Peer Monitoring with Partial Commitment*

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Abstract

We explore how a Principal uses peer monitoring to incentivize agents to acquire costly information. Peer monitoring presents a commitment challenge for the Principal, as each agent does not observe the behavior of others. We show that this interaction between partial commitment and limited liability leads to an inefficiently high monitoring rate: in the optimal contract, the Principal monitors with strictly positive probability and tasks are unnecessarily duplicated. Bundling multiple tasks reduces this inefficiency. In contrast, virtual monitoring is optimal and approximate efficiency obtains when the Principal has full commitment or liability is unlimited.

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1 Introduction

Technological advancement is changing how modern companies interact with their workforce. New information technology, such as crowdsourcing platforms, creates the opportunity for firms to access a flexible and inexpensive pool of temporary workers on-demand to complete simple information acquisition tasks. Millions of potential employees are available around the clock and may start work immediately. The allure of these online labor markets are their low frictions. But in the absence of conventional methods of supervising employees, tapping into this online workforce raises new incentive issues. We explore how a Principal uses peer monitoring to incentivize agents to acquire costly information when hiring in a spot market.

To fix ideas, suppose a Principal has a series of images, and each image is either a tree or a flower. The Principal hires agents to accurately categorize these images. Importantly, the effort cost of verifying each image is similar to the effort cost of completing the initial categorization. For example, verifying that an image is correctly categorized requires the same effort as categorizing the image.[1] If the agent exerts costly effort to view the image, he observes the correct category. The Principal does not observe whether an agent views the image and the agent does not directly care about whether his categorization is correct. After choosing whether to view the image and observing the category, the agent sends a message stating whether the image is a tree or a flower. The information an agent observes is not verifiable, but the Principal can assess the accuracy of an agent's message by assigning the same image to other agents and comparing the messages. Incentives are generated by conditioning an agent's payment upon this comparison, which we refer to as *peer-monitoring*.[2] A key feature of this environment is that the cost of monitoring is endogenous, since the incentive compatible wage payments depend crucially on the probability a second agent is assigned to each image.

[1] In contrast, verifying the accuracy of a computer program can be significantly less involved than writing the program in the first place.

[2] We discuss other types of monitoring, such as inserting known images into the pool of tasks, in the crowdsourcing application, and demonstrate that peer-monitoring is outperforms these commonly used monitoring schemes under a broad set of conditions.

Peer monitoring presents a commitment challenge for the Principal, as each agent does not observe the behavior of others. Therefore, in addition to ensuring a contract effectively monitors agents, the Principal must also ensure that the contract is credible. First, it seems unlikely that the Principal would be able to assess penalties for poor performance when agents can simply disappear from the platform. Therefore, the most severe punishment is withholding wages, and agents are protected by limited liability. Second, the Principal conditions an agent's payment on the messages of other agents, which an agent does not directly observe. Therefore, whether the Principal can commit to truthfully reveal the messages of other agents will affect the credibility of a contract. For example, if its possible to hide the messages of other agents and pay the agent a lower wage, the Principal will do so. We focus on a simple departure from full commitment in which the Principal can hide messages from an agent's peers but cannot fabricate the content of these messages (i.e. the messages a Principal observes are verifiable). We compare this partial commitment setting to one with full commitment.

Our main result is to derive the Principal's optimal contract. The optimal monitoring rate and wage scheme depend crucially on the commitment power of the Principal. Three key features emerge: (i) virtual monitoring, or monitoring with arbitrarily small probability, is optimal under full commitment while stochastic monitoring, or monitoring with strictly positive probability, is optimal under partial commitment, (ii) bundling – simultaneously assigning an agent multiple images – reduces agent rents and the inefficiency that arises from monitoring, and (iii) the optimal contract is approximately efficient under full commitment but not under partial commitment.

The Principal's commitment power determines what type of wage schemes are credible. When the Principal has full commitment, it can credibly commit to condition payment on whether or not the agent is monitored. In this case, the Principal will pay a large positive wage when an agent is monitored and matches on every assigned image, including an additional bonus for less likely image categories, and otherwise pay a wage of zero. This wage is inversely proportional to the probability of being paid, and therefore an agent's expected wage payment is independent of the monitoring rate.

In contrast, under partial commitment, the former wage scheme is not credible as the Principal would not truthfully reveal when an agent is monitored. Instead, the Principal pays a positive wage when the agent matches on all images that are monitored, including an additional bonus for less likely image categories. If the agent has any mismatches, he is paid nothing. Compared to full commitment, the agent is paid a lower wage with higher probability. Monitoring at a higher rate lowers the incentive compatible wage for each message but does not affect the probability of payment. Therefore, the expected wage payment to an agent is decreasing in the monitoring rate.[3]

The Principal chooses the optimal monitoring rate to minimize the expected wage payment per task, which is the product of the expected number of agents hired and the expected wage payment per agent. Monitoring more frequently increases the former. Under full commitment, the latter is independent of the monitoring rate. Therefore, *virtual monitoring*, or monitoring with arbitrarily low probability, is optimal. This contract is *approximately efficient*, since duplication occurs with arbitrarily small probability (in the efficient contract, a single agent exerts effort on each task).

Under partial commitment, the Principal faces a trade-off between efficiency and agent rents. Monitoring at a higher rate reduces the expected payment per agent but increases the expected number of agents hired. *Stochastic monitoring*, or monitoring with strictly positive probability, is optimal. The Principal incurs some inefficiency in order to reduce the rents paid to agents. Therefore, the optimal contract is *inefficient*, despite the fact that it is possible to implement the efficient action profile.[4]

Bundling, or grouping multiple images together for a single agent, is optimal under both partial and full commitment. Bundling ties the agent's wage for one image to his performance on all images, which partially relaxes the limited lia-

[3]In contrast, under full commitment, monitoring at a higher rate lowers the incentive compatible wage but is exactly offset by the higher probability of payment, rendering the expected wage payment independent of the monitoring rate.

[4]Note that while information aggregation is a commonly cited motivation for hiring multiple agents, in this model, there is no learning justification for the duplication of tasks: multiple agents are employed with strictly positive probability solely for incentive reasons. Thus, the paper provides a complementary explanation for why a principal may consult multiple agents before taking a decision.

bility constraint by allowing the Principal to punish an agent on multiple images when he deviates on one. This reduces the rents captured by the agents. Under partial commitment, bundling also reduces the optimal monitoring rate, which reduces the inefficiency that arises from monitoring. Monitoring and bundling are strategic substitutes: the Principal hires monitors less frequently as the number of images assigned to an agent increases. The efficiency loss relative to the contractible effort benchmark vanishes asymptotically.

Our main findings apply to settings with multidimensional tasks as well. For example, consider a multidimensional chore in which the Principal monitors individual components of the chore and punishes agents across all dimensions for poor performance on any dimension. Bundling these different tasks strengthens incentives relative to contracting each dimension separately.

Literature. In the standard agency model, an agent's action is not observable and he is subject to moral hazard. The Principal observes an informative signal about an agent's action and conditions payment on the realization of the signal (Holmstrom 1979; Mirrlees 1976, 1999). As long as the distribution of the signal varies with the chosen action, the Principal is able to align incentives so that the agent chooses the desired action.

A complication arises if the Principal is restricted in how severely it can punish an employee. Under limited liability, instead of punishing an agent by paying him $-x < 0$ when a bad signal obtains, the Principal transfers x to the agent at the outset and simply takes away this transfer upon observing a bad signal. Such an arrangement preserves incentives, but the agent captures rents (Bolton and Dewatripont 2005, Ch. 4). Agents in our model capture rents for the same reason. The size of these rents is endogenously determined by the Principal's choice of monitoring technology.

