highlight the non-monotonic relationship between complexity and expected quality of proposals obtained in the model.

The model may also be applied to better understanding the drivers of regulatory cycles and of over-complexification. In the regulatory context, interest groups lobbying for reforms map to proposers in the model, where proposer ability traces the alignment of an interest group with the public interest. Misaligned proposers produce bad quality complex reforms in the sense that they are detrimental to the public interest. Decision makers are the regulators who adopt or reject proposals from interest groups. Our mechanism points to a novel interpretation for endogenous cycles of complexification and simplification of regulation, emerging due to the strategic incentives of interest groups that propose regulatory changes. These endogenous cycles amplify cycles driven by exogenous changes in the economic environment.

Finally, the model also has implications for understanding the role played by checks and balances in democracies: in the absence of vetoing by a decision maker separate from the proposer, there could be excessive reforms. Moreover, complexity may endogenously evolve to one extreme or to the other. Separation between decision maker and proposer limits reformism and can also maintain an intermediate level of complexity.

The paper is organized as follows. Section 2 discusses the related literature, Section 3 presents the model, Section 4 characterizes the equilibrium for any given complexity of the environment, and Section 5 derives the dynamics of complexity. Section 6 discusses applications, Section 7 presents the natural single proposer-decider model against which we can compare our framework. Section 8 concludes, and all proofs are in the Appendix.

2 Related Literature

Our paper contributes to the literature on reform processes and legislative and regulatory complexity. The reforms in our model have the general feature of being incremental (Dewa-

tripont and Roland, 1992, 1995; Callander, 2011), in that policy change happens gradually – a proposer cannot propose something that massively increases or decreases complexity in one step. Incrementalism emerges endogenously in Kawai et al. (2018), in an evolutionary model where entanglements and interdependencies among policies make it very difficult to make grand reforms. Beside justifying incrementalism, entanglements and interdependencies also create a bias in favor of policy complexity, and in their framework policies that start complex tend to become ever more complex, whereas simple policies stay simple forever. In contrast, our framework does not consider entanglements and each policy domain could cycle endogenously between simplification and complexification equilibria. Taking reforms as incremental, we show how the complexity of the status quo, the need for reform, and the ability of policymakers affect reform adoption and we trace the resulting dynamics of complexity of the environment.

Central to our model is the view that complexity of the environment refers to the difficulty for the decision maker to discern the consequences of a proposed reform. This notion of complexity is introduced and analyzed in a general model in Asriyan et al. (2020). Here, the policymaking environment requires adjustment along two dimensions: First, we allow the consequences of a policy to depend on the state of the world, so that a more complex reform is not always costlier than a simpler reform; second, the complexity of the environment is history dependent, determined by the reforms adopted up to the current date. Implicitly, a more complex reform affects the future complexity of the legislative environment.

The strategic choice of reforms by a more informed proposer follows a similar logic as in models of strategic communication by experts (Crawford and Sobel, 1982). The proposer produces the reform most likely to be adopted by the decision maker given her prior. Yet, the proposer's offer only affects the type of reform adopted each period, while the evolution of complexity is not an explicit choice of the proposer. Moreover, we focus on the dynamics of complexity when agents are short-lived, so that reform proposals do not have reputational consequences. In the extension where proposers have longer tenures and a benefit to building reputation, equilibrium reform proposals are consistent with the intuition of the literature

on expert communication under reputational concerns (Ottaviani and Sørensen, 2006b,a): Proposers produce more complex reforms, as these increase their expected reputational payoff.[2] To keep focus on the evaluation of reforms and resulting complexity, we abstract away from the underlying market or electoral processes that generate the reputational concerns.[3]

We examine the feedback between reform proposals and the complexity of the environment for future regulators or legislators. This is in line with the type of questions studied in Gratton et al. (2020), who study the consequences of greater uncertainty (in the form of political instability) on quantity and quality of legislation. The evolution of the quantity and quality of legislation in turn affects the functioning of the bureaucracy and politicians' incentives. Like in their model, we study the dynamic consequences of regulation decisions, but without focusing on signaling incentives of bad politicians to 'enter' with a reform proposal. The political instability shock analyzed in that paper and the consequent incentive effects on bad politicians effectively correspond to a downward shock to the reduced form expected ability of proposers in our model.

In relation to the literature on regulatory reforms and lobbying, our framework maps to a model of informational lobbying, where the proposer is an industry-level interest group. This approach complements the literature focused on self-regulation (McCarty, 2017) or on quid-pro-quo lobbying conducted by individual companies.

Finally, the paper relates to the formal literature on checks and balances. Beside the classic Barro (1973), the closest related articles are Rogers (2003), Tsebelis (1999), and Gratton and Morelli (2018), who study the optimality of checks and balances under policy uncertainty when the veto player has the same signaling incentives as the proposer. To the best of our knowledge our model is the first to focus on the endogenous complexity consequences for the comparison between systems with and without checks and balances.

[2]In our extension, learning about proposer type is immediate after a complex reform is adopted. See Callander (2011) as part of the literature that has focused on learning motives, and Backus and Little (2020) on how learning can affect the 'stage zero' entry versus abstention of proposers.

[3]For the connection to electoral processes, see Levy et al. (2019) and Morelli et al. (2020) for models on the connection between the demand of populism and the strategic supply of simplistic policy platforms even when the state of the world may require a complex policy.

3 Model

Consider an environment in which a proposer can propose a reform to a status quo regulation. The proposal is adopted or rejected by a decision-maker (DM). The proposer's objective is to have his reform adopted. He derives a benefit normalized to 1 if his reform is adopted, and 0 otherwise.[4] A reform may either be simple (y^S) or complex (y^C). The benefit to the DM from each type of reform depends on the state of the world (θ). Additionally, the benefit from a complex reform also depends on the type of proposer. There are two possible states of the world, θ^S and θ^C, the latter occurring with known probability κ. There are also two proposer types, $P \in \{A, B\}$, where type A is high-ability and type B is low-ability. The probability of a proposer being of type A is π. The realized state of the world and proposer type are known to the proposer, but they are not observable to the DM.[5]

For the DM, the status quo regulation delivers a benefit normalized to 0. Adopting simple reform y^S yields a gain if it is implemented in state θ^S: it gives a net payoff $v > 0$ if the state is θ^S and a net loss $-l < 0$ if the state is θ^C. A reform y^C, on the other hand, requires high ability in addition to matching the state of the world: If the proposer is type A, then the DM receives the net benefit v in state θ^C, and $v - a \geq 0$ in state θ^S. If the reform comes from proposer B, the DM receives a net loss: $-l$ in state θ^C, and $-l - a$ in state θ^S. Below, we summarize the net gain to the DM from each reform, if it is adopted:

$\mathbf{y^S}$:

	A or **B**
$\theta^{\mathbf{S}}$	v
$\theta^{\mathbf{C}}$	$-l$

$\mathbf{y^C}$:

	A	**B**
$\theta^{\mathbf{S}}$	$v - a$	$-l - a$
$\theta^{\mathbf{C}}$	v	$-l$

Status Quo: 0.

Intuitively, a simple reform is a 'blanket' policy with no or few contingencies, which is good if

[4] In Section 5, we allow for the Proposer's payoff to differ based on the reform proposed, if the Proposer is longer-lived and there are reputation benefits from proposing one reform over another. We further discuss in Section 6 how this payoff assumption maps to politicians who are office motivated or to interest groups who benefit if their proposal is adopted.

[5] A low-ability proposer could additionally have worse information about the state of the world compared to a high-ability proposer. We show that the results of the model are qualitatively unchanged if we make this additional assumption. Details available upon request.

the state of the world does not require contingencies. The proposer's ability shouldn't matter much for simple, blanket policies; on the other hand, complex reforms contain multiple contingencies, and their payoff consequences depend greatly on the proposer's ability. Also, adopting a complex reform when it is not needed adds unnecessary administrative costs (a).

Reform Process. We consider the following reform process.

1. Nature chooses a state of the world, $\theta \in \{\theta^S, \theta^C\}$.
2. After observing θ and his type P, the proposer offers a reform proposal, either y^S (simple) or y^C (complex).
3. The DM receives a private signal $\rho \in \{s, c\}$ about θ, with $\Pr(\rho = \theta) = 1 - z$, where $z \in \left[z^{\min}, \frac{1}{2}\right]$, and $z^{\min} > 0$.[6] The value z is public information.
4. The DM makes decision d to approve ($d = 1$) or deny ($d = 0$) the reform.

Dynamics. The above reform process is repeated over an infinite horizon with periods $t \in \{0, 1, 2, ...\}$. A proposal that is approved in period t changes the status quo, and the complexity of the environment captured by the noise in the following period is given by

$$z_{t+1} = \begin{cases} \min\{z_t + \Delta, \frac{1}{2}\} & \text{after } y^C \text{ is approved,} \\ \max\{z_t - \Delta, z^{\min}\} & \text{after } y^S \text{ is approved,} \end{cases} \tag{1}$$

where $\Delta > 0$. The complexity of the environment therefore increases after y^C is approved, until the upper bound of $\frac{1}{2}$ is reached, at which point the noise is maximal, rendering the signal fully uninformative. Similarly, the complexity of the environment decreases after y^S is approved, until the lower bound of $z^{\min}$, at which the noise is minimal.

We assume that both the DM and the proposer live for only one period. In the next period, nature independently draws another DM and Proposer. Afterwards, in Section 5, we relax this assumption to allow for a DM with a longer tenure or for a longer-lived Proposer.

[6] We assume strict inequality because at $z = 0$ there is no imperfect information about the state of the world.

Complexity. Complexity in our model has two aspects. Complexity of reforms and complexity of the legislative environment. A complex reform adds more contingencies, and whether such contingencies match what is needed depends on both dimensions of asymmetric information; the outcome of a simple reform y^S instead only depends on the state of the world. The accumulation of complex reforms increases the complexity of the environment, and the simplest way to capture this fact is through the impact of a reform on the noise parameter z. A higher z means understanding the effects of a reform requires more expertise or a costlier analysis. The proposer can observe θ directly, the underlying assumption being that he has more expertise on the exact effects of the proposed reform. The process that governs the evolution of z establishes a direct connection between complex reforms y^C being adopted and z increasing.

Equilibrium Concept

Fixing any initial condition with a given triplet (κ, π, z), we define the PBE in the one-shot game as follows.

Definition 1 *A Pure Strategy Perfect Bayesian Equilibrium of the game is defined as a profile of strategies* $d : \{s, c\} \times \left\{y^S, y^C\right\} \times \left[z^{\min}, \frac{1}{2}\right] \to \{0, 1\}$ *for the DM and* $r_P : \left\{\theta^S, \theta^C\right\} \times \left[z^{\min}, \frac{1}{2}\right] \to \{y^S, y^C\}$ *for the proposer, and a system of beliefs* $\mu : \left\{y^S, y^C\right\} \times \left[z^{\min}, \frac{1}{2}\right] \to [0, 1]$ *for the DM such that (i) the proposer's strategy* $r_P(\theta, z)$ *is optimal given DM's strategy, (ii) the DM's approval strategy* $d(\rho, y, z)$ *is optimal given her belief, and (iii) the DM's belief* μ *must be consistent with Bayes' Rule whenever possible.*

Let $PBE(\kappa, \pi, z)$ denote the set of strategies that satisfy Definition 1. Since players live for only one period, a set of strategies in period t constitute a Perfect Bayesian Equilibrium if and only if they are in $PBE(\kappa, \pi, z_t)$. For each t, we select from $PBE(\kappa, \pi, z_t)$ the Pareto efficient equilibrium for the period-t players. This is equivalent to picking the best equilibrium from $PBE(\kappa, \pi, z)$ period-by-period. We denote this best equilibrium by $BPBE(\kappa, \pi, z)$.

Given our payoff assumptions, it is possible to obtain multiple equilibria that deliver the same maximal expected welfare. In that case, we select among those equilibria the one that

minimizes the future noise z, i.e., between two payoff equivalent actions, one where y^S is proposed and one where y^C is proposed, we select the equilibrium with y^S, stacking the deck against complexification.

Absorbing equilibrium Starting from noise z_0 in period $t = 0$, an initial $BPBE(\kappa, \pi, z_0)$ is *absorbing* if for each t and on-path z_t, we have $r_P(\cdot, z_0) = r_P(\cdot, z_t)$ and $d(\cdot, \cdot, z_0) = d(\cdot, \cdot, z_t)$. That is, the endogenous evolution of z_t for $t = \{1, 2, ..., \infty\}$ never challenges the existence conditions for that initial $BPBE$. If $BPBE(\kappa, \pi, z_0)$ is not absorbing it means that for some z_t potentially reached at some time t on the equilibrium path dictated by the initial equilibrium strategies, $BPBE(\kappa, \pi, z_t)$ does not call for the same best equilibrium strategy profile as $BPBE(\kappa, \pi, z_0)$.

4 Equilibrium

In each period t, we derive the $BPBE$ given (κ, π, z_t). To solve for the equilibrium, we first examine the DM's approval decision given a proposer's strategy. Afterwards, we derive the proposer choice given the DM's beliefs. Finally, we impose consistency between the proposer's strategies and the DM's beliefs. We obtain four types of pure strategy equilibria in which a reform is adopted with positive probability.[7] These equilibria can be Pareto ranked, and the ranking is stable across all (κ, π, z) where an equilibrium exists. The following Proposition describes each Pareto efficient equilibrium and its boundaries in the (κ, π) space, for a fixed z.

