HARVARD UNIVERSITY
DEPARTMENT OF ECONOMICS

General Examination in Macroeconomic Theory

SPRING 2016

You have **FOUR** hours. Answer all questions

Part A (Prof. Laibson): 60 minutes
Part B (Prof. Barro): 60 minutes
Part C (Prof. Farhi): 60 minutes
Part D (Prof. Rogoff): 60 minutes

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Macro Generals
May 2016
Please write legibly.

True, False, or Partially True (30 minutes): Please explain whether the following statements are True, False, or Partially True. You will be graded on the quality of your explanation.

a. (6 minutes) If $V(x)$ is a twice-differentiable convex function and x is an arithmetic Ito Process with no drift, then dV has negative drift.

b. (6 minutes) In a rational consumption model with exponential discounting, ln utility, and no risk,

$$\Delta \ln c_t = r - \rho,$$

where c_t is real consumption, r is the real interest rate and ρ is the exponential discount rate.

c. (6 minutes) If corporate taxes fall at date t, then the market price of capital is rising (either discretely or continuously) at date t.

d. (6 minutes) If $B(f)$ is a contraction mapping on a space of functions F, then for any $f \in F$, there exists an N sufficiently large, so that $B^n(f) = B^m(f)$ for all $n, m > N$.

e. (6 minutes) Marginal utility is a random walk if $\delta R = 1$, where δ is the discount factor and R is the risk free real rate of return.

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Asset pricing (30 minutes)

- Consider a static (one-period) endowment economy with identical agents.
- Each agent is endowed with x units of (deterministic) consumption.
- Each agent is also endowed with one unit of a stochastic consumption endowment with payout z, where $E(z) = 1$ and $V(z) = \sigma^2$. (Note that z may take negative values.)
- There is a pre-market in which the deterministic endowment x and the stochastic endowment z can be traded *before* the uncertainty on z is resolved. The equilibrium price of a unit of z is p units of x. Note that no consumption occurs in this pre-market, only asset trading. All consumption occurs after the uncertainty on z is resolved.
- If an agent buys q units of z in this pre-market at price p, her final (stochastic) consumption will be $c = (1 - qp)x + (1 + q)z$. Note that q can be positive or negative (or zero).

1. In equilibrium, all agents will (endogenously) choose to hold their original endowment of x and z. In other words, in equilibrium $q = 0$ for all agents. Explain why. What role does p play in this outcome?
2. Assume that the utility function in this economy, u, is increasing and strictly concave (and identical across agents). In equilibrium which relation will hold: $p < 1$, $p = 1$, or $p > 1$? Explain your reasoning.
3. Now assume that $u(c) = \theta c - \frac{\alpha c^2}{2}$, where $\theta > 0$ and $\alpha > 0$. (Note that u can take arguments of any sign, including $c < 0$.) Calculate the equilibrium price p (in the pre-market) as a function of x, θ, α, and σ^2.

Barro questions (7.5 minutes each, total of 60 minutes)
True-False-Uncertain.

Are the following statements true, false, or uncertain? Explain **briefly**, but your explanation determines your grade.

1. In Ramsey-type growth models, the saving rate falls during the transition to the steady state.

2. In Ramsey-type growth models, the fertility rate falls during the transition to the steady state.

3. International data since 1970 show a dramatic fall in world poverty rates but little change in the cross-country dispersion of per capita GDP.

4. Increased disaster risk can explain why expected real interest rates on rich countries' government bonds have been negative in recent years.

5. The large increase since 2007 in the ratio of U.S. public debt to GDP conflicts with the tax-smoothing model.

6. In varieties models of endogenous growth, a subsidy to R&D can generate the Pareto optimal long-run growth rate.

7. In quality-ladders models of endogenous growth, a rise in competition among providers of intermediate goods lowers the long-run growth rate.

8. In strategic-debt models, an increase in polarization (the gap between political parties in their preferred size of government) raises the fiscal deficit chosen by the incumbent party.

Farhi's part

May 9, 2016

1 Short Questions

Answer all the following questions. Some are True/False/Uncertain and are explicitly denoted as such. The others are direct questions. Explain and detail your answers VERY carefully. The QUALITY of your explanation determines your grade.

1. TRUE/FALSE/UNCERTAIN. In order to match the data, the RBC model requires a large elasticity of labor supply. This is consistent with the microeconomic evidence on the elasticity of labor supply.

2. Explain how the impulse responses for consumption and labor in the RBC model change when the persistence of the productivity shock increases. How is the amount of amplification of productivity fluctuations on output affected?

3. Explain what the the employment-lottery model is. Can this model generate an aggregate elasticity of labor supply which is larger than the individual elasticity of labor supply?

4. TRUE/FALSE/UNCERTAIN (for each of the following three statements). In the New-Keynesian model, real interest rates are counter-cyclical when monetary policy shocks are driving fluctuations. In the RBC model, real interest rates are pro-cyclical when productivity shocks are driving fluctuations. In both cases, real wages are pro-cyclical. All these properties are consistent with the data.