In multilateral contracting environments, issues arise with statistically distinguishing deviations by individual agents. If the signal only reveals aggregate information about the actions of the group, and not information about individual action choices, the principal must guard against a free-rider problem. Holmstrom (1982) emphasizes the role of group penalties: all agents are punished whenever bad signals obtain. Group penalties are natural in our model since identifying the deviator requires hiring additional agents, which is more costly than identifying that a deviation occurred.

The signal structure can be viewed as the Principal's monitoring technology. In much of the literature, the monitoring technology is exogenous. In contrast, the Principal chooses the monitoring technology in our paper. Early studies in which monitoring is a choice variable include Becker (1968), Kolm (1973) and Mirrlees (1974).[5] These papers suggest combining infinitesimal monitoring with arbitrarily harsh punishments. Legros and Matthews (1993) study a multilateral partnership problem in which each agent privately devotes effort to a common project. Agents are monitored by instructing one agent to choose an inefficient action with small probability p. The cost of monitoring is determined by the loss from playing this inefficient action, and depends on p. Under unlimited liability, efficiency is approximated by choosing p close to zero. These requires large fines for some realizations of output. Under limited liability, the smallest p that satisfies incentives is bounded away from zero and the optimal contract is inefficient. In our paper, whether limited liability creates an inefficiency depends on the Principal's commitment power: the optimal contract under full commitment is approximately efficient but the optimal contract under partial commitment is not.

Rahman (2012) also considers a costly monitoring setting. The first-best strategy profile is for an agent to exert effort and the Principal to never monitor. He examines when virtual monitoring is feasible, and shows that the first-best strategy profile can be approximated arbitrarily well by monitoring with low frequency and punishing the agent severely whenever the monitor reports that he shirked. In our setting, virtual monitoring is always feasible. The Principal can approximate the first-best action profile, provided there is no upper bound on transfers. However, this is achieved at great cost: wage payments are unboundedly large. With full commitment, the probability of paying these large wage payments shrinks proportionally, so that the expected wage payment is independent of the monitoring probability. In contrast, with partial commitment, the probability of paying a positive wage for a message is independent of the moni-

[5] The efficiency-wage theory of Shapiro and Stiglitz (1984) is sometimes portrayed as an example of endogenous monitoring. See, for example, Bolton and Dewatripont (2005, § 4.1.3).

toring probability, while the required wage payment becomes unboundedly large as the monitoring probability shrinks. Virtual monitoring is optimal under full commitment but not under partial commitment.

In our paper, bundling multiple tasks strengthens incentives by tying the payment for one part of the job to performance on all parts of the job. This dynamic is similar to that identified by Fuchs (2007) in a repeated setting. He shows it is optimal for the Principal to withhold payment until the final period.[6]

The paper proceeds as follows. An example is presented in Section **??**. The formal model is introduced in Section 2, while Section 3 derives the Principal's optimal contract under partial and full commitment. Section **??** develops an application to crowdsourcing platforms. Omitted proofs are in the Appendix.

2 Model

2.1 Set-up

A Principal is faced with a countably infinite stream of independent and identical tasks $t = 1, 2, ...$ and can delegate tasks to a countably infinite pool of agents $i = 1, 2, ...$ A task can be assigned to multiple agents, and multiple tasks can be assigned to a single agent. The following convention is maintained: objects pertaining to tasks are subscripted and objects pertaining to agents are superscript.

The Task. Each task t has an unknown state ω_t drawn from finite set Ω with common prior belief $\pi \in \Delta(\Omega)$. The Principal can hire agents to learn about the state. Let $n_t \in \{1, 2\}$ denote the number of agents hired for task t when the Principal delegates the task, and let $\mathcal{I}_t$ be the ordered set containing the identities of these agents.[7] Agents do not observe the number or identity of other agents hired for a task.

An agent i assigned to task t chooses an *unobservable* effort level $e^i_t \in \{0, 1\}$. Exerting effort ($e = 1$) perfectly reveals the state and is costly; we normalize this

[6] Abreu, Milgrom, and Pearce (1991) emphasize the reusability of punishments: one punishment can simultaneously provide incentives across many periods.

[7] Restriction attention to hiring one or two agents is without loss of generality in the setting we study, as the Principal would never find it optimal to hire more than two agents.

cost to $c = 1$. No effort ($e = 0$) yields no information about the state and is costless. Let $s_t^i \in \mathcal{S} \equiv \Omega \cup \{\emptyset\}$ be the information agent i observes about task t. After making an effort choice and observing information about the state, the agent sends a message to the Principal, $m_t^i \in \mathcal{S}$. Information is *not verifiable*: the agent can send any message in $\mathcal{S}$, regardless of the information he observes.

Upon receiving messages from all hired agents on a task, the Principal compiles a message profile $(m_t^i)_{i \in \mathcal{I}_t} \in \mathcal{M}$, where $\mathcal{M} \equiv \mathcal{S} \cup \mathcal{S}^2$ is the set of all possible message profiles, and chooses an action $A_t \in \Omega$. It receives a payoff of $v > 1$ if its action matches the state and zero otherwise.

The Contract. The Principal designs a contract for each agent, which comprises a set of tasks, a monitoring technology and a wage structure. Let $\mathcal{J} = \{t_j\}_{j=1}^J$ denote the set of tasks, where $J = |\mathcal{J}|$ is the *job size*, which, due to exogenous legal or technological constraints faced by the Principal, we assume to be finite, $J \leq \overline{J}$ for some $\overline{J} < \infty$. A contract has *bundling* if it has multiple tasks, $J \geq 2$ and *maximal bundling* if it has the largest feasible job size, $J = \overline{J}$.

Agents are monitored by comparing their messages to the messages of other agents assigned to the same task, which we refer to as *peer-monitoring*. The monitoring technology is the set of probability distributions over the number of agents hired for each task in an agent's job, $Q = (Q_j)_{t_j \in \mathcal{J}}$ where $Q_j \in \Delta(\{1, 2\})$. Let $\mathbf{q} = (q_1, ..., q_J)$ denote the monitoring profile for the contract, where $q_j = Q_j(2)$.

The wage structure depends on the agent's messages, as well as the messages from other agents on tasks in the agent's job. The Principal aggregates the message profiles for all tasks in $\mathcal{J}$ into a report $r = (m_{t_j})_{t_j \in \mathcal{J}} \in \mathcal{R}$ for the agent, where $\mathcal{R} \equiv \mathcal{M}^J$. An agent does not observe the messages of other agents. Therefore, if the agent's wage depends on the messages from other agents, the Principal must reveal these messages to the agent. Given a report r, the Principal chooses to reveal report $\tilde{r} \in \tilde{R}(r)$ to the agent, where $\tilde{R} : \mathcal{R} \rightrightarrows \mathcal{R}$ and $\tilde{R}(r) \subset \mathcal{R}$ is the set of reports that are feasible for the Principal to reveal to an agent with report r. The restrictions on $\tilde{R}$ determines the commitment environment for the Principal. Under *partial commitment*, the content of other agents' messages is verifiable, but their receipt is not. In other words, the Principal cannot fabricate messages from other agents, but it can hide them and claim not to have hired a second agent. Formally, say $r' \subset r$ if r' can be constructed by removing other agents' messages from r.[8]

Definition 1 (Partial Commitment)**.** *A Principal has* partial commitment *if* $\tilde{R}(r) = \{r' | r' \subseteq r\}$ *for all* $r \in \mathcal{R}$.

Partial commitment is realistic if the messages from other agents are verifiable but the burden of proof lies on the Principal to produce these messages. For example, the Principal can produce evidence to distinguish a report with a mismatch from a report with a match, but cannot produce evidence to distinguish a report with no monitoring from a report with monitoring. As a comparison, we also consider the case of *full commitment*, in which the content and receipt of messages from other agents are verifiable. Therefore, the Principal can commit to truthfully reveal the observed report.