Proposition 1 *Given any $z \in \left[z^{\min}, 1/2\right]$, there exist thresholds $\pi_1(\kappa, z), \pi_2(\kappa, z), \pi_3(\kappa, z)$ and $\bar{\kappa}(z)$ such that the pure strategy $BPBE(\kappa, \pi, z)$ has the following form:*

1. *(**Simplification**) If $\pi \geq \pi_3$, proposer A offers y^S after θ^S, and y^C after θ^C; proposer*

[7] In a very small space of parameters the $BPBE$ is in mixed strategies, but we ignore them for simplicity, as they do not add anything substantive (details available upon request).

B offers y^S in both states, and the DM approves the proposal:

$$r_A(\theta, z) = y^C \textit{ if and only if } \theta = \theta^C; \quad r_B(\theta, z) = y^S; \quad d(\rho, y, z) = 1. \tag{2}$$

2. *(**Matching**) If $\pi \in [\pi_2, \pi_3)$, both proposer types offer y^S after θ^S, and y^C after θ^C, and the DM approves the proposal:*

$$r_A(\theta, z) = r_B(\theta, z) = y^C \textit{ if and only if } \theta = \theta^C; \quad d(\rho, y, z) = 1. \tag{3}$$

3. *(**Complexification**) If $\pi < \pi_3$ and $\pi \in [\pi_1, \pi_2)$, proposer A offers y^C in both states, proposer B offers y^S after θ^S and y^C after θ^C, and the DM approves the proposal:*

$$r_A(\theta, z) = y^C; \quad r_B(\theta, z) = y^C \textit{ if and only if } \theta = \theta^C; \quad d(\rho, y, z) = 1. \tag{4}$$

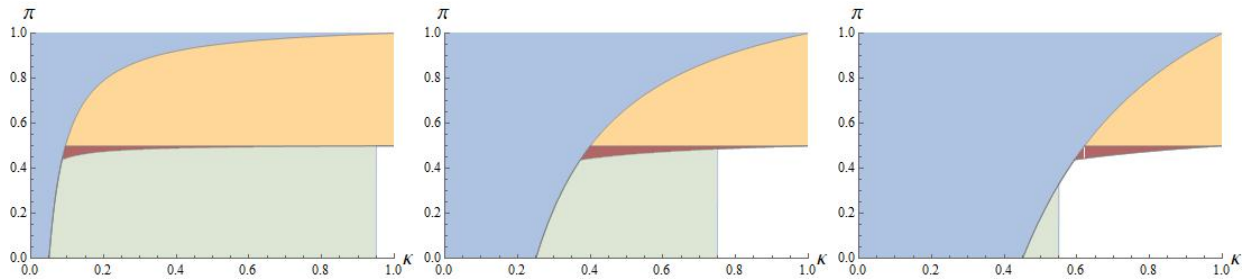
4. *(**Pooling**) If $\pi < \min\{\pi_1, \pi_3\}$, both proposer types offer y^S in both states, and the DM approves conditional on $\rho = s$ and $\kappa \leq \bar{\kappa}$:*

$$r_A(\theta, z) = r_B(\theta, z) = y^S; \quad d(\rho, y, z) = \mathbb{1}_{\{\rho = s \textit{ and } \kappa \leq \bar{\kappa}\}}. \tag{5}$$

These four different equilibria are illustrated in Figure 1 in the (κ, π) space given a low z (Panel a), a medium z (Panel b) or a high z (Panel c) . In the Simplification equilibrium, and the DM approves all proposals. Thus, the DM suffers a loss only if the proposer is B and the state is θ^C. The threshold π_3 reflects the maximum loss the DM can tolerate: the value at which the DM is indifferent between approving and rejecting the proposal when her evaluation indicates state θ^C is more likely, i.e., her signal is $\rho = c$:

$$\pi_3(\kappa, z) = \max\left\{0, 1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} \cdot \frac{z}{1-z}\right\}. \tag{6}$$

The threshold $\pi_3(\kappa, z)$ increases in κ, because in that case the DM has a higher belief that



(a) Low noise: $z = 0.1$ (b) Medium noise: $z = 0.45$ (c) High noise: $z = 0.45$

Figure 1: Illustrates the $BPBE$ in the parameter space (κ, π): in the blue area it is the Simplification equilibrium, in the yellow area it is the Matching equilibrium, in the red area it is the Complexification equilibrium, in the green area it is the Pooling equilibrium with $\kappa \leq \bar{\kappa}$, and in the white area it is the Pooling equilibrium with $\kappa > \bar{\kappa}$. This and all subsequent figures take $l = v = 1$ and $a = 0.5$.

the state is θ^C, in which she may suffer a loss. The threshold also decreases in z. Higher z gives a signal $\rho = c$ less weight in the DM's posterior belief about θ. Thus, the DM places lower weight on being in the state y^C and is willing to tolerate a higher probability that $P = B$.

The Matching equilibrium is the one where proposer B offers y^C after θ^C. In this equilibrium, y^S is guaranteed to produce a gain over the status quo. After y^C, however, the DM knows that the state must be θ^C, and that she registers a loss only if the proposer is a B type. Thus, π_2 captures the minimum probability that $P = A$ needed to sustain this equilibrium:

$$\pi_2 = \frac{l}{v + l}. \tag{7}$$

This threshold does not depend on κ or z, as the equilibrium play reveals the state θ to the DM. In the Complexification equilibrium, proposer A offers y^C in all states. The event in which the state is θ^C and the proposer is B, in which case the DM makes a loss, is less likely after observing y^C than in the previous equilibria, since now y^C is offered by A in all states. Given these proposer strategies, threshold π_1 is the value at which the DM is indifferent between approving and rejecting the proposal after a signal $\rho = c$:

$$\pi_1(\kappa, z) = \frac{l \cdot (1 - z) \cdot \kappa}{(v + l) \cdot (1 - z) \cdot \kappa + (v - a) \cdot z \cdot (1 - \kappa)}. \tag{8}$$

As with π_3, the threshold π_1 is increasing in κ and decreasing in z. The drivers are the same: the DM's probability of suffering a loss is higher when θ^C is more likely and when the signal $\rho = c$ is more precise.

In the Pooling Equilibrium, y^S is offered in both states by both proposer types. Thus, the proposer's type is irrelevant for the DM's payoff. What matters is whether y^S is adopted in the state of the world in which it brings a benefit. The DM approves the proposal as long as state θ^S is sufficiently likely, i.e., as long as κ is sufficiently low and the signal is $\rho = s$:

$$\kappa \leq \bar{\kappa}(z) = \frac{(1-z) \cdot v}{(1-z) \cdot v + z \cdot l}. \tag{9}$$

If $\kappa > \bar{\kappa}$, the DM expects a loss compared to the status quo and thus rejects the proposal.

The resulting equilibrium regions in the (κ, π) space lead to the following implications.

Corollary 1 *A low-ability proposer proposes a complex reform only if the expected quality of the proposer is intermediate, so that $\pi_1 < \pi < \pi_3$. When $\pi_1 < \pi < \min\{\pi_2, \pi_3\}$, a high-ability proposer also offers a complex reform when it is not necessary (in state θ^S).*

Both unnecessary and low-quality complex reforms are proposed when the DM expects the average ability of proposers to be intermediate. When ability is expected to be high, the DM approves all proposals, and there is no incentive for the proposer to offer bad or unnecessary complex reforms. When ability is expected to be low, complex proposals are rejected, so there is again no incentive to offer them. Yet, when expected ability is intermediate, DM believes that a complex proposal is sufficiently likely to come from proposer A. She therefore approves complex proposals, giving the proposer the incentive to offer them. The implication of this result is that complex reforms are likely to be low-quality or unnecessary when the decision maker faces a balanced mix in the pool of possible proposers. Instances of high complexity in regulation may therefore be associated with a relative balance between competing interest groups, some with reforms that benefit the DM and some opposing reforms. Moreover, the above results imply that observations of relative simplifications of reform proposals are consistent with either an increase or a decrease in the expected ability of proposers.

The three panels of Figure 1 indicate how the $BPBE$ changes with z. The Simplification and the Complexification equilibria exist for more parameter values (κ, π) at higher values of z. At a low z, a signal $\rho = c$ would deter the DM from approving, due to the expectation of a loss in state θ^C. As the noise increases, the DM receives less precise information and thus she places less weight on the signal.

The Matching and Pooling equilibria are the $BPBE$ for a smaller range of parameter values as z increases. For the Matching equilibrium, this is due to the Simplification equilibrium being sustainable where it was not before. For the Pooling equilibrium, this happens because the DM responds to less precise information from the signal by demanding a higher probability of θ^S, i.e., lower κ, in order to approve. These observations lead to the following insight. Each boundary between equilibrium regions in the (κ, π) space changes monotonically with noise z. Bounds $\pi_1(\kappa, z)$, $\pi_3(\kappa, z)$, and $\bar{\kappa}(z)$ decrease as z increases. The bound π_2 is independent of z. This allows us to derive implications for the effect of noise on reform adoption.

Corollary 2 *The probability that a reform is adopted increases in the complexity of the environment z when $\kappa \leq \frac{v}{l+v}$. It decreases in the complexity of the environment when $\kappa > \frac{v}{l+v}$.*

A more complex environment makes it more difficult for the DM to gather information about the implications of a reform proposal. With less precise information, the DM essentially has a more porous sifter through which to filter reform proposals. This results in more proposals getting approved when the DM attaches a prior probability that the state is θ^S greater than a threshold. With a high prior that the state is θ^S, an increase in z can make the DM switch from a signal-dependent approval strategy to accepting all proposals. Conversely, when the DM attaches a higher probability to state θ^C, an increase in z increases the likelihood of rejection.

5 The Dynamics of Complexity

5.1 Complexity cycles, complexity traps, and path dependence

In this Section, we consider the dynamics in the complexity of reforms and the complexity of the environment, given the lasting effects of reform decisions taken each period. As described in the model setup, adopting a reform in period t impacts the environment complexity in period $t+1$, z_{t+1}. Adopting policy y^S reduces z_{t+1}, while adopting policy y^C increases z_{t+1}.

Cycling The results in the previous section show that the regions in which each type of $BPBE$ is sustainable depend on z. In particular, the boundaries $\pi_1(\kappa, z)$, $\pi_3(\kappa, z)$, and $\bar{\kappa}(z)$ decrease as z increases. For any point in the space (κ, π), the endogenous change in z may lead to one of the following two possibilities on the dynamic path.

First, it may be that starting at any z_0, the $BPBE(\kappa, \pi, z_0)$ is absorbing, i.e., as defined in Section 3, for each t and on-path z_t, we have $r_P(\cdot, z_0) = r_P(\cdot, z_t)$ and $d(\cdot, \cdot, z_0) = d(\cdot, \cdot, z_t)$. This is the case, for instance, at any point (κ, π) satisfying $\pi_2 < \pi < \pi_3(\kappa, 1/2)$. The $BPBE$ is Matching for any z_t. In this case, the endogenous evolution of z_t implies a monotonic path for the average complexity of the environment given κ, π and player strategies: increasing if y^C is adopted more often and decreasing if y^S is adopted more often.

Second, it may be that the $BPBE(\kappa, \pi, z_0)$ is not absorbing for at least some values of z_0. This means that the player strategies in the $BPBE$ change along the endogenous path of z_t. For instance, consider a location (κ, π) near the boundary $\pi_3(\kappa, z)$. This boundary increases after policies y^C are adopted and it decreases after policies y^S are adopted. It means that our location may be in the Simplification $BPBE$ for some values z_t, it may move into a Matching $BPBE$ as z_t endogenously decreases, and then move back into the Simplification $BPBE$ as z_t endogenously increases. Such switches between equilibria, if frequent enough, lead the average z_t to oscillate around an intermediate value rather than monotonically increase or decrease. We explore next the conditions under which these switches result in cycling between two equilibria and an intermediate long-run average complexity of the environment.

Cycling between equilibria may only occur if the location (κ, π) moves between the Simplification and the Matching equilibria, or between the Simplification and the Complexification equilibria. Policy y^S is more likely to be adopted in the Simplification equilibrium than in the other two equilibria. Yet, noise z_t endogenously decreases after y^S is adopted, which in turn makes the Simplification equilibrium unsustainable for lower values of π. That is, the boundary $\pi_3(\kappa, z)$ increases, and locations (κ, π) move into the Matching (or Complexification) $BPBE$. In this new equilibrium, reform y^C is more likely, leading to higher z_t and a decrease in $\pi_3(\kappa, z)$. Thus, at a location (κ, π), cycling emerges whenever the equilibrium play in a $BPBE$ leads, on average, to the boundary between equilibria changing such that we switch to a new $BPBE$. In the new $BPBE$, the equilibrium play leads, on average, to the boundary changing such that we cross back into the original $BPBE$. A necessary condition for a location (κ, π) to be in the Simplification $BPBE$ for some values of z and in the Matching $BPBE$ for other values of z is that $\pi > \pi_3(\kappa, 1/2)$. Similarly, the necessary condition for a location (κ, π) to be in the Simplification $BPBE$ for some values of z and in the Complexification $BPBE$ for other values of z is $\pi > \pi_4(\kappa) \equiv \{\pi : \pi_1(\kappa, z) = \pi_3(\kappa, z), \text{for some } z \in \left[z^{\min}, \frac{1}{2}\right] |\kappa\}$. Finally, the values of κ and π must allow for sufficiently frequent adoptions of y^S in one equilibrium and y^C in the other, for cycling to emerge. The following Proposition describes these conditions on parameters as well as the expected frequency of fluctuations between higher and lower complexity, measured as

$$\lim_{t\to\infty} \mathbb{E}\left[\sum_{t=1}^{T} \frac{\{t : r_P(\cdot, z_{t-1}) \neq r_P(\cdot, z_t) \wedge d(\cdot, \cdot, z_{t-1}) = d(\cdot, \cdot, z_t)\}}{T} |z_0\right]. \tag{10}$$

Proposition 2 *Two types of cycles occur in equilibrium as z evolves endogenously:*

1. *The $BPBE$ cycles between the Simplification and the Matching equilibria if $\frac{1}{2} < \kappa < \frac{1}{2\pi}$ and $\pi_2 < \pi$. The expected frequency of fluctuations increases in κ.*

2. *The $BPBE$ cycles between the Simplification and the Complexification equilibria if*

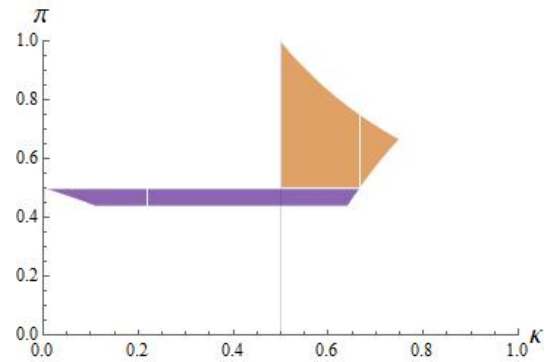


Figure 2: Illustrates the regions where cycling occurs between the Simplification and the Matching equilibria (orange) and between the Simplification and Complexification equilibria (violet).