5. Consider the New-Keynesian model with productivity shocks. Assume first that the labor tax is set to offset monopoly power. Characterize optimal monetary policy. Is commitment required to implement this outcome? Assume now that the labor tax is zero. Characterize optimal monetary policy under commitment and under no commitment, and explain why the two differ.

6. Consider the New-Keynesian model with cost-push shocks. Explain how to interpret cost-push shocks. Characterize optimal monetary policy under commitment and under no commitment, and explain why the two differ.

2 Problem

Consider an economy where the representative consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_t}{P_t}, N_t)$$

subject to a sequence of dynamic budget constraints

$$P_t C_t + M_t + Q_t B_t + T_t \leq M_{t-1} + B_{t-1} + W_t N_t$$

where C_t is consumption, N_t is labor, P_t is the price level, W_t is the nominal wage, M_t is nominal money holdings, B_t is short-term nominal bond holdings, and T_t is a lump-sum tax (which can be negative in the case of a lump-sum rebate). The period utility function is given by

$$U(C_t, \frac{M_t}{P_t}, N_t) = \log(C_t) + \log(\frac{M_t}{P_t}) - \frac{N_t^{1+\phi}}{1+\phi}$$

Firms are monopolistically competitive, each producing a differentiated good whose demand is given by $Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$, and has access to a linear production function

$$Y_t(i) = A_t N_t(i)$$

where productivity follows the following process

$$\frac{A_t}{A_{t-1}} = (1+\gamma_a) e^{\epsilon_t^a}$$

where ϵ_t^a is an i.i.d. normally distributed process with mean zero and variance σ_a^2. The money supply follows the following process

$$\frac{M_t}{M_{t-1}} = (1+\gamma_m) e^{\epsilon_t^m}$$

where ϵ_t^m is an i.i.d. normally distributed process with mean zero and variance σ_m^2.

1. Describe the optimality conditions for the problem facing the representative consumer.

2. (flexible prices) Assume that firms set the price in each period after observing the shocks to maximize profits $Y_t(i)(P_t(i) - W_t/A_t)$ subject to the demand schedule above. Derive the optimality condition associated with the firm's problem.

3. Solve exactly for the equilibrium levels of aggregate employment, output, and inflation.

4. Discuss how utility depends on the two parameters describing monetary γ_m and σ_m^2. Show that the optimal policy must satisfy the Friedman rule and discuss alternative ways of supporting that rule in equilibrium.

5. (sticky prices) Now assume that firms have to set their price one period in advance, i.e. before the realization of the shocks. In that case, they will choose a price in order to maximize the discounted profit $E_{t-1}[\beta(C_{t-1}/C_t)(Y_t(i)/P_t)(P_t(i) - W_t/A_t)]$ subject to the demand schedule above. Derive the optimality conditions of the firm's problem and solve exactly for the equilibrium levels of employment, output, and real money balances.

6. Evaluate expected utility at the equilibrium values of employment, output, and real money balances.

7. Consider the class of money supply rules where $\epsilon_t^m = \phi_\epsilon \epsilon_t^a + \phi_\nu \nu_t$, where is a normally distributed i.i.d. process with zero mean and unit variance, and independent of ϵ_t^a at all leads and lags. Notice that within the family of rules, monetary policy is fully described by three parameters γ_m, ϕ_ϵ, and ϕ_ν. Determine the values of those parameters that maximize expected utility, subject to the constraint of a non-negative nominal interest rate. Show that the resulting equilibrium under the optimal policy replicates the flexible price equilibrium analyzed above.

Ec2010D (part two): General Questions

Kenneth Rogoff

May 14, 2016

1 Speculative Exchange Rate Attacks

This example is based on Morris and Shin (1998). Imagine that the Central Bank holds reserves (fundamentals) of

$$\theta \in [0, 1]$$

The level of reserves, however, are not common knowledge. We assume instead that there are a continuum of individual traders of mass 1, each of whom is endowed with one unit of domestic currency, and receives a private signal x_i about the quality of the fundamentals,

$$x_i = \theta + \varepsilon_i \tag{1}$$

with $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. Define the precision of information instead of the noise, $\lambda_\varepsilon \equiv \sigma_\varepsilon^{-2}$.

The only information that is common knowledge among traders is the prior distribution of θ (that is, the primitive distribution that generates θ). Assume it is *uninformative*; in this case, this means all traders have the prior $\theta \sim N(\bar{\theta}, \lambda_\theta^{-1})$ with $\lambda_\theta \to 0$.