Definition 2 (Full Commitment)**.** *A Principal has* full commitment *if* $\tilde{R}(r) = \{r\}$ *for all* $r \in \mathcal{R}$.

In either case, the Principal cannot misrepresent an agent's own messages in the revealed report, as the agent directly observes his own messages.

The wage structure $W : \mathcal{R} \to \mathbb{R}$ maps the revealed report to a numeric value. We restrict attention to wage structures in which agents are protected by limited liability.

Condition 1 (Limited Liability)**.** $W(r) \geq 0$ *for all* $r \in \mathcal{R}$.

Let $C \equiv (\mathcal{J}, Q, W)$ denote the contract.

Agents and tasks are identical, and agents are anonymous, so there is no strategic element to assigning agents and tasks to contracts. Given a contract with job size J, the Principal fills the contract with J tasks and assigns it to the next available agent.

2.2 The Agent's Problem.

Fix a contract $C = (\mathcal{J}, Q, W)$ assigned to agent i, with job size $J = |\mathcal{J}|$.

[8] For example, if $\Omega = \{0, 1\}$ and $J = 1$, report $(1, \cdot)$ is a subset of $(1, 1)$ and $(1, 0)$ but not a subset of $(0, 0)$ or $(0, 1)$, where $\cdot$ corresponds to the case where there is no second agent.

Strategies. The agent's strategy is a distribution over effort profiles $\sigma^i \in \Delta(\{0,1\}^J)$ and a map from the set of signal profiles to a distribution over job message profiles $\mu^i : \mathcal{S}^J \to \Delta(\mathcal{S}^J)$.[9] Abusing notation, let σ^i_t be the probability that agent i exerts effort on task t. Denote the set of all strategy profiles for player i by Σ^i. Two strategies play a prominent role in our analysis. Let $(\overline{\sigma}^i, \overline{\mu}^i)$ denote the strategy that corresponds to exerting effort on all tasks and reporting information truthfully, $\overline{\mu}^i(\mathbf{s}) = \mathbf{s}\ \forall \mathbf{s} \in \mathcal{S}^J$. When the agent exerts high effort on all tasks, the probability of signal $\mathbf{s} \in \Omega^J$ is $\Pi_J(\mathbf{s})$. Let $(\sigma^i_0, \mathbf{m})$ denote the strategy that corresponds to exerting effort on no tasks and reporting message $\mathbf{m} \in \mathcal{S}^J$.

Payoffs. The agent is risk-neutral. His payoff depends on the wage payment and cost of effort (recall this is normalized to one), and is independent of the state on any task. Given revealed report $\tilde{r}^i$ and effort profile e^i, the payoff from the contract is

$$W(\tilde{r}^i) - \sum_{t \in \mathcal{J}^i} \mathbb{1}_{\{e^i_t = 1\}}.$$

Incentives. We are interested in equilibria in which agents exert high effort and report truthfully on all tasks, and the Principal truthfully reveals the observed report. In such an equilibrium, an agent's incentives are governed by the probability distribution over his reports and the wage payment for each report, but are independent of the other agents' contracts. Conditional on other agents exerting effort and reporting truthfully, the distribution over reports depends on the agent's strategy, the monitoring distribution and the distribution over the state space. Let $g : \Sigma^i \to \Delta(\mathcal{R})$ be the probability measure induced over reports when other agents play $(\overline{\sigma}^{-i}, \overline{\mu}^{-i})$, where $g(r|\sigma^i, \mu^i)$ is the probability of report r when agent i chooses strategy (σ^i, μ^i). The incentive constraint for agent i to play $(\overline{\sigma}^i, \overline{\mu}^i)$ is

$$\sum_{\mathcal{R}} W(r) g(r|\overline{\sigma}^i, \overline{\mu}^i) - J \geq \sum_{\mathcal{R}} W(r) g(r|\sigma^i, \mu^i) - \sum_{t \in \mathcal{J}} \sigma^i_t \qquad \forall (\sigma^i, \mu^i) \in \Sigma^i. \quad \text{(IC)}$$

[9] It is without loss of generality to define the message strategy as independent of realized effort, since each signal corresponds to a unique effort level.

An agent accepts a contract if and only if it is individually rational to do so,

$$\sum_{\mathcal{R}} W(r)g(r|\overline{\sigma}^i, \overline{\mu}^i) - J \geq 0. \tag{IR}$$

Under limited liability, (IR) is implied by (IC), as the agent earns a weakly positive payoff under the deviation to no effort on all tasks. An agent captures *rents* in contract C if his expected payoff is strictly positive.

2.3 The Principal's Problem

Strategies. At the task level, the Principal chooses how many agents to hire and an action choice, and at the agent level, the Principal chooses how to design each contract. A task-strategy is a distribution over the number of agents to hire for each task, $\eta_t \in \Delta(\{0, 1, 2\})$, and a map from the set of message profiles to the set of distributions over the action choice for each task, $\alpha_t : \mathcal{M} \to \Delta(\Omega)$. We restrict attention to stationary task strategies η in which $\eta_t = \eta$ for all t. This is without loss of generality, as every non-stationary task-strategy has a payoff-equivalent stationary task-strategy, given that tasks are symmetric and hiring multiple agents provides no additional information about the state.

Monitoring Consistency. The monitoring technology for a contract depends on the number of agents hired for each task in the contract. Therefore, the Principal's choice of monitoring technology is linked to the Principal's task-strategy η. The monitoring consistency condition requires that the Principal's monitoring technology is consistent with the number of agent-tasks generated by the Principal.

Condition 2 (Monitoring Consistency)**.** *Given stationary task-strategy η with $\eta(1) = 1 - p$ and $\eta(2) = p$, monitoring profile $(q_1, ..., q_J)$ is consistent if*

$$\frac{1}{J}\sum_{j=1}^{J} q_j \quad = \quad \frac{2p}{1+p}. \tag{1}$$

Given a monitoring rate profile, there is a unique stationary task-strategy η that satisfies Condition 2 (the converse is not true). Therefore, we can restrict attention to characterizing the Principal's optimal contract, and the stationary task-strategy that is consistent with this contract follows from monitoring consistency. The following example provides intuition for this condition.

Example 1. Suppose that the Principal hires two agents for a task with probability $p = 1/2$. In expectation this generates three agent-tasks for every two tasks and two of these three agent-tasks are assigned to two agents. From an agent's perspective, the probability that the task has been assigned to a second agent is $2/3$ and the probability that it has only been assigned to the current agent is $1/3$. Consistency requires monitoring rate $q = 2/3$. More generally, if the Principal offers a single contract with the same monitoring rate for all tasks, consistency requires monitoring rate $q = \frac{2p}{1+p}$.

Payoffs. The Principal's payoff on a task depends on whether its action matches the realized state of the world and the payments to agents. The payoff from delegating task t to set of agents $\mathcal{I}_t$, offering contracts $\{C^i\}_{i\in\mathcal{I}_t}$, receiving reports $\{r^i\}_{i\in\mathcal{I}_t}$ and choosing action A_t is

$$v \times \mathbb{1}_{\{A_t=\omega\}} - \sum_{i\in\mathcal{I}_t} W^i(r^i)/J^i,$$

where $W^i(r^i)/J^i$ is the per-task payment for an agent hired for task t. This ensures that payments are not double-counted across tasks.[10]

[10] It may seem more natural to define the Principal's payoff on a self-contained block of tasks and agents, in which all agents in the block are assigned to tasks in the block and vice versa. For any monitoring technology, job size and monitoring technology with $\eta(n) \in \mathbb{Q}$ for $n = 0, 1, 2$, where $\mathbb{Q}$ is the set of rational numbers, it is possible to form such a block. Choose T tasks such that the number of agent-tasks, $T\sum_{n=1}^{2}\eta(n)n$, and the number of agents, $I = T\sum_{n=1}^{2}\eta(n)n/J$, are integers. For example, if $\eta(1) = \eta(2) = 1/2$ and $J = 10$, setting $T = 20$ generates 30 agent-tasks to be completed by $I = 3$ agents. The Principal's payoff on this block is

$$v\sum_{t=1}^{20}\mathbb{1}_{\{A_t=\omega\}} - \sum_{i=1}^{3} W^i(r^i)$$

Maximizing the expected payoff per-task is equivalent to maximizing the expected payoff per block. Thus it is valid to define the Principal's objective function in terms of the per-task payoff.