$\max\{\frac{1-2\kappa}{2(1-\kappa)}, \pi_4\} < \pi < \pi_2$. *The expected frequency of fluctuations increases in* κ *if and only if* $\pi < \frac{1}{2}$.

Cycling is expected in equilibrium in a non-trivial region, illustrated in Figure 2. As the conditions above show, the expectation of cycling between equilibria is a stronger result than there just being switches possible between two equilibria. It requires that in expectation z_t oscillates around an intermediate value z^*.

The frequency of these cycles is higher when the region boundaries move on average faster in the direction that generates cycles. In our case, this means that y^S is proposed on average more often in the Simplification $BPBE$ and y^C is proposed on average more often in the Matching (or Complexification) $BPBE$. The frequency of y^S in the Simplification equilibrium decreases in $\kappa\pi$; the frequency of y^C in the Matching (or Complexification) equilibrium increases in κ (or $\kappa(1-\pi)$). Thus, there is a trade-off between increasing the average transition duration from Simplification to Matching (or Complexification) and increasing the reverse average transition duration.

The cycling described in Proposition 2 happens around the boundary $\pi_3(\kappa, z)$, which separates the Simplification $BPBE$ from the Matching (or Complexification) $BPBE$. Thus, for any location (κ, π), the cycling happens where $\pi_3(\kappa, z^*) = \pi$, which implies that the

environment's expected complexity must be

$$z^*(\kappa, \pi) = \frac{l \cdot \kappa \cdot (1-\pi)}{l \cdot \kappa \cdot (1-\pi) + v \cdot (1-\kappa)}. \tag{11}$$

We summarize this insight below and describe how z^* changes with κ and π.

Proposition 3 *Cycling happens around intermediate complexity $z^*(\kappa, \pi)$. Complexity z^* increases if the proposer is expected to be of lower ability (lower π) or if the state of the world is expected to favor complex reforms (higher κ).*

When the proposer is more likely to be low-ability, or the state is more likely to be θ^C, the cycling happens around a higher complexity level, that is, when the DM has less precise information. This comes in contrast with the standard intuition that a DM would be more likely to adopt a potentially costlier policy y^C when she has more precise information. The result emerges because cycling here happens due to the change in the proposer's strategies, while the DM approves the proposal non-contingently on her signal. Thus, cycling happens because the proposer adapts his strategy to ensure approval. For cycling to exist, the DM must choose to not make use of her signal. If the DM is more likely to suffer a loss, either due to lower π or higher κ, the information provided by the signal is more valuable. For her to not use this information, the signal must be less precise. This insight highlights that the cycling here is driven by the strategic interaction between the DM and the proposer. As we show in Section 7, any cycling that could emerge with a singular uninformed proposer-decider relies on the signal being used, leading to a decrease in complexity when the proposer is more likely to be low-ability.

Path Dependence. If different values of z_0 correspond to a different absorbing $BPBE(\kappa, \pi, z_0)$, then we have path dependence for long-run outcomes. The following proposition describes under what conditions this is the case, and under what conditions the system converges to the three possible long run average levels of complexity regardless of initial conditions.

Proposition 4 *On the equilibrium path, the expected complexity of the environment z_t evolves as follows:*

1. *Increases towards $\frac{1}{2}$ if $\pi_2 < \pi < \pi_3(\kappa, \frac{1}{2})$ or $\kappa \cdot \pi > \frac{1}{2}$.*

2. *Decreases towards $z^{\min}$ if $\pi_3(\kappa, \frac{1}{2}) < \pi < \max\{\frac{1-2k}{2(1-k)}, \pi_4\}$, or if $\kappa < \frac{1}{2}$ and $\pi > \pi_2$.*

3. *Oscillates around $z^*(\kappa, \pi)$ in the cycling regions described in Proposition 2.*

4. *Exhibits path dependence if $\pi < \min\{\pi_1(\kappa, \frac{1}{2}), \pi_2\}$. It decreases towards $z^{\min}$ if the $BPBE(\kappa, \pi, z_0)$ is Pooling and $\kappa \leq \bar{\kappa}$; it stays at z_0 if the $BPBE(\kappa, \pi, z_0)$ is Pooling and $\kappa > \bar{\kappa}$; it increases towards $\frac{1}{2}$ if the $BPBE(\kappa, \pi, z_0)$ is Complexification.*

We illustrate these regions in Figure 3 in the (κ, π) space In each region, we compute which policy is expected to be adopted more often, given the equilibrium strategies. These depend on the proposer's identity and the state of the world. If the proposer is more likely to be high-ability (high π) or the state of the world is more likely to be θ^C (high κ), then proposal y^C is more likely to be offered and adopted.

Next, we explore further the path dependence region described in Proposition 4.

Corollary 3 *Path dependence exists when the proposer is likely to be low-ability and the state is likely to favor complex reforms. Depending on the starting complexity of the environment, the system may enter a 'complexity trap' towards maximum complexity or an 'inaction trap' where complexity cannot change.*

The initial complexity of the environment determines the path along which z evolves, whether in the direction of simplification (if in the Pooling Equilibrium with low z), complexification (if in the Complexification Equilibrium) or maintaining the status quo (if in the Pooling equilibrium with high z). In this region, there is a high probability of a loss from the reform: a high likelihood of state θ^C and proposer B. Thus, starting in an environment with low complexity, the DM can rely on the signal and accept proposals y^S when $\rho = s$. This further simplifies the environment. Starting with high complexity of the environment,

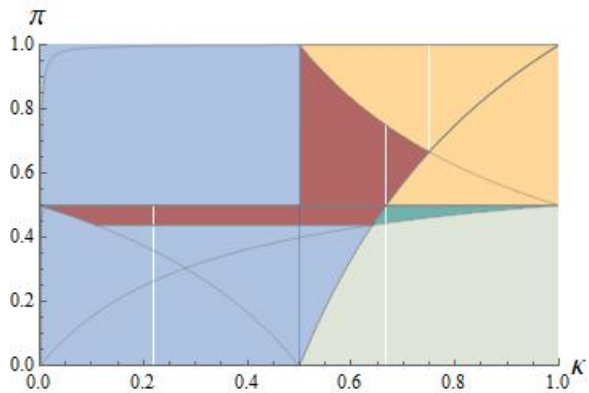


Figure 3: Illustrates the long-run expected evolution of z. In the blue region, z is expected to decrease towards $z^{\min}$. In the yellow region, z is expected to increase towards $\frac{1}{2}$. In the red region, long-run z oscillates around z^*. In the dark green and the light green regions, expected evolution depends on the starting z_0: if at z_0 the location (κ, π) is in the Complexification equilibrium (the dark green region), then z is expected to increase towards $\frac{1}{2}$, if the location is in the Pooling equilibrium and $z \leq z^D$, then z decrease towards $z^{\min}$, and if $z > z^D$, then z stays at z_0.

the DM does not follow the noisy signal. If π is sufficiently high, she approves any proposal, including a complex one, which then creates even more complexity in the environment. If π is sufficiently low, she rejects any proposal, and the status quo remains in place.

In the next subsections, we extend the model to allow for longer lived decision-makers and longer-lived proposers.

5.2 Dynamics with Longer-Lived Decision Makers

So far, we analyzed the case in which each DM and each Proposer live for only one period. In this section, we modify the model to allow for the DM to serve for two consecutive periods (i.e., two terms). The rest of the model is unchanged. Thus, in each of the two periods of her tenure, the DM encounters a new Proposer.

The equilibrium in this extended model may be derived by backwards induction. In the second period of the DM's tenure, the game is the same as described in the previous sections. Proposition 1 describes the $BPBE$ given any (κ, π, z). In the first period of her tenure, the DM anticipates next period's outcome given the endogenous evolution of z. For instance, if the proposed reform is y^S, approval would reduce next period's complexity of

the environment to $z - \Delta$. Given this update, she can expect next period's equilibrium play corresponding to $(\kappa, \pi, z - \Delta)$. This results in an expected payoff that we denote by $U(\theta, \pi, z - \Delta)$. Rejection of the proposed reform, however, keeps the same z in the next period. This leads to an expected payoff next period of $U(\theta, \pi, z)$. The expected payoffs after approval versus rejection differ only if the change in z leads to a change in equilibrium play or in expected payoffs. Otherwise, the DM expects the same payoff for either z or $z - \Delta$. Given the equilibrium regions described in Proposition 1, the $BPBE$ changes when z goes to $z - \Delta$ only for locations (κ, π) around the boundaries $\pi_1(\kappa, z)$ and $\pi_3(\kappa, z)$. For instance, for (κ, π) just above π_3 curve but below π_2, approving y^S leads to a Pooling $BPBE$ in the next period, as z decreases, while rejecting it keeps the location in a Simplification $BPBE$ next period. The latter equilibrium has a higher expected payoff than the Pooling equilibrium. Rejecting the reform therefore yields $\omega = 0$ plus a relative gain of $DU \equiv U(\theta, \pi, z) - U(\theta, \pi, z - \Delta) > 0$. The problem then becomes equivalent to the problem of a one-term DM with a higher outside option $\omega' = DU$. To approve the reform, the DM requires a higher posterior belief about the proposer being an A-type or the state being θ^S. This results in a higher value for $\pi_3(\kappa, z)$ in the $DM's$ first term. Analyzing the critical locations in the neighborhood of π_1 yields a similar insight. We summarize it in the following proposition.

Proposition 5 *If the DM serves for two consecutive periods, then in the DM's first term:*

1. *If $\pi \geq \pi_2$, the equilibrium is the same as for a one-term DM;*

2. *If $\pi < \pi_2$, then $\pi_1(\kappa, z)$, $\pi_3(\kappa, z)$, and $\bar{\kappa}(z)$ are higher than under a one-term DM.*

As suggested by the above discussion, the main effect of adding a term for the DM comes through the effective increase in the DM's outside option around the boundaries π_1, π_3, and $\bar{\kappa}$. This increase is strict only for values of $\pi < \pi_2$, as the Simplification and Matching equilibria yield the same expected payoff for the DM. The implication for the evolution of the environment's complexity is summarized in the following corollary.

Corollary 4 *The expected complexity of the environment is (weakly) lower in the long-run when each DM serves for two periods compared to the case when each DM serves one term.*

In some policy areas (around $\pi_3(\kappa, z)$), complexity does not decrease after the DM's first term, whereas it would decrease in the model with one-term DMs. In other policy areas (around $\pi_1(\kappa, z)$), complexity is expected to decrease after the DM's first term, compared to the case of a one-term DMs.

5.3 Dynamics with Longer-Lived Proposers

In this section, we consider the complementary case in which the proposer is long-lived, while the DM's tenure is one period, as in the main model. We assume that the proposer lives for two periods. In the second period, he derives a benefit proportional to his reputation, $\mu \cdot V$, where μ is the second-period DM's belief about the proposer's type, given the history of proposals and outcomes. The payoff V is the reduced form representation of the benefit of a promotion or an outside option that the proposer can access given his reform record. We assume that $V < 1$, so that the Proposer still prefers to have his reform adopted each period.

The benefit that comes from reputation means that proposer A gains more from offering a complex reform than a simple reform, whenever both these reforms have the same likelihood of approval. Thus, in the Simplification and Matching equilibria, proposer A is no longer indifferent between y^S and y^C. Proposal y^C, if approved, allows for the outcome to reveal the proposer's type. Hence, proposer A can gain V in the second period. For proposer B, however, the benefit from building reputation has the opposite effect. He now weakly prefers offering y^S, if both proposal types have the same probability of acceptance. The implication is that the prevalence of complex reforms increases. Moreover, the Pure Strategy $BPBEs$ can no longer be Pareto ranked, as the B-type Proposer now benefits more from Pooling on y^S, the equilibrium which does not allow the DM to learn his type. Thus, in the next result, we select in each period t the Best PBE for the period-t DM.

Proposition 6 *If the proposer lives for two periods and has reputational concerns, then the pure strategy $BPBE(\kappa, \pi, z)$ for the decision-maker in the first period of the Proposer's tenure is the Complexification equilibrium if $\pi \geq \pi_1(\kappa, z_t)$ and the Pooling equilibrium if $\pi < \pi_1(\kappa, z_t)$.*

The above result highlights that reputational concerns lead to more unnecessarily complex reforms, chosen by proposer A in order to signal his type. The equilibrium consequence is that proposer B also offers more complex reforms, in order to ensure approval. The result is higher complexity and more low quality reforms.