If a trader i decides to attack ($I_i = 1$) she gets

$$\begin{aligned} -c, \text{ if } R &= 0 \\ 1 - c, \text{ if } R &= 1 \end{aligned} \tag{2}$$

where $R = 1$ stands for "regime change" (abandon the peg). The Central Bank abandons the peg whenever the size of the attack, or mass of agents that decide to sell their unit of domestic currency for 1 unit of reserves, is larger than reserves,

$$\begin{aligned} R &= 1 \Leftrightarrow A = \int_0^1 I_i di > \theta \\ R &= 0 \text{ otherwise} \end{aligned} \tag{3}$$

Since the only heterogeneity among individual traders is the signal they receive, we can characterize "threshold strategies" by

$$\begin{aligned} I_i &= 1 \Leftrightarrow x_i \leq x^* \\ I_i &= 0 \text{ otherwise} \end{aligned} \tag{4}$$

The game therefore consists in solving for the threshold signal x^*, and the (possibly multiple) equilibria of the game ($R = 1$ or 0), as a function of the fundamentals. In order to solve for these, it is useful to remember that the Bayesian posterior expectation of θ given the normally distributed signal x_i and uninformative prior is given by

$$\theta | x_i \sim N(x_i, \lambda_\varepsilon^{-1})$$

1. Show that the size of the attack is given by

$$A = \int_0^1 I_i di = \Phi\left(\sqrt{\lambda_\varepsilon}(x^* - \theta)\right) \tag{5}$$

   and therefore there exists a cutoff fundamental θ^* defined by

$$\theta^* = \Phi\left(\sqrt{\lambda_\varepsilon}(x^* - \theta^*)\right) \tag{6}$$

   for which $R = 1$ (the peg is abandoned) when $\theta < \theta^*$ and $R = 0$ (the peg is maintained) when $\theta \geq \theta^*$.

2. Characterize the indifference condition of the "marginal trader" (who receives $x_i = x^*$). Show it satisfies

$$\Phi\left(\sqrt{\lambda_\varepsilon}(x^* - \theta^*)\right) = 1 - c \tag{7}$$

3. For this part a brief intuitive answer is sufficient: (6) and (7) clearly define unique cutoffs,

$$\begin{aligned} \theta^* &= 1 - c \\ x^* &= \frac{\Phi^{-1}(1-c)}{\sqrt{\lambda_\varepsilon}} + (1 - c) \end{aligned}$$

   Is the equilibrium unique? What if $\sigma_\varepsilon^2 = 0$? What if $\sigma_\varepsilon^2 > 0$ but extremely (arbitrarily) small?

2 Global Imbalances

Consider a two-country, two-period economy with a single homogenous good. Foreign variables are denoted with an *. In each country there is a continuum of households of mass 1 with symmetric CRRA preferences and risk aversion parameter ρ; household i's preferences are thus

$$U^i = \frac{(c_1^i)^{1-\rho} - 1}{1-\rho} + \mathbb{E}_1 \left[\frac{(c_2^i)^{1-\rho} - 1}{1-\rho} \right]$$

Each household recieves a common endowment of $\bar{X}$ in period 1 and a stochastic endowment X_2^i in period 2. There is no aggregate risk and no growth, i.e. $\mathbb{E}_1 [X_2^i] = \bar{X}$. The stochastic endowment has a Pareto distribution with shape parameter $\alpha > 1$ and lower bound $\frac{\alpha-1}{\alpha}\bar{X}$, so that it is distributed over $\left[\frac{\alpha-1}{\alpha}\bar{X}, \infty\right)$ with probability density function

$$f_{X_2}(x) = \alpha \left(\frac{\alpha - 1}{\alpha} \bar{X} \right)^{\alpha} \frac{1}{x^{\alpha+1}}$$

The two countries differ in terms of the level of development of their financial markets:

- Home: Households have access to a full set of state-contingent bonds so that markets are complete. Households also have access to a riskless bond with return $1 + r$.
- Foreign: Households have access only to a riskless bond with return $1 + r^*$.

1. For the time being assume that the two countries are in financial autarky. What is the equilibrium autarky rate for the riskless bond in Home?
2. Show that the autarky rate for the riskless bond in Foreign is
$$1 + r^* = \frac{1}{\beta} \times \frac{\alpha + \rho}{\alpha} \left[\frac{\alpha - 1}{\alpha} \right]^{\rho}$$
3. Assume from now on that countries have log-utility. Which country has a higher interest rate? Why?
4. Use a Metzler diagram to represent the autarky interest rate in both Home and Foreign (i.e. plot two charts, each with the interest rate on the vertical axis and aggregate savings on the horizontal axis).
5. Assume now that the two countries can trade a riskless bond, but nothing else. Use the Metzler diagram to show what the equilibrium interest rate looks like when there is trade in the riskless bond and show how this relates to the trade balance.
6. Based on your previous results, characterize qualitatively the world interest rate r^W (relative to the autarky interest rate), and first-period consumption in both countries (relative to autarky). What is the trade balance in the first period?

3 Essay Questions

Please give SHORT answers to ANY THREE of the following four short-answer essay questions. Each of your three answers is equally weighted.

(a) Can you list four distinct reasons why global real interest rates on advanced country government bonds might be at exceptionally low levels? Just a sentence or two on each reason can be enough for full credit.

(b) What are the two key explanatory variables (for trade between two countries) in the standard gravity model of trade? In what sense does the model work well empirically and why might it be surprising? Which of the two key variables in gravity models is easier to rationalize in terms of standard economic models of international trade?

(c) Why is it a puzzle that shocks to the real exchange rate seem to take many years to dissipate?

(d) What research evidence is there from goods and financial markets to support the view that the position of the US dollar in global financial markets is fairly unique?