Contract Design. The Principal offers a set of contracts $\mathcal{C}$ to maximize its expected per-task payoff, subject to the agents' incentive and individual rationality constraints and the Principal's limited liability, monitoring consistency and credibility constraints. The Principal solves

$$\max_{(\alpha,\mathcal{C})} \mathbb{E}\left[v \times \mathbb{1}_{\{A_t=\omega\}} - \sum_{i\in\mathcal{I}_t} W^i(r^i)/J^i\right] \tag{*}$$

subject to IC and IR for all i, and Conditions 1 and 2. In this framework, the decision to not delegate tasks to agents corresponds to setting $J^i = 0$ for all i and $\mathcal{I}_t = \emptyset$ for all t.

We define several types of monitoring in the context of the optimal contract. Let $\mathcal{Q}(\varepsilon) = \{Q | q_j \geq \varepsilon \ \forall j\}$ be the set of monitoring profiles with a monitoring rate of at least ε on each task. We say that *virtual monitoring* is optimal if, for any $\varepsilon > 0$, when the Principal is restricted to the set of contracts with monitoring profiles in $\mathcal{Q}(\varepsilon)$, the optimal contract sets $q_j = \varepsilon$ for all $j = 1, ..., J$. *Stochastic monitoring* is optimal if there exists an $\varepsilon > 0$ such that when the Principal is restricted to the set of contracts with monitoring profiles in $\mathcal{Q}(\varepsilon)$, the optimal contract sets $q_j > \varepsilon$ for all $j = 1, ..., J$.

Delegation is Efficient. The surplus from delegating a task to an agent equals the value of learning the correct state minus the cost of effort, $v - 1$. If the Principal does not hire an agent, it chooses the action corresponding to the most likely state, which yields an expected surplus of $\bar{\pi}v$, where $\bar{\pi} = \max_\omega \pi(\omega)$ is the probability of the most likely state. We assume that delegation is efficient, which corresponds to $v > 1/(1 - \bar{\pi})$.

Unlimited Liability. If there is no restriction on negative transfers, the Principal can punish agents with arbitrarily severe punishments. In the optimal contract, the Principal sets $W(r) < 0$ for reports r that only occur under shirking profiles, ensuring that the expected payoff from any effort profile with shirking is negative.

Lemma 1 (Unlimited Liability)**.** *Under both partial and full commitment, the* *optimal contract has virtual monitoring, no bundling and punishes mismatches. For any $\varepsilon > 0$,*

$$W(r) = \begin{cases} 1 & r \in \Omega \cup \{(m^i, m^{-i}) \in \Omega^2 | m^i = m^{-i}\} \\ 1 - 1/(\varepsilon(1-\overline{\pi})) & r \in \{(m^i, m^{-i}) \in \Omega^2 | m^i \neq m^{-i}\} \end{cases}$$

when restricted to $\mathcal{Q}(\varepsilon)$.

Therefore, when there is no restriction on negative wages, neither unobservable effort nor partial commitment create inefficiencies. The optimal wage structure satisfies the agent's individual rationality constraint with equality and agents do not earn any rents. Approximate efficiency and zero rents are achieved independent of the job size, so there is no benefit to bundling.

3 The Optimal Contract

3.1 Towards the Optimal Wage Structure

With limited liability, using negative transfers to punish shirking is not possible. The Principal must dissuade shirking by providing agents with rents. As in Shapiro and Stiglitz (1984), incentives are generated by the threat of losing these rents if caught deviating. Agents can guarantee themselves a positive expected payment by shirking on all tasks and sending a message to the Principal *as if* they exerted effort and acquired signals. Therefore, any contract that satisfies the incentive constraint for high effort and truthful messages also satisfies individual rationality.

Fix the job size J. Under strategy $(\overline{\sigma}^i, \overline{\mu}^i)$, the set of messages that an agent sends with positive probability is Ω^J, with $k \equiv |\Omega|^J$ elements. Partition the set of reports into $\mathcal{R}_M$, in which the agent sends a message $m \in \Omega^J$ and matches on all monitored tasks (this set includes profiles with tasks that are not monitored), and $\mathcal{R}_N$, in which a mismatch occurs on a monitored task or an agent sends a message $m \notin \Omega^J$.[11] When agents play $(\overline{\sigma}, \overline{\mu})$, any $r \in \mathcal{R}_N$ occurs with proba-

[11]For example, if $J = 1$ and $\Omega = \{0, 1\}$, then $\mathcal{R}_M = \{0, 1, 00, 11\}$ and $\mathcal{R}_N = \{\emptyset, 01, 10, \emptyset 0, \emptyset 1, 0\emptyset, 1\emptyset\}$.

bility zero, since the probability of a mismatch or a message outside Ω^J is zero. For any deviation $(\sigma^i, \mu^i) \neq (\overline{\sigma}^i, \overline{\mu}^i)$, mismatches occur with positive probability. Therefore, it is never optimal to offer a positive transfer for a report in $\mathcal{R}_N$.

Lemma 2. *Under limited liability, the optimal wage structure satisfies* $W(r) = 0\ \forall r \in \mathcal{R}_N$.

Proof. Using the partition over reports, rewrite the incentive constraint as

$$\sum_{r\in\mathcal{R}_N} W(r)\left[g(r|\overline{\sigma}^i,\overline{\mu}^i) - g(r|\sigma^i,\mu^i)\right] + \sum_{r\in\mathcal{R}_M} W(r)\left[g(r|\overline{\sigma}^i,\overline{\mu}^i) - g(r|\sigma^i,\mu^i)\right] \geq \left(J - \sum_{t\in\mathcal{J}^i}\sigma_t^i\right)$$

For any $r \in \mathcal{R}_N$, $g(r|\overline{\sigma}^i,\overline{\mu}^i) = 0 \leq g(r|\sigma^i,\mu^i)$ for all (σ^i,μ^i). Setting $W(r) > 0$ for $r \in \mathcal{R}_N$ lowers the left hand side of the incentive constraint, which is never optimal. □

Partition $\mathcal{R}_M$ into $\{R(m)\}_{m\in\Omega^J}$, where $R(m)$ is the set of possible reports that occur with positive probability when an agent sends message $m \in \Omega^J$ and agents play $(\overline{\sigma},\overline{\mu})$. This is the set of reports generated by message m, excluding mismatch reports.[12] We say a wage structure is *simple* if it pays a wage of 0 on any report in $\mathcal{R}_N$ and for each message $m \in \Omega^J$, there exists a $w > 0$ such that for any $r \in R(m)$, the agent is either paid w or 0.

Definition 3 (Simple Wage Structure)**.** *A wage structure is* simple *if it can be represented as* $(\mathbf{w}, \boldsymbol{\rho})$*, where* $\mathbf{w} = (w_1, ..., w_k) \in \mathbb{R}_+^k$ *is a vector of wage payments and* $\boldsymbol{\rho} = (\rho_1, ..., \rho_k)$ *is a vector of sets of reports with* $\rho_j \subset R(m_j)$ *for each* $m_j \in \Omega^J$*, such that, given* $k = |\Omega|^J$,

$$W(r) = \begin{cases} w_j & \textit{if } r \in \rho_j,\ j = 1, ..., k \\ 0 & \textit{if } r \in \mathcal{R} \setminus \bigcup_{j=1}^k \rho_j. \end{cases}$$

In such a wage structure, the Principal pays at most $k+1$ different amounts, even though there are $k(k+1)$ different reports in $\mathcal{R}$. Note that a simple wage structure satisfies limited liability and pays zero on $r \in \mathcal{R}_N$, as deemed optimal in Lemma 2.