In the second period of the proposer's tenure, if the Complexification equilibrium was played before, then the proposer's type is revealed. Then, the DM approves any proposal if $P = A$ and only approves a proposal y^S from $P = B$ if state θ^S is sufficiently likely, that is, if $\kappa < v/(v + l)$. If the Pooling equilibrium was played in the previous period, then there is no learning about the Proposer's type, and we return to the stage game presented in the main model.

An implication of the above results is that longer-tenures and reputational benefits for proposers do not lead to a different path of reforms in the region of the (κ, π) space where there is path-dependence. Reputational concerns do not change the 'inaction trap' described in Corollary 3; however, they expand the complexity trap. It is in the cases where simple reforms would be beneficial and approved by the DM that the high-ability proposer offers instead a complex reform that results in the highest reputational payoff.

6 Applications

In this section, we discuss how our model captures the reform process in the legislative and the regulatory domains, and how the implications of the model shed light on several empirical puzzles regarding the evolution of complexity and its relationship to efficiency and growth.

6.1 Legislative Complexity, Bureaucratic Efficiency and Growth

In the legislative context, it is common for the production of legislation to involve a better-informed agent who proposes reforms, which must be approved by a less-informed decision maker. In the United States, both at the state and federal levels, the proposer is oftentimes a bureaucrat, who has expertise on the topic, that is, better information on the relevant state of the world θ (Bendor and Meirowitz, 2004).[8] Moreover, the bureaucrat is tasked with the implementation of any adopted reforms, and therefore his ability P is consequential for the reform's outcome. Therefore, π captures the expected capacity of the bureaucracy to implement a complex reform. The decision-maker is a politician, who can vote to approve or reject the reform. The politician is electorally accountable for the reform's effects, while the bureaucrat is not, which explains their different objectives. The politician's electoral benefit depends on the reform's outcome, so both on the economic value of the reform and bureaucracy's capacity. A bureaucrat who is motivated by career concerns (Alesina and Tabellini, 2007) may find implementing the reform valuable, but he may not be directly impacted by the outcome of that reform.

In the context of parliamentary systems, like in many European countries, the proposer is usually a politician, in the legislature or in the executive (Laver et al., 1996). The decision-maker is the relevant majority leader in the parliament, who controls the vote over its approval. The proposer politician may have the sole interest of getting a bill passed if he is strongly office-motivated, and showing legislative activity signals competence to voters or furthers his career prospects (Canes-Wrone et al., 2001; Gratton and Morelli, 2018; Gratton et al., 2020). The majority party leadership instead may be evaluated by voters based on the reform's outcome. The outcome depends both on the economic state (θ) and the competence of the proposing politician (P).

Finally, simple reforms are the ones that contain few or no contingencies, while complex

[8]For instance, Cates (1983), as quoted in Ting (2009), provides the following anecdote: faced with a proposal to reform Social Security in 1950, Senator Eugene Millikin (R,CO) complained that [t]he cold fact of the matter is that the basic information is alone in possession of the Social Security Agency. There is no private actuary...that can give you the complete picture...I know what I am talking about because I tried to get that.

reforms add many contingencies. This means that a simple reform requires less bureaucratic competence in drafting and implementing it, while a complex reform is more sensitive to the bureaucratic competence.

A number of empirical studies have examined the effect of complex reforms on the quality of legislative outcomes. Studies from different institutional contexts and time periods show potentially opposing effects of increasing complexity. On the one hand, higher legislative complexity has been shown to accompany lower quality legislation and worse bureaucratic efficiency. Gratton et al. (2020) examine the production of legislation in Italy during the First Republic (1948-1992) and the Second Republic (1992-2017). They show that higher political instability in the Second Republic is associated with lower quality and more complex legislation compared to the First Republic. They rationalize these findings by noting that higher political instability shortens the expected political horizon of legislators. This means that voters are called to evaluate the performance of legislators before their legislative proposals are fully implemented. This in turn incentivizes incompetent politicians to propose bad quality legislation, in order to appear hard-working and competent to voters. Therefore, as in our model, proposers derive a benefit if their reform is adopted, regardless of its contents. The increase in the production of low quality laws is then shown to have increased the complexity of the legislation and decreased bureaucratic efficiency. This Kafkaesque loop determines endogenously a reduction of the expected quality of proposals.

On the other hand, higher legislative complexity in terms of reforms containing more contingent clauses and detail have been shown to accompany higher efficiency and economic growth in the context of the U.S. states over the period 1965-1998 (Ash et al., 2019). They find that the effect is larger when there is higher economic uncertainty, i.e., the state of the world in which adding more provisions or contingencies is socially beneficial (higher κ). They rationalize these findings by noting that state-level legislation in the U.S. is competitive, which leads to better information about which reforms are needed given what has worked in other states. In our model, this would map to a higher expected quality of proposals, π.

At first glance, the above results present a puzzle as to when reforms that increase

legislative complexity are desirable. Our model sheds light on this puzzle. Consider an industry for which both in Italy and the U.S. in the late 80's there is the same relatively high likelihood κ that complex reforms are needed. Consider also starting in both countries from the same value π, at which we are in the Simplification region described in Proposition 1, close to the π_3 curve. A political instability shock like the one documented for the Italian case in the early 90's (and the consequent loop) generated a drop in π, bringing the polity in the bad Complexification region. As pointed out in Corollary 1, it is exactly for intermediate values of π that we have unnecessary and low quality complexifications with negative welfare consequences. In the case of U.S. states, high inter-state competition in the 20th century generated contagion and learning that increased the expected quality of proposals, π, bringing it higher up in the Simplification region. This determined an increase in legislative complexity as well as welfare. We summarize this insight in the following remark, where we assume that the $DM's$ payoff corresponds to the social benefit of reforms.

Remark 1 *Positive shocks that increase the expected quality of proposers (and proposals) can increase both legislative complexity and welfare. Negative shocks that decrease the expected quality of proposers (and proposals) can increase legislative complexity and reduce welfare.*

An additional reason to believe that complexifications are more likely to be bad when the shock is a negative political one, is that in such a case the shorter expected political horizon increases the dependence of reputation concerns on reform activity, hence altering the complexification incentives in the manner described in Proposition 6.

6.2 Regulatory Complexity and Reform Cycles

In the context of regulation, the rule-making process generally involves expert lobby groups proposing reforms to be adopted by regulators (Grossman and Helpman, 2001; Bertrand et al., 2018; Levy and Razin, 2013). In terms of our model, the proposer maps to an interest group who wants its reform proposal to be adopted by the decision maker, a regulator. The assumed information asymmetry between the proposer and the decision-maker captures

that, in many industries, the high degree of specialization and higher complexity of the environment (higher z) makes industry insiders better informed than regulators on the effects of their proposal (McCarty, 2017). An interest group's current stance on an issue may be aligned or misaligned with the public interest (which we assume the regulator is seeking to serve), and this positioning is many times not transparent. For instance, Bertrand et al. (2018) show that non-profit groups are tied to firms through difficult to trace links (donations by charitable arms of U.S. corporations), and these non-profits submit proposals for rules that favor their donors. Then, a misaligned interest group maps in our model to proposer B, whose drafting of the reform is detrimental to the public interest. An aligned interest group maps to proposer A, whose drafting of the reform is beneficial to the public interest. As assumed throughout the paper, a simple reform (y^S) contains few or no contingencies, while a complex reform (y^C) adds many contingencies.

Our model delivers several insights regarding the evolution of regulation and the complexity of the regulatory environment.

Remark 2 *Cycles between complexification and simplification of regulation may emerge:*

1. *exogenously, due to shocks to κ, the likelihood that complexity is needed given the economic or technological state.*

2. *endogenously, due to the evolution in the complexity of the regulatory environment, z, that changes the incentives of interest groups to propose complex versus simple reforms.*

First, a change in economic or technological conditions may change the likelihood that more regulatory complexity is beneficial (κ increases). Such shocks may shift the equilibrium play from one in which reform proposals more often are simple (the Simplification region) to one in which complex reform proposals are more likely (the Matching or Complexification region). For instance, periods of financial innovation have been shown bring about simplification of regulation, while periods of crisis are accompanied by increases in regulatory contingencies. This pattern has been documented empirically in both the United States and in Europe (Almasi et al., 2018; Barth et al., 2012; Dagher, 2018). It is worth mentioning

that the shift described above cannot happen in the model if the proposer is highly likely to be aligned with the regulator (high π). Empirically, Dagher (2018) finds that the aforementioned pattern is not encountered around the Swedish banking crisis of the 1990s. Consistent with our model, he points to political institutions which grant less access to private interests (which corresponds to high π in our model).

Second, keeping economic or technological conditions fixed, cycling between complexification and simplification of regulation can emerge endogenously due to the incentives of interest groups and regulators, as derived in Proposition 2. The complexity of the regulatory environment, z, evolves endogenously, and it changes the interest groups' incentives. When z is small, both aligned and misaligned interest groups are more likely to propose complex reforms. As these contingencies are adopted, the regulatory environment becomes more complex, and interest groups switch to proposing simple rules. These proposals are adopted, they simplify the regulatory environment, thus generating a cycle. This dynamic complements the result in Asriyan et al. (2020) that aligned proposers may complexify regulation, while misaligned proposers may simplify regulation.

Finally, Proposition 2 shows that endogenous cycling happens for moderate values of both π and κ. These are the situations in which there is higher uncertainty about the alignment of interest groups and about the appropriateness of making regulatory changes. In industries in which these conditions are met, the lobbying process gives rise to cycles of simplification and complexification that complement those due to the electoral motivations (McCarty et al., 2013) or due to economic shocks (Fernández-Villaverde et al., 2013).

7 Comparison with a singular Proposer-Decider

Our model can contribute to the discussion on the desirability of checks and balances, since the proposer P, whether a politician or an interest group, cannot have his policy implemented unless it is approved by a gatekeeper, DM. In this section, we compare this setting to having a singular proposer-decider.

The case in which there is a single decision-maker who is uncertain about her competence at implementing a reform, $\pi \in (0, 1)$, is equivalent to assuming that the decision of whether to undertake a reform is taken before the identity of the reformer is revealed. Moreover, the singular decision maker case requires to assume that (s)he cannot observe θ. She only receives signal ρ, with noise z. After observing signal ρ, the decision maker chooses the policy that maximizes her expected utility (y^S, y^C, or the status quo).[9]

The decision-maker's optimal choice at each (κ, π) is illustrated in Figure 4. If the probability of a loss from reform (state θ^C and B competence) is low, then the decision-maker adopts reform y^S regardless of signal, as this reform is most likely to match the state and deliver a benefit. The region of the parameter space where this decision is optimal is illustrated in dark blue. If the probability of state θ^C is high, and competence is expected to be high, the decision-maker chooses y^C regardless of signal, as she expects this reform to match the state and produce a gain. This region of the parameter space is illustrated in dark orange. For intermediate values of κ, there is high uncertainty about the state of the world, and the decision maker relies on her signal to choose policy. She chooses policy y^S after signal $\rho = s$. After $\rho = c$, she expects state θ^C to be more likely. In that case, she wants to implement policy y^C only if she is likely to be competent, i.e., π is sufficiently high (the yellow region in the figure). Otherwise, maintaining the status quo is preferable (the light blue region in the figure). Finally, if κ is very high, the regulator expects state θ^C. If there is high uncertainty about her competence (intermediate π), then she uses the signal to choose y^C after $\rho = c$ and maintain the status quo otherwise (the light orange region in the figure). If she expects to not be competent, then a reform is expected to produce a loss, and she therefore maintains the status quo regardless of signal (the white region in the figure). Cycling can emerge with a single decision-maker, but in a different form and for different reasons than in the main model. There is cycling between implementing a reform

[9]Our main model and the model with a singular proposer-decider may be viewed as two ends of a spectrum. At one end, the proposer alone produces the reform proposal that is then offered for evaluation, while at the other end the decision maker alone produces it. For an analysis of when there may be joint contributions to production, see Kartik et al. (2020).

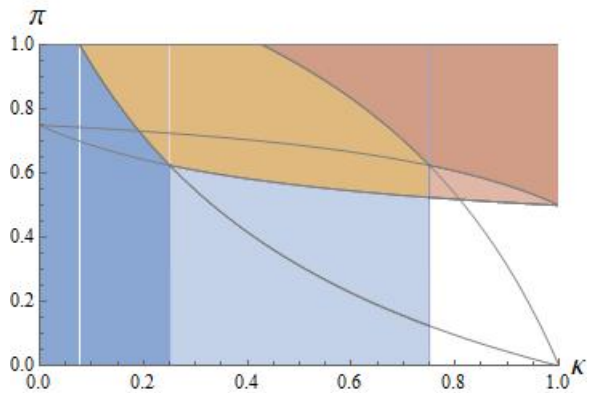


Figure 4: Illustrates the single decision-maker's policy choice given $z = 0.25$. In the dark blue region, y^S is chosen regardless of signal, in the light blue region, y^S is chosen after $\rho = s$ and the status quo is maintained after $\rho = c$. In the dark orange region, y^C is chosen regardless of signal, in the light orange region, y^C is chosen after $\rho = c$ and the status quo is maintained after $\rho = s$. In the yellow region, y^S is chosen after $\rho = s$ and y^C is chosen after $\rho = c$. In the white region, the status quo is maintained regardless of signal.

and implementing no reform, and the cycling is contingent on the signal $\rho = c$.[10] This result has a straightforward intuition: as the decision maker receives more precise information, she acts on that information to implement a more risky, complex reform. As the information becomes less precise, the decision-maker maintains the safe status quo. In contrast to the cycling obtained in the main model, this cycling is driven by the decision maker conditioning reform y^C on the precision of her information. This also implies that the $z^{*S}(\kappa, \pi)$ around which cycling happens in this case increases in its arguments. As π decreases, the decision-maker faces a higher likelihood of a loss from reform y^C, and therefore requires a more precise signal in order to adopt this reform. In our main model the cycling emerges when the decision maker is not making decisions contingent on her signal, while here the cycling is driven by the decision maker acting contingent on her signal. This leads to the contrasting result: For a (κ, π) in the intersection of region $\mathcal{U}$ and this cycling region, cycling that happens at high complexity in the main model happens at low complexity here, and vice-versa.

