



HARVARD UNIVERSITY
DEPARTMENT OF ECONOMICS

General Examination in Microeconomic Theory

Fall 2015

You have **FOUR** hours. Answer all questions

Part A (Glaeser)

Part B (Maskin)

Part C (Hart)

Part D (Green)

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT BOTH YOUR EXAM NUMBER AND PROFESSOR ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

In the Sovereign Nation of Legalia, anyone who – accidentally or on purpose – enters onto someone else's property uninvited and does damages must pay a large fixed fine. In the neighboring country of Economica, the punishment would be based on the damages done.

1. Provide a simple model in which Economica's legal system is superior to that of Legalia. Explain how your answer would differ if the damage is accidental or planned.
2. How would your answer change if the trespasser was only caught with some probability less than one?
3. Can you adjust the damages system to make Economica again superior to Legalia?
4. How would your answer change if property owners first invested to improve their area.
5. How would your answers change if justice could be subverted? Please do not write a model for this part of the question. Just discuss.

	A	B	C	D
a	-2,-2	0,-1	0,-1	0,-1
b	-1,0	2,2	5,1	1,1
c	-1,0	1,5	4,4	1,1
d	-1,0	1,1	1,1	3,3

Table 1

- (A) Find all rationalizable strategies of the game in Table 1.
- (B) Find all Nash equilibria (including those in mixed strategies) of the game in Table 1.
- (C) Suppose that the game of the Table 1 is repeated infinitely many times. Find the set of payoff pairs (v_1, v_2) for which there exists a discount factor $\delta < 1$ and a subgame perfect equilibrium of the repeated game whose discounted average payoffs are (v_1, v_2) with discount factor δ .
- (D) Suppose that that game of Table 1 is repeated T times, where $T < \infty$. Show that if T is big enough, there exists a subgame perfect equilibrium of the repeated game (with no discounting) for which the average payoffs are almost $(4, 4)$.
- (E) Reformulate the claim of part (D) (which is stated somewhat loosely) so that it is *precise*.

1. (a) Define the core of an exchange economy. Prove that a Walrasian equilibrium lies in the core.
- (b) Consider a competitive economy with one (type of) consumer and two (types of) firms, which the consumer owns. There are two dates 0 and 1, two states of the world, and all uncertainty is resolved at date 1. There is one good at each date. The consumer cares only about consumption at date 1: her utility function is $\pi \log x_1 + (1-\pi) \log x_2$, where x_i is date 1 consumption in state i and π is the probability of state 1. The consumer has an endowment of one unit of the good at date 0. The two firms have constant returns to scale technologies. Firm 1 can transform 1 unit of the date 0 good into (3,1) (i.e., 3 units of the good at date 1 in state 1 and 1 unit of the good at date 1 in state 2) and firm 2 can transform 1 unit of the date 0 good into (1,3).

Compute the Arrow-Debreu equilibrium for this economy under the assumption that both firms operate. Show that this is indeed the equilibrium for π in some range. Characterize the equilibrium also when π is outside this range.

2. A risk neutral principal can hire a risk neutral agent to carry out a task. The agent can exert effort or not. The principal cannot observe the agent's effort. With effort the task succeeds with probability $\frac{7}{8}$ and fails with probability $\frac{1}{8}$. Without effort the task succeeds with probability $\frac{3}{4}$ and fails with probability $\frac{1}{4}$. The principal's revenues = Σ if the project succeeds, and zero if it fails. Effort causes the agent a disutility equal to c . (No effort is costless.) The agent's consumption cannot fall below zero in any state of the world. The agent has a zero outside option.
- (i) Suppose the agent has zero wealth. Write down the conditions for an incentive scheme to elicit effort from the agent. What is the lowest expected cost incentive scheme for the principal that satisfies these conditions and the agent's participation constraint? What is the agent's utility under this incentive scheme?
- (ii) For what values of Σ and c is it worthwhile for the principal to elicit effort from the agent? Compare this to the first best outcome.
- (iii) Suppose that the agent has positive wealth A . How do your answers to (i) and (ii) change?

Answer all three questions. They are worth 20 point each.

1. a. Which of the postulates in Arrow's Theorem are violated by the use of the Kemeny Method for aggregating n ordinal rankings into a societal ranking of alternatives.

b. Give a numerical example to illustrate your answer to part a.

2. There are n people. They can form bilateral links (think of them as friendships) if they want to. A link between players i and j is worth x_{ij} , which can be either positive or negative.

a. The people decide to use the Shapley Value to divide the total value that they can generate by forming links. Write the Shapley Value allocation as a function of the set of x_{ij} .

b. Assume that any coalition can form and make its own decision about which links among its members it wants to form. Show that the Shapley Value allocation of part a. is in the core of this transferable utility game.

c. What is the nucleolus of this game?

3. There is a region that is divided into n pieces of land, each owned by one person. This region has a benevolent government, meaning that the government wants to be as efficient as possible, maximizing the surplus that the people receive.

The people each value their own piece at v_i . The v_i are, from the point of view of the government, independent random variables, each drawn independently from a distribution F that has mean μ . The government contemplates taking all the land from the people and using it for some public purpose. The value, as seen by the government, for this taking of land is V .

a. One possibility is for the government to take the land and compensate each person μ if $V > n\mu$, but if $V < n\mu$ it will leave these pieces in the hands of their original owners. In what sense is this an optimal decision rule.

b. Now suppose that the government tries to do better than the decision rule of part a. The government runs a VCG mechanism to elicit the parameters v_i . Each person is asked to state the value of their parameter, but because these are private information they might not do so honestly. Let the stated valuations be w_i . In the VCG mechanism the government lets the owners keep their land if $\sum w_i > V$ but takes the land if $\sum w_i < V$. According to the rules of the VCG mechanism, only the pivotal people pay anything to the government; all others do not make or receive a monetary transfer. In what sense is this mechanism efficient? Does this mechanism necessarily improve on that of part a in every instance? Does it improve on part a in expectation if not in every instance?