[12] For example, if $J = 1$, then $R(1) = \{1, 11\}$ and $R(0) = \{0, 00\}$.

3.2 The Optimal Contract

Partial Commitment Our main result is a characterization of the optimal contract under limited liability and partial commitment. We show that the interaction between partial commitment and limited liability creates monitoring inefficiency, in that tasks are unnecessarily duplicated.

Theorem 1. *Under limited liability and partial commitment, the optimal contract that delegates tasks to agents has a symmetric stochastic monitoring technology* $\mathbf{q}^* = (q, ..., q)$ *for some* $q \in (0, 1)$*, maximal bundling and a simple wage profile in which the agent is paid a positive wage on reports in which he matches on all monitored tasks and zero on all other reports,*

$$W(r) = \begin{cases} \frac{J}{x_j(\sum_{j=1}^{k} \Pi_J(m_j)/x_j - 1)} & \text{if } r \in R(m_j), \text{ for each } m_j \in \Omega^J \\ 0 & \text{if } r \in \mathcal{R}_N, \end{cases}$$

where $x_j = \prod_{t=1}^{J}(1 - q + q\pi(m_{jt}))$ *is the probability of a report in* $R(m_j)$ *when the agent shirks and sends message* m_j*. There exists a* $\overline{v}$ *such that for* $v > \overline{v}$*, it is optimal to delegate tasks to agents.*

The optimal wage structure takes the form of paying the agent a positive wage when the agent matches on all tasks on which he is monitored. If the agent has any mismatches, he is paid nothing. The agent is paid the same wage for a message independent of how many tasks are monitored. For example, if the agent is assigned 10 tasks and sends message m, he is paid the same wage when he is monitored and matches on anywhere from 0 to 10 of these tasks. As the monitoring rate q decreases, the agent is paid a higher wage for each message, but in contrast to full commitment, the probability of being paid this wage is independent of q.

The wage for each message is inversely proportional to the probability $\Pi_J(m)$ of state profile m. Therefore, a less likely state profile corresponds to a higher wage payment. This can be viewed as a bonus for identifying less likely states. For example, if an agent is asked to review 10 images and 5% of images need to be flagged while the remaining 95% do not, the agent's wage is increasing in the number of images he flags.

The optimal contract uses stochastic monitoring, and is therefore inefficient. The size of the inefficiency depends on the dispersion of the distribution over states and the maximum job size. Bundling lowers the per-task cost of monitoring by reducing the required wage on each message. This leads to a lower optimal monitoring rate and reduces the efficiency loss from stochastic monitoring. As is standard, limited liability allows the agent to capture rents. Bundling also reduces these rents by allowing the Principal to link punishment across multiple tasks. Therefore, the Principal strictly benefits from bundling and selects the maximum feasible job size.

Agents are treated symmetrically, as are tasks within an agent's job. Intuitively, monitoring technologies with asymmetric monitoring probabilities either across tasks or agents make inefficient use of the Principal's monitoring ability, since the expected wage payment at the optimal wage structure is convex in q. Fixing a task strategy, and therefore the expected number of agents per-task, the monitoring consistency condition pins down the feasible monitoring technologies. Given an asymmetric monitoring technology $\mathbf{q}$, there exists a symmetric monitoring technology $\mathbf{q}'$ that hires the same expected number of agents but pays a lower average expected wage per agent.

Full Commitment. The following theorem characterizes the optimal contract under limited liability and full commitment to contrast with the efficiency loss that arises under partial commitment.

Theorem 2. *Under limited liability and full commitment, the optimal contract that delegates tasks to agents uses a virtual monitoring technology, maximal bundling and a simple wage profile in which the agent is paid a positive wage on reports in which he is monitored and matches on every task and zero on all other reports. There exists a $\overline{v}$ such that for $v > \overline{v}$, it is optimal to delegate tasks to agents.*

The optimal wage structure takes the form of paying the agent a large positive wage when the agent is monitored and matches on all tasks. For example, if the agent is assigned 10 tasks, and is monitored and matches on all 10, the agent is paid a positive wage. But if the agent is monitored and matches on 9 or fewer tasks, or has any mismatches, the agent is paid nothing. As the monitoring rate ε decreases, the agent is paid a higher wage with lower probability. For each message $m \in \Omega^J$, there is a single report on which the agent is paid a positive wage.

The optimal contract uses virtual monitoring, and is therefore approximately efficient. However, the agent captures rents. This is because of limited liability, which guarantees a positive payoff to shirking in any contract that pays a positive wage on at least one report. In expectation, each agent is paid $\frac{|\Omega|^J}{|\Omega|^J-1}$ per-task wage on reports in which he matches, while the per-task cost of effort is $c = 1$. Therefore the Principal still benefits from bundling and selects the maximum feasible job size.

3.3 Outline of Proof of Theorem 1

This section provides an outline of the proof of Theorem 1 through a series of lemmas. An analogous outline is provided for Theorem 2 in Appendix A.

Optimal Wage Structure. Partial commitment translates to a condition on the monotonicity of W, which is not satisfied in the optimal wage structure for full commitment. The wage structure characterized in Lemma 5 only pays agents a positive wage on reports $\{R_J(m)\}$ in which an agent is monitored and matches on every task. Every report in $R(m)$ is a subset of the report $R_J(m)$. Therefore, under partial commitment, it is not credible for the Principal to pay a positive wage for $R_J(m)$ and zero for other reports in $R(m)$ (recall the credibility constraint requires $W(r) \geq W(\tilde{r})$ for all $r \subset \tilde{r}$). More generally, it is not credible to condition payment on the realization of monitoring by paying higher wages for reports with more monitoring.

We first derive the optimal wage structure for a fixed monitoring rate $\mathbf{q}$ and job size J.

Lemma 3. *Fix the monitoring rate* $\mathbf{q}$ *and job size* J*. Under limited liability and partial commitment, the optimal wage structure to enforce* $(\overline{\sigma}, \overline{\mu})$ *is simple and* *takes the form* $(\boldsymbol{\rho}^*, \mathbf{w}(\boldsymbol{\rho}^*))$*, where*

$$\begin{aligned}\boldsymbol{\rho}^* &= (R(m_1), ..., R(m_k)) \\ \mathbf{w}(\boldsymbol{\rho}^*) &= \frac{J}{\sum_{j=1}^{k} \mathit{\Pi}_J(m_j)/x_j - 1} \cdot (1/x_1, ..., 1/x_k)\end{aligned}$$

such that for each $m_j \in \Omega^J$*,* $x_j = \prod_{t=1}^{J}(1 - q_t + q_t\pi(m_{jt}))$ *is the probability of a report in* $R(m_j)$ *when the agent shirks and sends message* m_j *and* $\mathit{\Pi}_J(m_j)$ *is the probability of state profile* m_j*. If* $r \in R(m_j)$ *for some* m_j*, then* $W(r) = J/x_j(\sum_{j=1}^{k} \mathit{\Pi}_J(m_j)/x_j - 1)$ *and otherwise,* $W(r) = 0$*.*

Lemma 3 establishes that the optimal wage structure is simple and takes the form of paying the agent the same wage on every profile in which the agent matches on any task on which he is monitored, regardless of the number of monitored tasks, and otherwise paying the agent a wage of zero. The wage for reports generated by message m is inversely proportional to the likelihood that the agent matches on all monitored tasks when the agent shirks and reports m. For any monitoring rate $\mathbf{q}$ and job size J, this wage will be lower than the wage for the same message under full commitment, but the Principal will pay this wage with higher probability.