Given our assumption that the regulator is benevolent, and her payoff equals the social

[10] When z is high, the signal $\rho = c$ is less informative, and the decision-maker is not willing to take a risk of implementing policy y^C, as it only delivers a benefit if the state is θ^C. She then maintains the status quo after $\rho = c$. After signal $\rho = s$, the decision-maker still implements y^S, as state θ^S is sufficiently likely. Implementing y^S decreases the average noise z, which in turn makes the signal more informative. This induces the decision-maker to implement y^C after $\rho = c$.

welfare, it is clear that checks and balances deliver higher expected social welfare than a single uninformed decision maker. Compared to the single decision maker, checks and balances allow for more information to be accessed before a decision is made. This is because the proposer has private information, which is reflected in the equilibrium strategies, and the decision maker forms beliefs that are consistent with those strategies. In sum, checks and balances allow for some access to information (because of the informed proposer) and some rejection of proposals which are expected to bring a loss. They limit the downsides of a single uninformed decision maker. In fact, the solution of the single decision maker could be implemented in the system with checks and balances, as it is in the Pooling region. Yet, strategies with different proposer types proposing different policies yield higher expected welfare.

A more elaborate comparison can be made in terms of the dynamics of complexity. A system of checks and balances does not lead to everywhere higher or to everywhere lower expected complexity of the environment (z) in the long-run compared to a single uninformed decision-maker. Intuitively, on the one hand, checks and balances reduce long-run complexity because policy y^C is implemented less often for most parameter values. With an informed proposer, policy y^C may be offered only after θ^C, and the decision maker may reject proposals for high values of κ and low values of π. On the other hand, checks and balances increase long-run complexity because they support equilibria where y^C is offered more often, i.e., the Complexification equilibrium. The informed proposer chooses y^C more often in order to get approval from the decision maker given her beliefs. There is a debate on whether shifting the authority over approving the details of reforms from legislators to regulators will result in more simplification (as argued by Teles, 2013) or whether removing checks and balances increases instability (as argued by Besley and Mueller, 2018), and by extension complexity, as the environment becomes more uncertain. Our results bring a note of caution to both these theses. As shown above, long-run complexity comparisons depend on the fundamentals.

This insight complements that of Gratton and Morelli (2018). In their model, checks and balances reduce the frequency with which bad reforms are approved (type I errors),

but they also increase the frequency with which good reforms are rejected (type II errors). We also obtain the result that checks and balances help decrease the frequency of type I errors, outside the Complexification region, where she approves y^C less often. Yet, checks and balances increase the frequency of type I errors inside the Complexification region, where the decision maker approves y^C when it follows state θ^S or proposer B.

Finally, we wish to emphasize a feature of the singular decision-maker case in terms of the relationship between the expected ability of proposals and endogenous complexity: in contrast with our finding in proposition 3, in a singular decision maker system without checks and balances z_∞ (weakly) increases in π. Any cycling that could emerge with a singular uninformed proposer-decider relies on the signal being used, leading to a decrease in complexity when the proposer is more likely to be low-ability.

8 Concluding Remarks and Future Directions

In this paper we have analyzed a large class of situations where decision making typically involves an informed but potentially low-ability proposer and a decision maker, typically less informed. In particular, we have endowed the proposer with a choice between proposing new details (in a law or regulation) or proposing elimination of details or contingencies. When the proposer cares about passing her proposal no matter what, her proposal can sometimes be a more detailed law even though the situation is such that this complexification is not beneficial for the general public.

We have first characterized the equilibrium between proposer and decision maker for every pair of common state and private type asymmetric information parameters, and then we have studied the implications of such equilibrium reforms for the endogenous evolution of complexity. In the long run, for a large set of intermediate parameter values, the endogenous level of complexity fluctuates around an intermediate level. This level is increasing in the probability that the proposer is low-ability. This finding highlights that even if complexity relates to the common state information asymmetry, the additional fear of captured or

incompetent proposers actually has negative spillovers on complexity itself.

We showed that the proposed model allows us to nest and reconcile a number of recent findings in legislative politics and regulation studies, and could be useful for institutional design.

In future research, the model may be connected to the literature on endogenous incompleteness of contracts (Tirole, 1999). Adopting a complexification reform can be mapped to making a contract more complete. This may be beneficial or it may be detrimental, depending on the alignment of interests between proposer and decision maker. A regulator may decide not to introduce a proposed completion of a contract for lack of trust that the benefits from the additional contingencies will truly outweigh the costs of writing them and the costs of enforcing them. For instance, the regulator may think that the costs may be on everybody whereas the benefits might be concentrated in the proposing interest group only. Endogenous incompleteness is more likely to emerge when there is a higher likelihood that additional contingencies are beneficial (high κ), the regulator faces high uncertainty (π is not too high), and there is high noise (z is higher). This insight is distinct from that obtained when endogenous incompleteness is due to writing costs alone. In Battigalli and Maggi (2002), where the focus is on endogenous incompleteness due to writing costs, greater uncertainty could cause an increase in the likelihood that more contingencies are beneficial, thus decreasing endogenous incompleteness. Making the link between contract incompleteness and our framework leads to the conjecture that higher uncertainty in the form of higher complexity of the environment could make contracts more incomplete. This in turn may have the collateral effect of transferring more power to some institutions over others, for instance giving more power and discretion to the bureaucracy.

Another important direction for future research is the comparative politics or comparative institutions direction. In the US, policy reforms are delegated to a bureaucratic agency. In Europe, the European Commission or other bureaucratic agencies decide on regulation. But in many other countries the legislators delegate less, and the decision maker is a member of parliament. For this case, we must consider also accountability, coalition formation, etc.

Relatedly, we could extend the analysis to reform complementarities. A complexification reform could induce positive returns when combined with other changes proposed by other proposers on connected issues. This complementarity would reduce the incentive of the regulator to adopt a complex policy, given the uncertainty about the possibility that the complementary reform will also occur.

Some research could also be dedicated to the connection between complexity cycles and the business cycle. At a time in which complexity of the environment is very high it may be good to simplify decision making. Especially in the case in which complementarities exist or coordination is necessary, like in the case of the Covid-19 crisis, it would seem that complex designs could lose all their potential strength if adopted in isolation.

Lastly, our framework has focused on incremental reforms only. A next step would be to also consider the alternative of radical reforms or revolutions. The incremental dynamics of reforms could then be contrasted to the dynamics of major policy changes. In line with the view of Acemoglu and Robinson (2019), our model suggests that the continuous process of incremental reforms may stop when the perception is that more elaborate reforms would be necessary but the elite of proposers is perceived to be bad or captured (low trust in institutions and low confidence in expertise in their terminology). Hence, such situations are exactly those where the world of incremental reforms stops and the chapter of institutional regime change begins.

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A Proofs

A.1 Proof of Proposition 1

The decision-maker's approval strategy :

Given a proposal y, signal ρ and noise z, the DM approves it if

$$\mathbb{E}\left[v\left(\theta, y\right)|y, \rho\right] - w \geq 0. \tag{12}$$

We examine this approval condition after each possible combination of y and ρ:

1. After y^S and $\rho = s$:

$$\frac{\left[\Pr\left(y^S|\theta^H, A\right)\cdot\pi + \Pr\left(y^S|\theta^H, B\right)\cdot(1-\pi)\right]\cdot\kappa}{\left[\Pr\left(y^S|\theta^L, A\right)\cdot\pi + \Pr\left(y^S|\theta^L, B\right)\cdot(1-\pi)\right]\cdot(1-\kappa)} \leq \frac{v}{l}\cdot\frac{1-z}{z} \tag{13}$$

2. After y^S and $\rho = c$:

$$\frac{\left[\Pr\left(y^S|\theta^H, A\right)\cdot\pi + \Pr\left(y^S|\theta^H, B\right)\cdot(1-\pi)\right]\cdot\kappa}{\left[\Pr\left(y^S|\theta^L, A\right)\cdot\pi + \Pr\left(y^S|\theta^L, B\right)\cdot(1-\pi)\right]\cdot(1-\kappa)} \leq \frac{v}{l}\cdot\frac{z}{1-z} \tag{14}$$

3. After y^C and $\rho = s$:

$$\frac{1}{1+\Psi\left(1-z\right)} - \frac{c}{v+l}\cdot\frac{1}{1+\Upsilon\left(1-z\right)} \geq \frac{l}{v+l}, \tag{15}$$

where

$$\Psi\left(1-z\right) \equiv \frac{z\cdot\kappa\cdot\Pr\left(y^C|\theta^H, B\right) + (1-z)\cdot(1-\kappa)\cdot\Pr\left(y^C|\theta^L, B\right)}{z\cdot\kappa\cdot\Pr\left(y^C|\theta^H, A\right) + (1-z)\cdot(1-\kappa)\cdot\Pr\left(y^C|\theta^L, A\right)}\frac{1-\pi}{\pi}, \tag{16}$$

$$\Upsilon\left(1-z\right) \equiv \frac{z\cdot\kappa\cdot\left[\Pr\left(y^C|\theta^H, A\right)\cdot\pi + \Pr\left(y^C|\theta^H, B\right)\cdot(1-\pi)\right]}{(1-z)\cdot(1-\kappa)\cdot\left[\Pr\left(y^C|\theta^L, A\right)\cdot\pi + \Pr\left(y^C|\theta^L, B\right)\cdot(1-\pi)\right]}. \tag{17}$$

4. After y^C and $\rho = c$:

$$\frac{1}{1+\Psi\left(z\right)} - \frac{c}{v+l}\cdot\frac{1}{1+\Upsilon\left(z\right)} \geq \frac{l}{v+l}, \tag{18}$$

where

$$\Psi\left(z\right) \equiv \frac{(1-z)\cdot\kappa\cdot\Pr\left(y^C|\theta^H, B\right) + z\cdot(1-\kappa)\cdot\Pr\left(y^C|\theta^L, B\right)}{(1-z)\cdot\kappa\cdot\Pr\left(y^C|\theta^H, A\right) + z\cdot(1-\kappa)\cdot\Pr\left(y^C|\theta^L, A\right)}\frac{1-\pi}{\pi}, \tag{19}$$

$$\Upsilon\left(z\right) \equiv \frac{(1-z)\cdot\kappa\cdot\left[\Pr\left(y^C|\theta^H, A\right)\cdot\pi + \Pr\left(y^C|\theta^H, B\right)\cdot(1-\pi)\right]}{z\cdot(1-\kappa)\cdot\left[\Pr\left(y^C|\theta^L, A\right)\cdot\pi + \Pr\left(y^C|\theta^L, B\right)\cdot(1-\pi)\right]}. \tag{20}$$

Pure Strategy Equilibria We consider all the possible pure strategy equilibria where there is a positive probability of acceptance:

1. Equilibrium where B proposes y^S, and A proposes y^S after θ^S and y^C after θ^C

The DM approves with probability 1 after observing y^C. After observing y^S, the DM's response is

- after signal $\rho = s$, approve if

$$1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} \cdot \frac{1-z}{z} \leq \pi, \tag{21}$$

- after $\rho = c$, approve if

$$1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} \cdot \frac{z}{1-z} \leq \pi. \tag{22}$$

Thus, the DM approves regardless of signal (and this equilibrium exists) if

$$\pi \geq 1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} \cdot \frac{z}{1-z}. \tag{23}$$

The DM's expected payoff given this equilibrium play is

$$U^{(1)} - \omega = \kappa \cdot \pi \cdot v - \kappa \cdot (1-\pi) \cdot (l-a) + (1-\kappa) \cdot v. \tag{24}$$

2. Equilibrium where all proposers chose y^S after θ^S and y^C after θ^C After observing y^S, the DM approves with probability one. After observing y^C, the DM approves if

$$\pi \geq \frac{l}{v+l}. \tag{25}$$

The DM's expected payoff given this equilibrium play is

$$U^{(2)} - \omega = \kappa \cdot \pi \cdot v - \kappa \cdot (1-\pi) \cdot l + (1-\kappa) \cdot v. \tag{26}$$

3. Equilibrium where proposer A chooses y^C for all θ and proposer B chooses y^S if θ^L and y^C if θ^H.

After observing y^S, the DM approves with probability 1. After observing y^C, the DM approves in the following cases:

- after $\rho = c$, if

$$\pi \geq \pi^{mch} = l \cdot \frac{(1-z) \cdot \kappa}{(1-z) \cdot \kappa \cdot (v+l) + (v-a) \cdot z \cdot (1-\kappa)}. \tag{27}$$

- after $\rho = s$, if

$$\pi \geq \pi^{mcl} = l \cdot \frac{z \cdot \kappa}{z \cdot \kappa \cdot (v + l) + (v - a) \cdot (1 - z) \cdot (1 - \kappa)}. \tag{28}$$

Since $\frac{1-z}{z} \geq \frac{z}{1-z}$, we have

$$\pi^{mch} \geq \pi^{mcl}. \tag{29}$$

Thus, this equilibrium exists if $\pi \geq \pi^{mch}$. The DM's expected payoff given this equilibrium play is

$$U^{(3)} - \omega = \kappa \cdot \pi \cdot v - \kappa \cdot (1 - \pi) \cdot l + (1 - \kappa) \cdot \pi \cdot (v - a) + +(1 - \kappa) \cdot (1 - \pi) \cdot v. \tag{30}$$

4. Pooling on y^S for all θ:
The DM's approval condition reduces to

1. If $\rho = c$:

$$(1 - z) \cdot (1 - k) \cdot v - z \cdot k \cdot l \geq 0. \tag{31}$$

2. If $\rho = s$:

$$z \cdot (1 - k) \cdot v - (1 - z) \cdot k \cdot l \geq 0. \tag{32}$$

Thus, an equilibrium with pooling on y^S (regardless of θ) and probability one of approval exists if

$$k \leq k^{pool} \equiv \frac{z \cdot v}{z \cdot v + (1 - z) \cdot l} \tag{33}$$

An equilibrium with pooling on y^S and approval conditional on $\rho = s$ exists if

$$k \leq k^{cp} \equiv \frac{(1 - z) \cdot v}{(1 - z) \cdot v + z \cdot l}, \tag{34}$$

and the DM's off path belief is that a deviation has come from proposer B.

The DM's expected payoff given the equilibrium play with conditional approval is

$$U^{(4)} - \omega = -z \cdot \kappa \cdot l + (1 - z) \cdot (1 - \kappa) \cdot v. \tag{35}$$

5. Pooling on y^C regardless of θ
If the proposer only offered y^C the DM's approval decision reduces to:

- After $\rho = c$:

$$\pi \geq \frac{l}{v + l} + \frac{c}{v + l} \cdot \frac{1}{1 + \frac{(1-z) \cdot \kappa}{z \cdot (1-\kappa)}}, \tag{36}$$

- After $\rho = s$:

$$\pi \geq \frac{l}{v + l} + \frac{c}{v + l} \cdot \frac{1}{1 + \frac{z \cdot \kappa}{(1-z) \cdot (1-\kappa)}}. \tag{37}$$

If y^S were proposed (off-equilibrium), then the DM's expected payoff from accepting, given her belief about who deviated is

$$-\widehat{\kappa} \cdot l + (1 - \widehat{\kappa}) \cdot v, \tag{38}$$

so a deviation is not profitable as long as

$$\widehat{\kappa} > \frac{v}{v + l},$$

where $\widehat{\kappa} = \Pr(\theta^C | y^S)$ is the belief that a deviation is undertaken in state θ^C. Then, an equilibrium with pooling on y^C and positive probability of acceptance exists and

- If
$$\pi \in \left[\frac{l}{v+l} + \frac{c}{v+l} \cdot \frac{1}{1 + \frac{(1-z)\cdot\kappa}{z\cdot(1-\kappa)}}, \frac{l}{v+l} + \frac{c}{v+l} \cdot \frac{1}{1 + \frac{z\cdot\kappa}{(1-z)\cdot(1-\kappa)}} \right], \tag{39}$$
the DM accepts contingent on $\rho = c$;

- If
$$\pi \geq \frac{l}{v+l} + \frac{c}{v+l} \cdot \frac{1}{1 + \frac{z\cdot\kappa}{(1-z)\cdot(1-\kappa)}}, \tag{40}$$
the DM accepts regardless of signal.

The DM's expected payoff given this equilibrium play is

$$U^{(5)} - \omega = \pi \cdot v - (1 - \pi) \cdot l - (1 - \kappa) \cdot c. \tag{41}$$

6. Equilibrium where proposer A offers y^S after θ^S and y^C after θ^C, and proposer B offers y^C in all states

After y^S, the DM accepts with probability 1. After, y^C the DM approves if

- after signal $\rho = c$:
$$\pi \geq \frac{l \cdot (1 - z) \cdot \kappa + (l + c) \cdot z \cdot (1 - \kappa)}{(v + l) \cdot (1 - z) \cdot \kappa + (l + c) \cdot z \cdot (1 - \kappa)}. \tag{42}$$

- after signal $\rho = s$:
$$\pi \geq \frac{l \cdot \kappa + (a + l) \cdot \frac{1-z}{z} \cdot (1 - \kappa)}{(v + l) \cdot \kappa + (a + l) \cdot \frac{1-z}{z} \cdot (1 - \kappa)}. \tag{43}$$

Thus, the DM approves regardless of signal if

$$\pi \geq \frac{l \cdot \kappa + (a + l) \cdot \frac{1-z}{z} \cdot (1 - \kappa)}{(v + l) \cdot \kappa + (a + l) \cdot \frac{1-z}{z} \cdot (1 - \kappa)}. \tag{44}$$

The DM's expected payoff given this equilibrium play is

$$U^{(6)} - \omega = \kappa \cdot \pi \cdot v - \kappa \cdot (1-\pi) \cdot l + (1-\kappa) \cdot \pi \cdot v + -(1-\kappa) \cdot (1-\pi) \cdot (l+c). \tag{45}$$

Ranking on Equilibria Notice then that $U^{(1)} \geq U^{(2)} > U^{(3)} > U^{(6)} > U^{(4)}$. Also, $U^{(6)} > U^{(5)}$, and Then,

$$\pi_3 \equiv 1 - \frac{v}{l} \cdot \frac{1-\kappa}{\kappa} \cdot \frac{z}{1-z}. \tag{46}$$

$$\pi_2 \equiv \frac{l}{v+l}. \tag{47}$$

$$\pi_1 \equiv \frac{l \cdot (1-z) \cdot \kappa}{(1-z) \cdot \kappa \cdot (v+l) + (v-a) \cdot z \cdot (1-\kappa)}. \tag{48}$$

Also, let

$$\pi^{pool} \equiv \frac{l}{v+l} + \frac{c}{v+l} \cdot \frac{1}{1 + \frac{z \cdot \kappa}{(1-z) \cdot (1-\kappa)}} \tag{49}$$

$$\pi^{compl} \equiv \frac{l \cdot \kappa + (a+l) \cdot \frac{1-z}{z} \cdot (1-\kappa)}{(v+l) \cdot \kappa + (a+l) \cdot \frac{1-z}{z} \cdot (1-\kappa)} \tag{50}$$

Notice that $\pi_2 \leq \pi^{pool}$ and $\pi_2 \leq \pi^{compl}$ for any κ and z. Thus, the parameter values for which equilibria 5 and 6 exist are also the parameter values for which equilibrium 2 exists. Notice also that at k^{pool}, we have $\pi_3(k^{pool}) = 0$, and π_3 increases in k. Thus the equilibrium with pooling at y^S at approval probability one exists only when equilibrium 1 also exists. However, equilibrium 4 with approval conditional on $\rho = s$ exists since $\pi(k^{cp}) > 0$.

Thus, the Best Perfect Bayesian equilibrium may take forms 1-4, with the boundaries between these regions given by $\pi_1 - \pi_3$.

Other proposer strategies. Notice that the other possible pure strategies are not part of an equilibrium in which there is a positive probability of acceptance. Specifically, if all proposers offer y^S after θ^H and y^C after θ^L, the DM would surely reject after y^S. Thus, this cannot be an equilibrium. Similarly, consider the case where proposer A chooses y^S in all states and proposer B chooses y^S after θ^S and y^C after θ^C. After y^C, the DM rejects with probability 1. Thus, this cannot be an equilibrium.

A.2 Proof of Corollary 1

Follows directly from Proposition 1.

A.3 Proof of Corollary 2

The boundary of the region where proposals are accepted with probability one is given by

$$\pi^B \equiv \min\{\pi_1, \pi_2, \pi_3\}. \tag{51}$$

From (46) - (48), it follows that

$$\frac{\partial \pi_3}{\partial z} \leq 0; \frac{\partial \pi_2}{\partial z} = 0; \frac{\partial \pi_1}{\partial z} \leq 0. \tag{52}$$

Also, the upper bound $\bar{\kappa}$ for the Pooling region changes as follows:

$$\frac{\partial \bar{\kappa}}{\partial z} = \frac{-v \cdot l}{((1-z) \cdot v + z \cdot l)^2} < 0. \tag{53}$$

Then, π^B weakly decreases in z at each κ. Thus, the boundaries of the Simplification and the Complexification equilibria expand. At $z = \frac{1}{2}$, $\pi_3 \geq 0$ for $\kappa \leq \frac{v}{l+v}$. Thus, in the region where $\kappa \leq \frac{v}{l+v}$, $\pi^B \geq 0$ and it increases. The $BPBE$ becomes ones in which the proposal is accepted with probability one, instead of the Pooling equilibrium, where the proposal is accepted with probability $z \cdot \kappa + (1-z) \cdot (1-\kappa)$.

If $\kappa > \frac{v}{l+v}$, the bound $\bar{\kappa}$ decreases as z increases:

$$\frac{\partial \bar{\kappa}}{\partial z} = \bar{\kappa} \cdot \frac{-v}{1-z}. \tag{54}$$

The bound π_1 decreases as z increases:

$$\frac{\partial \pi_1}{\partial z} = \pi_1 \cdot \frac{-(v-a) \cdot (1-\kappa)}{1-z}. \tag{55}$$

Thus, the region where the proposal is rejected changes approximately by $-\frac{\partial \bar{\kappa}}{\partial z} \cdot \pi_1(\bar{\kappa}, z) + \frac{\partial \pi_1}{\partial z} \cdot \bar{\kappa}$. Notice that

$$\frac{\frac{\partial \bar{\kappa}}{\partial z} \cdot \pi_1(\bar{\kappa}, z)}{\frac{\partial \pi_1}{\partial z} \cdot \bar{\kappa}} = \frac{v}{(v-a) \cdot (1-\kappa)} > 1. \tag{56}$$

Thus, the region of the parameter space (κ, π) where the proposal is rejected expands as z increases. At the maximum z, $\bar{\kappa} = \frac{v}{l+v}$. Thus, the region where the proposal is rejected expands as z increases, from a lower bound $\bar{\kappa} \to 1$ as $z^{\min} \to 0$ to $\frac{v}{/} v + l$ when $z = \frac{1}{2}$.

A.4 Proof of Proposition 2

Notice that the space (κ, π) may be divided into the following regions

Absorbing. Consider the Simplification $BPBE$. The boundary $\pi_3(\kappa, \pi)$ weakly decreases in z. Thus, the $BPBE$ is absorbing and the same for any $z_0 \in [z^{\min}, 0.5]$ if $\pi \geq \pi_3(\kappa, z^{\min})$. As $z^{\min} \to 0$, $\pi_3(\kappa, z^{\min}) \to 1$, meaning that this region contracts to 0.

Consider the Matching $BPBE$. The lower bound $\pi_2 = \frac{l}{l+v}$ does not change with z. However, the upper bound of this region, given by π_3 decreases in z. Thus, the region where $\pi \in \left[\pi_2, \pi_3(\kappa, \frac{1}{2})\right)$ is a Matching $BPBE$ for all $z \in [z^{\min}, 0.5]$. It is therefore absorbing for all $z \in [z^{\min}, 0.5]$.

Consider the Complexification $BPBE$. The upper bound $\pi_2 = \frac{l}{l+v}$ does not change with z. The lower bound for this region is given by $\pi_1(\kappa, z)$, which decreases in z. Thus, the $BPBE$ is the Complexification equilibrium for all z if $\pi \in \left[\pi_1(\kappa, z^{\min}), \pi_2\right)$. As $z^{\min} \to 0$, the interval for π contracts to 0.

Consider the Pooling $BPBE$. The boundary of the region where Pooling is the $BPBE$ expands as z decreases: π_1, π_3, and $\bar{\kappa}$ all increase. The value of $\bar{\kappa}$ is $\frac{v}{l+v}$ at $z = 0.5$ and converges to 0 as $z^{\min} \to 0$. Thus, for any (κ, π), there exists a value z' at which the BPBE is Pooling with approval conditional on signal for $z < z'$ and Pooling with rejection for $z > z'$. Hence, the $BPBE$ is absorbing.

Not absorbing. Consider the regions where the $BPBE$ is not absorbing. This implies that for each (κ, π) in these regions, there exists $z'(\kappa, \pi) \in (z^{\min}, 0.5)$ at which (κ, π) is on the boundary between two different $BPBEs$.

Consider first the boundary $\pi_3(\kappa, z)$, with $z'(\kappa, \pi)$ defined as $\pi = \pi_3(\kappa, z')$. For $\pi_3(\kappa, z) \geq \pi_2$, the boundary is between the Simplification $BPBE$, when $z \geq z'$ and the Matching $BPBE$, when $z < z'$. In both of these equilibria, either y^S or y^C may be proposed along the equilibrium path. Thus, we can construct an path of possible realizations of θ and P such that y^C is proposed when $z < z'$ and y^S when $z > z'$. Along such a path, the $BPBE$ switches between Simplification and Matching, and it is hence not absorbing. Each location (κ, π) at which $\pi = \pi_3(\kappa, z') > \pi_2$ is therefore not absorbing.

Let κ^M be defined as the value at which $\pi_3(\kappa^M, \frac{1}{2}) = \pi_1(\kappa^M, \frac{1}{2})$. For any $\kappa \leq \kappa^M$, let π_4 be defined as the value at which the following equality is satisfies for some z in $\left[z^{\min}, \frac{1}{2}\right]$: $\pi_3(\kappa, z) = \pi_1(\kappa, z)$. Given the expressions for $\pi_3(\kappa, z)$ and $\pi_1(\kappa, z)$, we obtain

$$\pi_4 = 1 - \frac{\sqrt{((v)^2 - l \cdot c)^2 + 4 \cdot l \cdot (v)^3} - ((v)^2 + l \cdot c)}{2 \cdot l \cdot (v - a)}. \tag{57}$$

Hence, for $\pi \geq \pi_4$ and z, the $BPBE$ is Simplification if $z \geq z'$ and it is Complexification if $z < z'$. Thus, if $\pi_3(\kappa, z) \in [\pi_4, \pi_2)$, then the boundary is between the Simplification $BPBE$, when $z \geq z'$ and the Complexification $BPBE$, when $z < z'$. In both of these equilibria, both y^S and y^C may be proposed along the equilibrium path. Thus, as above, we can construct an path of possible realizations of θ and P such that y^C is proposed when $z < z'$ and y^S when $z > z'$. Along such a path, the $BPBE$ switches between Simplification and Complexification, and it is hence not absorbing. The same analysis holds for each location (κ, π) at which $\pi = \pi_3(\kappa, z') \in [\pi_4, \pi_2)$.