The intuition for paying the same wage on all reports in $R(m)$ is as follows. Let $R_l(m) \subset R(m)$ be the set of reports in which the agent sends message m and is monitored and matches on l tasks. Since the difference between the probability of a report in $R_l(m)$ under working and shirking is increasing in l, the Principal wants to pay a higher wage for reports in which the agent is monitored on more tasks. However, this is precisely what violates credibility. Therefore, the partial commitment requirement is binding and the Principal cannot condition on the realization of monitoring to strengthen incentives.

Optimal Monitoring and Bundling. As in the case of full commitment, the optimal monitoring rate is determined by minimizing the expected per-task wage bill. Fix monitoring rate $\mathbf{q}$ and job size J. Under the wage structure characterized in Lemma 3, with probability $p_j(\rho_j^*) = \mathit{\Pi}_J(m_j)$ an agent sends message $m_j \in \Omega^J$ and is paid $w_j = J/x_j(\sum_{j=1}^{k} \mathit{\Pi}_J(m_j)/x_j - 1)$. The expected wage *per agent* is

$$\mathbf{p}(\boldsymbol{\rho}^*) \cdot \mathbf{w}^* = J \left(\frac{\sum_{j=1}^{k} \frac{\mathit{\Pi}_J(m_j)}{x_j}}{\sum_{j=1}^{k} \frac{\mathit{\Pi}_J(m_j)}{x_j} - 1} \right) \tag{2}$$

where $\mathbf{p}(\boldsymbol{\rho}^*) \equiv (p_1(\rho_1^*), ..., p_k(\rho_k^*))$ and x_j is the probability of a report in ρ_j^* when the agent deviates to always shirking and sending message m_j. Note that x_j depends on $\mathbf{q}$ and J. Therefore, under partial commitment, the expected wage per agent depends on the monitoring rate. More frequent monitoring lowers each x_j, and therefore, the expected wage for an agent, and the expected wage approaches infinity as the monitoring rate becomes small.

Suppose the Principal offers a contract with symmetric monitoring rate $\mathbf{q} = (q, ..., q)$ (we will establish that symmetric monitoring is optimal). Given monitoring consistency, the expected number of agents hired for each task is $\frac{2}{2-q}$. Therefore, the expected wage *per-task* is

$$\left(\frac{2}{2-q} \right) \left(\frac{\sum_{j=1}^{k} \frac{\mathit{\Pi}_J(m_j)}{x_j(q,J)}}{\sum_{j=1}^{k} \frac{\mathit{\Pi}_J(m_j)}{x_j(q,J)} - 1} \right). \tag{3}$$

where we write $x_j(q, J)$ to explicitly show the dependence of the expected per-task wage on (q, J). The optimal monitoring rate and job size minimize (3). Since (3) is decreasing in J, the Principal bundles the maximum number of tasks i.e. maximal bundling. However, now there is a trade-off between the expected number of agents hired and the expected wage per agent – the former is increasing in q while the latter is decreasing in q. Minimizing (3) with respect to q results in a strictly positive optimal monitoring rate i.e. stochastic monitoring.

Lemma 4 (Optimal Monitoring and Bundling)**.** *The optimal contract under limited liability and partial commitment has symmetric stochastic monitoring and maximal bundling.*

Proof. Lemma 3 and the above characterization of the expected wage per-task establish the optimality of maximal bundling and stochastic monitoring in the class of contracts with symmetric monitoring. Lemma 11 (in the Appendix) establishes the optimality of symmetric monitoring. □

3.4 Cost of Monitoring

The equilibrium cost of monitoring is endogenously determined by the optimal wage structure and monitoring rate. Under partial commitment, this cost of monitoring is strictly positive. Therefore, the inefficiency that arises from limited liability is strictly positive when the Principal has partial commitment. In contrast, under unlimited liability or full commitment, the equilibrium cost of monitoring is arbitrarily close to zero. Therefore, limited liability and partial commitment only lead to inefficiencies when they are jointly required.

Corollary 1 (Costly Monitoring)**.** *Under limited liability and partial commitment, the cost of monitoring in the optimal contract is strictly positive, while under limited liability and full commitment, the cost of monitoring in the optimal contract is arbitrarily small.*

Proof. Follows immediately from $q^* > 0$ under limited liability and partial commitment and from the optimality of virtual monitoring under full commitment. □

3.5 Comparative Statics

Full Commitment. Increasing the job size strengthens the effect of bundling: shirking becomes less attractive since the agent needs to produce acceptable output for more tasks. Therefore, the per-task wage for each report and the expected wage payment per agent is decreasing in the maximum job size. As the prior becomes more extreme, an agent is more likely to match when shirking and sending the message corresponding to the more likely state, and therefore must be paid a higher wage to deter this deviation. Therefore, the wage for the more likely states and the expected wage payment per agent both increase. The following Corollary outlines comparative statics on the optimal wage.

Corollary 2 (Wage)**.** *Let W^* be the optimal wage structure and $\theta = \prod_{j=1}^{|\Omega|} \pi(\omega_j)$ be a measure of the dispersion of the distribution over the state space. Under limited liability and full commitment,*

1. *If $\Pi_J(m)$ is increasing (decreasing) in dispersion θ, then $W^*(r)$ is decreasing (increasing) in dispersion θ for all $r \in R_J(m)$.*

2. *The expected wage payment per agent is decreasing in dispersion θ and decreasing in the maximum job size.*

Proof. The claims follow immediately from the wage derived in Lemma 5. $\square$

The optimal monitoring probability decreases with the maximum job size. With a larger job size, it is optimal to hire fewer agents for each task and pay each agent a lower per-task wage. Taken together, the rents that the Principal pays to an agent vanish as the job size becomes large.

Corollary 3 (No Asymptotic Rents)**.** *Under limited liability and full commitment,*

$$\lim_{J\to\infty} \mathbf{p}(\boldsymbol{\rho}^*) \cdot \mathbf{w}(\boldsymbol{\rho}^*)^T/J = 1,$$

Proof. The claim follows immediately from an agent's expected wage per-task, which is $\mathbf{p}(\boldsymbol{\rho})^* \cdot \mathbf{w}^*/J = |\Omega|^{\bar{J}}/(|\Omega|^{\bar{J}} - 1)$. $\square$

Partial Commitment. A more extreme prior increases the likelihood that a shirker will match when monitored, and the Principal monitors at a higher rate to reduce the rents captured by agents. Monitoring and bundling are substitutes: when more tasks are assigned to an agent, each task can be monitored less intensively. This contrasts with the optimal contract with full commitment, in which the optimal monitoring technology (virtual monitoring) is independent of the prior and job size.

Corollary 4 (Monitoring Rate)**.** *Let $\theta = \prod_{j=1}^{|\Omega|} \pi(\omega_j)$ be a measure of the dispersion of the state space. Under limited liability and partial commitment, the optimal monitoring rate q^* is increasing in θ and decreasing in the maximum job size.*

Proof. The claims follow immediately from Lemma 3 and Theorem 1. $\square$

Virtual monitoring is optimal asymptotically as the job size grows large. Therefore, the efficiency loss relative to the contractible effort benchmark vanishes and asymptotically, the optimal contract is approximately efficient. Similar comparative statics to Corollary 2 hold for the wage under partial commitment.