For $\pi < \pi_4$, the $BPBE$ is Pooling if $z \leq z'$, and it is Simplification if $z > z'$. In the Pooling equilibrium, proposals y^C are not approved. If $z \leq z'$, we are in the Pooling equilibrium. Once there, z may only further decrease. Hence, the $BPBE$ is absorbing for $z < z'$. If $z_0 > z'$, then any realized sequence of θ and P results in either (i) $z_t \leq z'$ in some period t, in which case the $BPBE(\kappa, \pi, z_t)$ is absorbing, or (ii) $z_t > z'$ for all $t > 0$, in which case the $BPBE(\kappa, \pi, z_t)$ is absorbing as well.

Consider next the boundary $\pi_1(\kappa, z)$, with $z'(\kappa, \pi)$ now defined as $\pi = \pi_1(\kappa, z')$. The case where $\pi_1(\kappa, z) = \pi_3(\kappa, z)$ was discussed above. For $\kappa > \kappa^M$, the $BPBE$ is Pooling if $z \leq z'$, and it is Complexification if $z > z'$ and $\pi < \pi_2$. In the Pooling equilibrium, proposals y^C are not approved and thus z may only decrease (or stay the same). Hence, the $BPBE$ is absorbing for $z < z'$. If $z_0 > z'$, then any realized sequence of θ and P results in either (i) $z_t \leq z'$ in some period t, in which case the $BPBE(\kappa, \pi, z_t)$ is absorbing, or (ii) $z_t > z'$ for all $t > 0$, in which case the $BPBE(\kappa, \pi, z_t)$ is absorbing as well.

Consider $z_0 \in \left(z^{min}, \frac{1}{2}\right)$ and a location (κ, π) such that $\pi_3(\kappa, z_0) \leq \pi$. Then, z_t, for $t = 1, 2, ...$ is expected to evolve as follows:

$$z_t = z_{t-1} + \Pr(y^C, d = 1) \cdot \Delta - \Pr(y^S, d = 1) \cdot \Delta. \tag{58}$$

Thus, z_t is expected to decrease if $\Pr(y^C, d = 1) < \Pr(y^S, d = 1)$, and it is expected to increase if $\Pr(y^C, d = 1) > \Pr(y^S, d = 1)$. For the Simplification $BPBE$, given the equilibrium strategies, $\Pr(y^C, d = 1) < \Pr(y^S, d = 1)$ reduces to

$$\kappa \cdot \pi < \kappa \cdot (1 - \pi) + (1 - \kappa), \tag{59}$$

or

$$\kappa < \frac{1}{2\pi}. \tag{60}$$

If at period t the noise z_t decreases to z^* such that $\pi \leq \pi_3(\kappa, z^*)$, the Simplification $BPBE$ is not longer achievable. The equilibrium play switches to another $BPBE$. If this $BPBE$ is Matching, then given the equilibrium strategies, $\Pr(y^C, d = 1) > \Pr(y^S, d = 1)$ reduces to

$$\kappa > 1 - \kappa. \tag{61}$$

Thus,

$$\kappa > \frac{1}{2}. \tag{62}$$

If condition (62) is satisfied, z_t is expected to increase, which lowers $\pi_3(\kappa, z_t)$. Then, for $z_t \geq z^*$, $\pi \geq \pi_3(\kappa, z^*)$, and the $BPBE$ switches to Simplification. Thus, under conditions (60) and (62), we obtain cycling between Simplification and Matching.

In the Complexification $BPBE$, given the equilibrium strategies, $\Pr(y^C, a = 1) > \Pr(y^S, a = 1)$ reduces to

$$\kappa + (1 - \kappa) \cdot \pi > (1 - \kappa) \cdot (1 - \pi). \tag{63}$$

Thus,

$$\kappa > \frac{1-2\pi}{2(1-\pi)}. \tag{64}$$

Then, under conditions (60) and (64), we obtain cycling between Simplification and Complexification, i.e., when $\pi > \max\{\pi_4, \pi_3\}$.

Frequency of Fluctuations The frequency of fluctuations is highest when the number of periods needed to cross from one region to the other and back is minimized, i.e., when $\Pr(y^S|\text{Simplification})$ and $\Pr(y^C|\text{Matching/Complexification})$ are maximized. For cycling between Simplification and Matching, $\Pr(y^S|\text{Simplification}) = 1-\kappa\cdot\pi$ and $\Pr(y^C|\text{Matching}) = \kappa$. Then, $1-\kappa\cdot\pi+\kappa$ decreases in π and it increases in κ.

For cycling between Simplification and Complexification, $\{\Pr(y^S|\text{Simplification}) = 1-\kappa\cdot\pi$ and $\Pr(y^C|\text{Complexification}) = \kappa+\pi\cdot(1-\kappa)$. The frequency of fluctuations is then increasing (decreasing) in κ when $\pi < (>)\frac{1}{2}$, and it is increasing (decreasing) in π when $\kappa < (>)\frac{1}{2}$.

A.5 Proof of Proposition 3

From (11), it follows immediately that $\frac{\partial z^*}{\partial \pi} < 0$ and $\frac{\partial z^*}{\partial \kappa} > 0$.

A.6 Proof of Proposition 4

As shown in the proof to Proposition 2, in the Simplification $BPBE$, z decreases on average if condition (60) is satisfied, i.e., $\kappa\cdot\pi < \frac{1}{2}$. Then, z increases on average if $\kappa\cdot\pi > \frac{1}{2}$. Finally, the noise is expected to stay at the status quo value if $\kappa\cdot\pi = \frac{1}{2}$.

In the Matching $BPBE$, from (62) expected z_t increases. The Matching $BPBE$ is absorbing if $\pi_2 = \frac{l}{l+v} < 1 - \frac{v}{l}\cdot\frac{1-\kappa}{\kappa} = \pi_3(\kappa, 1/2)$. This can be re-written as

$$\kappa > \frac{l+v}{l+v+l} > \frac{1}{2}. \tag{65}$$

In the Complexification BPE, if Condition (64) is satisfied, average z to increase. Otherwise, average z decreases. In the Pooling BPE, only proposals y^S are made, and they are approved conditional on $\rho = s$. Every period, z_t is thus expected to decrease with probability $(1-z)\cdot(1-\kappa)+z\cdot\kappa$.In the Rejection $BPBE$, no proposal is approved, thus $z_t = z_0$, for all $t > 0$.

Finally, in the regions where there is cycling, the cycling happens around the boundary between regions, i.e., around the value of z at which the location (κ, π) is on the boundary. Since cycling only happens between the Simplification region and another region (either Matching or Complexificaton), the boundary is given by $\pi = \pi_3(\kappa, z)$. Thus, the location (κ, π) is on the boundary at noise z^* defined implicitly by $\pi = \pi_3(\kappa, z^*)$.

A.7 Proof of Corollary 3

Follows directly from Proposition 4.

A.8 Proof of Proposition 5

Consider the problem in the first period of the $DM's$ tenure. The DM approves y^C given (κ, π, z)if $u\left(y^C|\rho\right) + U(\kappa, \pi, z + \Delta)$. The DM approves a simple proposal if $u\left(y^S|\rho\right) + U(\kappa, \pi, z + \Delta)$.

Notice that the $DM's$ expected continuation value is the same if $z + \Delta$ and $z - \Delta$ are in the same equilibrium region in the next period. Thus, given (κ, π), if z is Δ away from the boundaries π_1, π_2, π_3, or $\overline{\kappa}(z)$, then the continuation value for the DM does not differ based on whether y^C, y^S, or ω (rejection) is the implemented policy. In that case, the DM has the same problem as a one-term DM. Notice also that if $\pi_1(\kappa, \pi) > \pi_2$, then the expected utility for the DM is the same regardless of the region (because the DM receives $-l$ after proposer B and θ^C, regardless of which policy is proposed). Thus, if $\pi \geq \pi_2$, the DM has the same expected utility in the second period regardless of what is chosen in the first period.

If $\pi < \pi_2$, then we have the following cases:

1. If $\pi > \pi_3(\kappa, z), \pi < \pi_1(\kappa, z)$ and $\pi < \pi_3(\kappa, z - \Delta)$: if a proposal is rejected, the DM is in the Simplification region in the next period. If a complex proposal is accepted, the DM is in the Simplification region. If a simple proposal is accepted, the DM moves to the Pooling region.

   In the Simplification equilibrium, a proposal y^C only comes from type A in state θ^C. Thus it pays off v. A proposal y^S may come from either type. The boundary π_3 gives the minimum π above which the DM would accept y^S after a signal $\rho = c$ in the one-shot game. Thus, at π_3, the DM is indifferent between accepting and rejecting y^S in the one-shot game. If the DM accepts y^S, z decreases and the second period expected utility becomes $[\kappa \cdot (z - \Delta) \cdot (-l) + (1 - \kappa) \cdot (1 - z + \Delta) \cdot v]$. If the DM rejects y^S, the noise z does not change and so we have expected utility in the second period $[\kappa \cdot \pi \cdot v + \kappa \cdot (1 - \pi) \cdot (-l) + (1 - \kappa) \cdot v]$. The difference in expected utilities in the next period period (between acceptance and rejection) is

   $$-(z - \Delta) \cdot [\kappa l + (1 - \kappa) v] - \kappa \cdot [\pi v - (1 - \pi) l]$$

   If $\Delta = 0$, this difference is negative. Moreover, the difference is increasing in Δ. Hence, there exists a maximum value of Δ such that if Δ is below this threshold, the difference is negative. This is equivalent to saying that the outside option of the DM has increased. Then, boundary $\widehat{\pi}_3$ at which the DM is indifferent between accepting and rejecting y^S must satisfy $\widehat{\pi}_3(\kappa, z) > \pi_3(\kappa, z)$. Since the outside option is higher, the DM requires a higher probability that the proposer is the A-type.

2. If $\pi < \pi_3(\kappa, z)$ and $\pi > \pi_3(\kappa, z + \Delta)$: if a proposal is rejected, the DM is in the Pooling

region in the next period. Only simple proposals are made in equilibrium, so this we stay in the Pooling region as z decreases.

3. If $\pi > \pi_1(\kappa, z)$ and $\pi < \pi_1(\kappa, z - \Delta)$: if a proposal is rejected, the DM is in the Complexification region in the next period. If a complex proposal is accepted, the DM is in the Complexification region. If a simple proposal is accepted, the DM moves to the Pooling region. In the Complexification equilibrium, a proposal y^S only comes from type B in state θ^S. Thus it pays off v. The total expected value to the DM from approving is $v - z \cdot [\kappa l + (1-\kappa) v] + \Delta \cdot [(1-\kappa) v + \kappa l]$. The total expected value from rejecting is $0 + [\kappa \cdot \pi \cdot v + \kappa \cdot (1-\pi) \cdot (-l) + (1-\kappa) \cdot (1-\pi) v + (1-\kappa) \pi (-l)]$. This implies $\widehat{\pi}_1(\kappa, z) > \pi_1(\kappa, z)$.

4. If $\pi > \pi_3(\kappa, z), \pi > \pi_1(\kappa, z)$ and $\pi < \pi_3(\kappa, z - \Delta)$: if a proposal is rejected, the DM is in the Simplification region in the next period. If a complex proposal is accepted, the DM is in the Simplification region. If a simple proposal is accepted, the DM moves to the Complexification region. If the DM rejects y^S, the noise z does not change and so we have expected utility in the second period $[\kappa \cdot \pi \cdot v + \kappa \cdot (1-\pi) \cdot (-l) + (1-\kappa) \cdot v]$. If the DM accepts y^S, z decreases and the second period expected utility becomes $[\kappa \cdot \pi \cdot v + \kappa \cdot (1-\pi) \cdot (-l) + (1-\kappa) \cdot (1-\pi) v + (1-\kappa) \pi (-l)]$. This is clearly lower than under Simplification. Thus, $\widehat{\pi}_3(\kappa, z) > \pi_3(\kappa, z)$.

5. If $\pi_1(\kappa, z) < \pi < \pi_3(\kappa, z)$, and $\pi > \pi_3(\kappa, z + \Delta)$: if a proposal is rejected, the DM is in the Complexification region in the next period. If a complex proposal is approved, the DM moves to the Simplification region. If the DM rejects y^C, the noise z does not change and so we have expected utility in the second period

$$[\kappa \cdot \pi \cdot v + \kappa \cdot (1-\pi) \cdot (-l) + (1-\kappa) \cdot (1-\pi) v + (1-\kappa) \pi (-l)].$$

If the DM approves y^C, z increases and the second period expected utility becomes $[\kappa \cdot \pi \cdot v + \kappa \cdot (1-\pi) \cdot (-l) + (1-\kappa) \cdot v]$. If y^S is proposed, z decreases and the equilibrium play next period is Complexification. Thus, approval of either y^C or y^S dominates rejection.

A.9 Proof of Corollary 4

Notice that for $\pi \geq \pi_2$, the $BPBE$ regions are the same for a two-term DM as for a one-term DM. For $\pi < \pi_2$, $\pi_1(\kappa, z)$ and $\pi_3(\kappa, z)$ in the first term of a two-term DM are lower than the corresponding boundaries for the one-term DM. Thus, in the first period of a two-term DM, the Pooling $BPBE$ expands in the (κ, π) space, while the Simplification and Complexification $BPBE$ regions contract. Given Proposition 4, the change from the Simplification $BPBE$ to the Pooling $BPBE$ does not change the long-run complexity. The change from the Complexification $BPBE$ to the Pooling $BPBE$ decreases long-run complexity from $\frac{1}{2}$ to either $z^{\min}$ or z_0. Thus, overall, expected long-run complexity decrease.