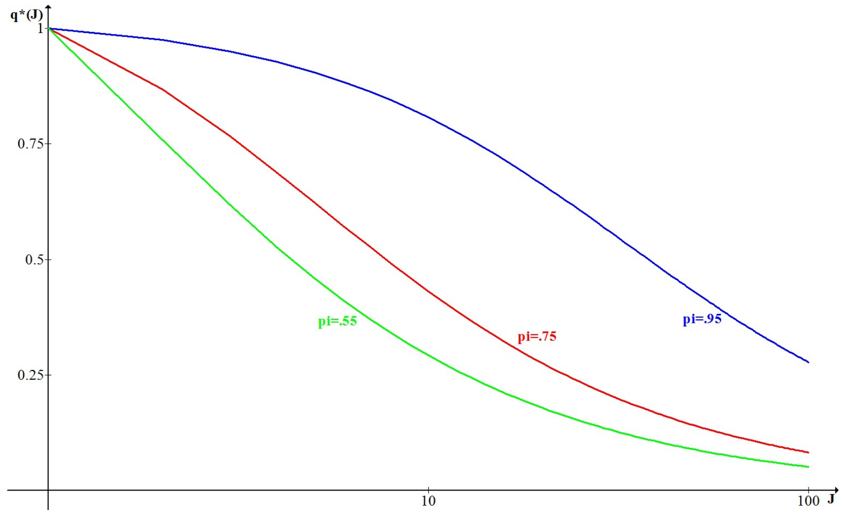

FIGURE 1. The optimal monitoring rate is decreasing in J.

3.6 Individual Monitoring.

Another method of quality assurance, which we refer to as *individual monitoring*, involves including tasks for which the Principal already knows the correct answer. An agent's payment is based on whether his messages match on the known tasks. In our setting, the optimal peer-monitoring contract strictly outperforms the optimal individual monitoring contract.

Theorem 3. *The optimal individual monitoring contract is dominated by the optimal peer-monitoring contract.*

The intuition for the result is as follows. Suppose the Principal seeds an agent's job with tasks for which it already knows the state. Agents are assigned J tasks and the Principal optimizes over how many known tasks to include within this set. Let n be the optimal number. An agent's job is judged on the basis of the agent's performance on the subset of known tasks. Payment is provided if and only if the agent performs satisfactorily on all n known tasks. An agent must be compensated for the cost of effort on all J tasks, as well as receive whatever rents are required to dissuade shirking. Let w_G denote the optimal wage. In equilibrium, the Principal pays each agent w_G and learns the state of $J - n$ new tasks. If $2J$ tasks are assigned to two agents, the cost is $2w_G$ while the benefit is

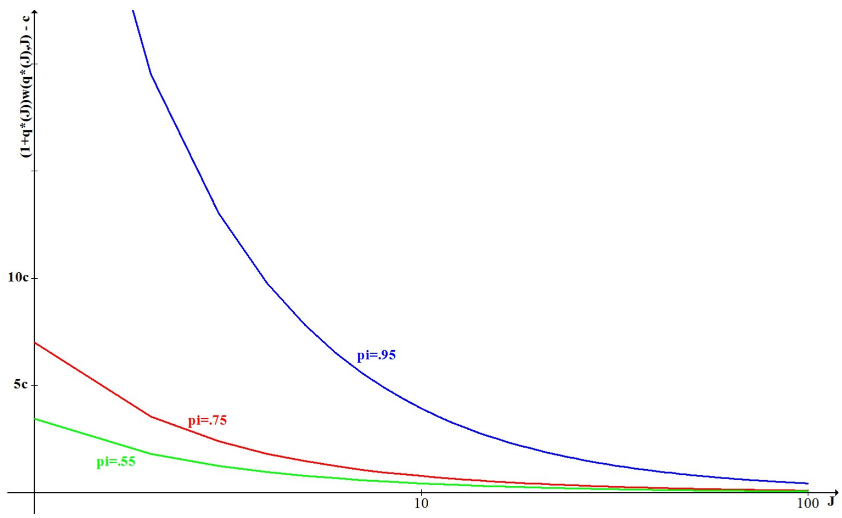

FIGURE 2. The efficiency loss is decreasing in J.

$2(J-n)$.

Alternatively, the Principal could have the two agents overlap on n of the J assigned tasks (i.e. peer-monitoring). The wage that incentivizes effort is the same, so the expected wage bill is the same as in the individual monitoring contract, $2w_G$. But now the Principal is learning the state for $J+J-n=2J-n$ new tasks. The benefit is strictly higher than in the individual monitoring contract, as the same wage bill is spread over a greater number of new tasks. Individual monitoring contracts are inefficient because they monitor agents independently, compared to the optimal peer-monitoring contract, which jointly monitor agents.

However, individual monitoring is useful for full implementation. Any peer-monitoring contract always has no effort equilibrium in which all agents choose zero effort. Inserting known tasks into an agent's job can eliminate this undesirable equilibrium. The Principal can use a combination of peer and individual monitoring for full implementation.

4 Discussion and Conclusion

The insights from Section 3 also apply to multidimensional tasks. Suppose an agent is assigned a task with multiple components. For example, an agent must complete a tax return with additional schedules for each source of non-wage income. Theorems 2 and 1 suggests that the Internal Revenue Service should jointly monitor all tax forms and impose the harshest possible penalty upon uncovering any irregularities.

An entertaining example of bundling incentives on multidimensional tasks is provided by the rock band Van Halen. Like many musical acts, the band's performance contract with event venues is a long, complicated document specifying hundreds of individual items. Within the 53-page rider is an obscure provision often taken as prima facie evidence of rock excess: a bowl of M&M's is to be provided with all brown candies removed. As the band's lead signer explained in his autobiography, the unusual request performed a monitoring function:

> Van Halen was the first band to take huge productions into tertiary, third-level markets. We'd pull up with nine eighteen-wheeler trucks, full of gear, where the standard was three trucks, max. And there were many, many technical errors [...] The contract rider read like a version of the Chinese Yellow Pages because there was so much equipment, and so many human beings to make it function. So just as a little test [...] in the middle of nowhere, was: "There will be no brown M&M's in the backstage area, upon pain of forfeiture of the show, with full compensation." So, when I would walk backstage, if I saw a brown M&M in that bowl ... well, line-check the entire production (Roth 1997, pp. 97-98).

The model is also robust to several other extensions, including asymmetric Principal payoffs and almost perfect signals.

Asymmetric Principal Payoffs. Suppose the Principal's payoffs are asymmetric across states and actions. Let $\alpha_\omega \geq 0$ be the Principal's payoff from matching the state when the state is ω and $\beta_\omega \leq 0$ be the Principal's payoff from failing to match state ω. For example, in an image screening task, α_1 (β_1) corresponds to permitting (prohibiting) the sharing of harmless content and α_0 (β_0) represents removing (failing to remove) an objectionable item. The structure of the optimal contract is unchanged – only the Principal's participation constraint changes. With payoffs α_ω and β_ω, the Principal's expected payoff from selecting $A_t = 0$ ($A_t = 1$) without hiring any agents is $(1-\pi)\alpha_0 + \pi\beta_1$ ($(1-\pi)\beta_0 + \pi\alpha_1$). The value of the proposed contract must exceed both of these values in order for the Principal to participate.

Noisy Signals. In the previous analysis, we assumed that effort perfectly reveals the state for each task. Suppose that signals are imperfect but sufficiently precise so that in the contractible effort benchmark the Principal still wants to hire a single agent. In the perfect signal model, agents always produce matching messages and obtain payment with certainty in equilibrium. This is not the case with imperfect signals. Even if all agents exert effort, agents sometimes send different messages for the same task.

There are three levers the Principal can use to introduce leniency into the contract. It can set a more tolerant match rate for reports, it can bundle fewer tasks in each job or it can monitor each task less intensively. The effect of each lever is the same: an agent is able to produce matching output on fewer tasks and still receive a positive wage. For sufficiently precise signals, the Principal finds it optimal to monitor less frequently, and otherwise the optimal contract is similar to the case of perfect signals. Incentive concerns push the Principal to inefficiently hire multiple agents for a single task. The Principal now receives an additional (small) learning benefit from hiring these additional agents. The Principal does not pay on mismatched reports and bundles as many tasks together as possible.