A.10 Proof of Proposition 6

Consider the case in which the proposer's payoff is given by

$$V^p(y|\theta, z_t) = \mathbb{1}_{d(\kappa,\pi,z_t)=1} + Pr(P = A|\kappa, \pi, y_{t-1}, u_{t-1}) \cdot V, \tag{66}$$

where V is the reputational payoff the proposer derives and u_{t-1} is the payoff to the DM in the previous period. We assume that $1 > V$, such that the Proposer still has strict preference for getting his proposal approved each period. In the second-period of the proposer's tenure, we have

$$Pr(P = A|y_{t-1}) = \begin{cases} 1, \text{ if } y_{t-1} = y^C \text{ and } u_{t-1} \in \{v - a, v\}, \\ 1, \text{ if } y_{t-1} = y^S \text{ and } r_A(\theta, z_{t-1}) = y^S,\ r_B(\theta, z_{t-1}) = y^C, \\ \pi, \text{ if } y_{t-1} = y^S \text{ and } r_A(\theta, z_{t-1}) = r_B(\theta, z_{t-1}), \\ 0, \text{ otherwise.} \end{cases} \tag{67}$$

Consider therefore the proposals in the first period of the proposer's tenure. If the DM approves with probability one unconditional on signal or reform type, then Proposer A has a strict gain from proposing y^C over y^S. Thus, any equilibrium with probability of approval 1 must have $r_A(\theta, z_t) = y^C$. Proposer B has a strict gain from proposing y^S if and only if $r_A(\theta, z_{t-1}) = r_B(\theta, z_{t-1})$. Otherwise, he is indifferent, as his type is reveled under any other equilibrium. Thus, the only possible equilibria where the DM approves with probability one are the Complexification equilibrium and the equilibrium where all players offer y^C: $r_A(\theta, z_{t-1}) = r_B(\theta, z_{t-1}) = y^C$ and $d(\rho, y^C) = 1, d(\rho, y^S) = 0$. This latter equilibrium is Pareto dominated by the Complexification equilibrium for the period$-t$ players. If the DM approves conditional on reform type and/or signal, then Pooling on y^S is an equilibrium. Moreover, $d(\rho, y^S) = 1$ if $\kappa \leq \frac{v \cdot z}{v \cdot z + l \cdot (1-z)}$. We have $d(\rho = s, y^S) = 1$ and $d(\rho = c, y^S) = 0$ if $\kappa \in \left(\frac{v \cdot z}{v \cdot z + l \cdot (1-z)}, \frac{v \cdot (1-z)}{v \cdot (1-z) + l \cdot z}\right]$. Finally, $d(\rho, y^S) = 0$ if $\kappa > \frac{v \cdot (1-z)}{v \cdot (1-z) + l \cdot z}$. Finally, notice that the Pooling equilibrium given the B-type a higher expected payoff if $\kappa \leq \frac{v \cdot z}{v \cdot z + l \cdot (1-z)}$. Thus, the raking of equilibria where Complexification dominates Pooling assumes a higher weight on the payoff of the DM relative to that of the Proposer.

A.11 Proof of Remark 1

Follows as an application of Corollary 1, assuming in one case a jump from $\pi_0 > \pi_2$ to $pi_1 \in (\pi_1, \pi_2)$ and in the other case a jump from $\pi_0' < \pi_1$ to $pi_1' > \pi_2$.

A.12 Proof of Remark 2

Follows from Proposition 2.

A.13 The Single-Decision Marker's Policy Choice

After signal $\rho = s$, the decision-maker gets the following gain over the status quo:

- if she implements y^S:

$$v \cdot \frac{(1-z)\cdot(1-\kappa)}{(1-z)\cdot(1-\kappa)+z\cdot\kappa} - l \cdot \frac{z\cdot\kappa}{(1-z)\cdot(1-\kappa)+z\cdot\kappa}. \tag{68}$$

- if she implements y^C:

$$v\cdot\pi - l\cdot(1-\pi) - a\cdot\frac{(1-z)\cdot(1-\kappa)}{(1-z)\cdot(1-\kappa)+z\cdot\kappa}. \tag{69}$$

After signal $\rho = c$, the decision-maker gets the following gain over the status quo:

- if she implements y^S:

$$v \cdot \frac{z\cdot(1-\kappa)}{z\cdot(1-\kappa)+(1-z)\cdot\kappa} - l \cdot \frac{(1-z)\cdot\kappa}{z\cdot(1-\kappa)+(1-z)\cdot\kappa}. \tag{70}$$

- if she implements y^C:

$$v\cdot\pi - l\cdot(1-\pi) - a\cdot\frac{z\cdot(1-\kappa)}{z\cdot(1-\kappa)+(1-z)\cdot\kappa}. \tag{71}$$

Since $z \leq \frac{1}{2}$, notice that

$$\frac{(1-z)\cdot(1-\kappa)}{(1-z)\cdot(1-\kappa)+z\cdot\kappa} \geq \frac{z\cdot(1-\kappa)}{z\cdot(1-\kappa)+(1-z)\cdot\kappa}. \tag{72}$$

Thus, the decision-maker:

1. implements y^S regardless of signal if

$$\pi \leq \pi^{sb}(\kappa, z) \equiv \frac{z\cdot(1-\kappa)}{z\cdot(1-\kappa)+(1-z)\cdot\kappa}\cdot\frac{v+l+c}{v+l} \text{ and} \tag{73}$$

$$\kappa \leq \kappa^{rbh} \equiv \frac{z\cdot v}{z\cdot v+(1-z)\cdot l}. \tag{74}$$

2. implements y^C regardless of signal if

$$\pi \geq \pi^{cb}(\kappa, z) \equiv \frac{(1-z)\cdot(1-\kappa)}{(1-z)\cdot(1-\kappa)+z\cdot\kappa}\cdot\frac{v+l+c}{v+l} \text{ and} \tag{75}$$

$$\pi \geq \pi^{rbl}(\kappa, z) \equiv \frac{l}{l+v} + \frac{c}{l+v}\cdot\frac{(1-z)\cdot(1-\kappa)}{(1-z)\cdot(1-\kappa)+z\cdot\kappa}. \tag{76}$$

3. implements y^S after $\rho = s$ and y^C after $\rho = c$ if

$$\pi \in \left(\pi^{sb}(\kappa, z), \pi^{cb}(\kappa, z)\right) \text{ and} \tag{77}$$

$$\pi \geq \pi^{rbh}(\kappa, z) \equiv \frac{l}{l+v} + \frac{c}{l+v} \cdot \frac{z \cdot (1-\kappa)}{z \cdot (1-\kappa) + (1-z) \cdot \kappa} \text{ and} \tag{78}$$

$$\kappa \leq \kappa^{rbl} \equiv \frac{(1-z) \cdot v}{(1-z) \cdot v + z \cdot l}. \tag{79}$$

4. implements y^S after $\rho = s$ and maintains status quo after $\rho = c$ if

$$\kappa \in \left(\kappa^{rbh}, \kappa^{rbl}\right) \text{ and } \pi \leq \pi^{rbh}. \tag{80}$$

5. implements y^C after $\rho = c$ and maintains status quo after $\rho = s$ if

$$\pi \in \left(\pi^{rbh}(\kappa, z), \pi^{rbl}(\kappa, z)\right), \text{ and } \kappa > \kappa^{rbl}. \tag{81}$$

6. maintains status quo after any signal if

$$\pi < \pi^{rbh}(\kappa, z) \text{ and } \kappa > \kappa^{rbl}. \tag{82}$$

Cycling between regions happens if endogenous changes in z move a location (κ, π) between two regions. Notice that π^{cb}, π^{rbh}, and κ^{rbh} decrease in z, while π^{sb}, π^{rbh}, and κ^{rbl} increase in z. Moreover, notice that $\kappa^{rbh} \leq \frac{1}{2} \leq \kappa^{rbl}$ and $\pi^{rbl} \geq \pi^{rbh} \geq \frac{l}{l+v}$. These properties imply that cycling between regions can occur only starting from the region $\kappa \in \left(\frac{1}{2}, \kappa^{rbl}\right)$ and $\pi \in \left(\pi^{rbh}(\kappa, z^{\min}), \pi^{rbh}\right)$, at some $z \in (z^{\min}, \frac{1}{2})$. Then, starting from such a point (κ, π), the decision-maker implements y^S after $\rho = s$ and maintains the status quo otherwise. Thus, expected z falls. This in turn reduces π^{rbh}. Then, at some z^*, $\pi^{rbh}(\kappa, z^*) \leq \pi$, i.e., the location crosses into the region where the decision-maker implements y^S after $\rho = s$ and y^C after $\rho = c$. The average z is expected to increase if $\rho = c$ is more likely than $\rho = s$, i.e., if $z \cdot (1-\kappa) + (1-z) \cdot \kappa > (1-z) \cdot (1-\kappa) + z \cdot \kappa$. This reduces to the condition that $\kappa > \frac{1}{2}$.

In the region where there is cycling at location (κ, π), it happens around z^{**} where

$$\pi = \pi^{rbh}(\kappa, z^{**}). \tag{83}$$

Given (79),

$$z^{**}(\kappa, \pi) = \frac{(\pi \cdot (v+l) - l) \cdot \kappa}{(l + c - \pi \cdot (l+v)) \cdot (1-\kappa) + (\pi \cdot (l+v) - l) \cdot \kappa} \tag{84}$$

Then, $\frac{\partial z^{**}(\kappa,\pi)}{\partial \pi} > 0$ and $\frac{\partial z^{**}(\kappa,\pi)}{\partial \kappa} > 0$. Moreover, as $\pi \to \frac{l}{l+v}$, $z^{**} \to 0$. As $\pi \to \frac{l+c}{l+v}$, $z^{**} \to \frac{1}{2}$.

Consider now comparing z^{**} to the z^* from the main model. Let (κ, π) be a location in the parameter space that satisfies the conditions for cycling both in the main model and

in the model with a single decision maker. Then, from (11), $z^*(\kappa, \pi)$ decreases in π. As $\pi \to \frac{l}{l+v}$, $z^* > z^{**} \to 0$. As $\pi \to \frac{l+c}{l+v}$, $z^* < z^{**} \to \frac{1}{2}$. Hence, there exists $\pi^* \in \left[\frac{l}{l+v}, \frac{l+c}{l+v}\right]$,

$$\pi^* = \frac{2 \cdot l \cdot (v+l) + (v)^2 + l \cdot c - \sqrt{\gamma}}{l \cdot (v+l)}, \tag{85}$$

where $\gamma = (2 \cdot l \cdot (v+l) + (v)^2 + l \cdot c)^2 - 4 \cdot l \cdot (v+l) \cdot (l \cdot a + (l)^2 + l \cdot v)$. For $\pi < \pi^*$, we have $z^* > z^{**}$. For $\pi > \pi^*$, we have $z^* < z^{**}$.

Expected welfare The problem with a single decision-maker has policy y^C as part of the solution only for $\pi \geq \pi^{rbh} \geq \frac{l}{l+v}$. For those values of π, in the main model, y^C is proposed only after θ^C. Policy y^C delivers a higher expected payoff when used only after θ^C than when used after any θ or after $\rho = c$. Thus, in the region in which policy choice is contingent on signal, the Simplification / Matching equilibrium yields higher welfare, as it offers y^C only in the state θ^C. For $\pi < \pi^{rbh}$, the outcome with a single-decision maker can be achieved in an equilibrium of our main model (the Pooling equilibrium). Yet, the main model allows for the Complexification equilibrium in a region which the decision-maker would implement the play from the Pooling or the Rejection regions. Since these equilibria are possible the main model for those parameter values, it must be the case that the DM expects higher welfare under the Complexification equilibrium.

Long-run complexity of the legislative environment Consider each of the regions:

1. where she implements y^S regardless of signal, z decreases on average until it reaches the lower bound $z^{\min}$.

2. where she implements y^C regardless of signal z increases on average until it reaches the upper bound $\frac{1}{2}$.

3. where she implements y^S after $\rho = s$ and y^C after $\rho = c$, the average z increases if

$$z \cdot (1-\kappa) + (1-z) \cdot \kappa \geq (1-z) \cdot (1-\kappa) + z \cdot \kappa, \tag{86}$$

   i.e., if $\kappa \geq \frac{1}{2}$, and average z decreases otherwise.

4. where she implements y^S after $\rho = s$ and maintains status quo after $\rho = c$, average z decreases.

5. where she implements y^C after $\rho = c$ and maintains status quo after $\rho = s$, average z increases.

6. where maintains status quo after any signal, z remains at its initial value z_0.

Outside the cycling region described in the single decision-maker's problem, if average z decreases, then it decreases until it reaches the lower bound $z^{\min}$. If average z increases, then

it increases until it reaches the upper bound $\frac{1}{2}$. In the cycling region, the cycling happens around the bound π^{rbh}, so at the z^* at which $\pi^{rbh}(\kappa, z^*) = \pi$.

Comparing the main model to the single decision-maker, the Complexification region in the main model has $z_\infty > z^{\min}$, while in the case of a single decision-maker, the same region (with $\pi < \frac{l}{l+v}$ has $z_\infty = z^{\min}$. Then, at $\kappa = 1/2 + \epsilon$, with $\epsilon \to 0$, and $\pi \in \left(\frac{l}{l+v}, \pi^{rbh}\right)$, the main model is in the Simplification region with $z_\infty = z^{\min}$, while with a single decision-maker we are in the cycling region with $z_\infty > 0$.