In conclusion, new information technology permits firms and workers to interact through spot labor markets. Compared to conventional labor markets, spot markets offer significant advantages for a firm. A flexible and scalable workforce is available to start work immediately and no preexisting relationship with a worker is presumed nor is the promise of a continuing relationship required.

But the minimal interaction between a supervisor and her employees raises new challenges. The supervisor must provide adequate supervision to ensure workers are exerting effort. Workers are compensated for their effort, but the exertion of effort is costly and unobservable, and the quality of a worker's output cannot be verified directly.

With traditional reputation mechanisms inapplicable and the threat of large penalties for poor performance unavailable, the supervisor creates incentives for effort by periodically hiring additional workers to duplicate some of the tasks she has already assigned. Wages are then made contingent upon satisfactory performance on all tasks.

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A Appendix

A.1 Outline of proof of Theorem 2

Optimal Wage Structure. We first derive the optimal wage structure to enforce high effort and truthful reporting for a fixed monitoring rate $\mathbf{q}$ and job size J. Any wage structure that enforces high effort and truthful reporting must satisfy the following conditions:

1. For each $m \in \Omega^J$, there exists a report $r \in R(m)$ such that $W(r) > 0$.
2. For each task t, there exists a report r with $W(r) > 0$ such that an agent is monitored and matches on task t.

If (1) does not hold, then an agent would never report m, and if (2) does not hold, then an agent would never exert effort and report truthfully on task t.

The characterization proceeds as follows. We calculate the optimal simple wage structure $(\mathbf{w}^*, \boldsymbol{\rho}^*)$ to deter deviations to the set of strategies $\{(\sigma_0, m_j)\}_{j=1}^{k}$ in which an agent shirks on all tasks and sends message $m_j \in \Omega^J$. We then show that $(\mathbf{w}^*, \boldsymbol{\rho}^*)$ also deters deviations to arbitrary strategy $(\sigma^i, \mu^i) \in \Sigma^i$, which makes it optimal in the class of simple wage structures. Finally, we show that $(\mathbf{w}^*, \boldsymbol{\rho}^*)$ is optimal in the set of all wage structures that satisfy limited liability and full commitment.

Given message $m \in \Omega^J$, let $R_J(m) \subset R(m)$ be the report where an agent is monitored and matches on all J tasks. For example, if $J = 1$ and $\Omega = \{0, 1\}$, then $R_1(1) = \{11\}$.

Lemma 5 (Optimal Wage Structure)**.** *Fix the monitoring rate* $\mathbf{q}$ *and job size* J*. Under limited liability and full commitment, the optimal wage structure to enforce* $(\overline{\sigma}, \overline{\mu})$ *is simple and takes the form* $(\boldsymbol{\rho}^*, \mathbf{w}^*)$*, where*

$$\begin{aligned} \boldsymbol{\rho}^* &= (R_J(m_1), ..., R_J(m_k)) \\ \mathbf{w}^* &= \frac{J}{(|\Omega|^J - 1)\overline{q}} \cdot (1/\Pi_J(m_1), ..., 1/\Pi_J(m_k)) \,. \end{aligned}$$

where for each $m_j \in \Omega^J$*,* $\Pi_J(m_j)$ *is the probability of state profile* m_j *and* $\overline{q} =$

$\prod_{t=1}^{J} q_t$ *is the probability of being monitored on all tasks. If report* $r \in R_J(m_j)$ *for some* m_j*, then* $W(r) = J/(|\Omega|^J - 1)\overline{q}\Pi_J(m_j)$ *and otherwise,* $W(r) = 0$*.*

Lemma 5 establishes that the optimal wage structure is simple and takes the form of paying the agent a positive wage only when he is monitored and matches on all tasks. The intuition is as follows. The likelihood ratio of the probability of a report under working compared to shirking and sending message m, $g(r|\overline{\sigma}^i, \overline{\mu}^i)/g(r|\sigma_0, m)$, is highest for the report in which the agent is monitored on all tasks. Therefore, paying a positive wage only on $R_J(m)$ generates the strongest incentives. The magnitude of the wage is driven by the ratio of the probability of a report in $R_J(m)$ under high effort to the probability under no effort and sending message m, $1/\Pi_J(m)$. This ensures that deviations to no effort and each message $m \in \Omega^J$ are equally profitable. Recall all proofs not presented in the text can be found in the Appendix.

Optimal Monitoring and Bundling. Next, we derive the optimal monitoring technology and bundling level. In a high effort and truthful reporting equilibrium, the Principal learns the true state for each task under any contract that satisfies the agent's incentive constraint. Any incentive compatible contract provides the Principal with the same information, and the Principal chooses the optimal contract to minimize the cost of acquiring this information. Therefore, the optimal monitoring rate is determined by minimizing the Principal's expected per-task wage bill.

Fix monitoring rate $\mathbf{q}$ and job size J and let $\overline{q} = \prod_{t=1}^{J} q_t$ be the probability of being monitored on every task. Under the wage structure characterized in Lemma 5, an agent receives report $r \in \rho_j^*$ and is paid $w_j^* = J/(|\Omega|^J - 1)\overline{q}\Pi_J(m_j)$ with probability $p_j(\rho_j^*) = \overline{q}\Pi_J(m_j)$ for each $j = 1, ..., |\Omega|^J$, and is paid 0 with probability $1 - \overline{q}$. Therefore, the expected wage *per agent* is

$$\mathbf{p}(\boldsymbol{\rho}^*) \cdot \mathbf{w}^* = J\left(\frac{|\Omega|^J}{|\Omega|^J - 1}\right) \tag{4}$$

where $\mathbf{p}(\boldsymbol{\rho}^*) \equiv (p_1(\rho_1^*), ..., p_k(\rho_k^*))$ is the probability of each set of reports in ρ^*. Importantly, (4) is independent of $\mathbf{q}$.

The Principal seeks to minimize the expected per-task wage, which depends on the expected number of agents hired for each task and the expected wage per agent. Suppose the Principal offers a single contract with monitoring rate $\mathbf{q}$. Given monitoring consistency, the Principal must use task strategy $\eta(2) = \left(\frac{1}{J}\sum_{j=1}^{J} q_j\right) / \left(2 - \frac{1}{J}\sum_{j=1}^{J} q_j\right)$. The expected number of agents hired is $1+\eta(2)$. Therefore, the expected wage *per-task* is

$$\left(\frac{2}{2 - \frac{1}{J}\sum_{j=1}^{J} q_j}\right)\left(\frac{|\Omega|^J}{|\Omega|^J - 1}\right) \tag{5}$$

The Principal's optimal monitoring rate and job size minimizes (5). Since (5) is increasing in each q_j and decreasing in J, the Principal chooses the smallest monitoring rate possible on each task – virtual monitoring – and bundles the maximum number of tasks – maximal bundling.

Lemma 6 (Optimal Monitoring and Bundling)**.** *The optimal contract under limited liability and full commitment has virtual monitoring and maximal bundling.*

Proof. Follows immediately from the above characterization and Lemma 5. ☐

A.2 Proofs of Lemmas

Proof of Lemmas 3 and 5. The proof follows from Lemmas 7 and 10.

Lemma 7. *Fix a vector of sets of reports $\boldsymbol{\rho} = (\rho_1, ..., \rho_k)$, monitoring rate $\mathbf{q}$ and job size J. The optimal simple wage structure to deter deviations from high effort and truthful reporting to strategies $\{(\sigma_0, m)\}_{m\in\mathcal{S}^J}$ is $\mathbf{w}(\boldsymbol{\rho}) = (w_1, ..., w_k)$, where*

$$w_j(\rho_j) = \left(\frac{J}{\left(\sum_{l=1}^{k}\frac{p_l}{x_l} - 1\right)}\right)\frac{1}{x_j},$$

$p_j \equiv g(\rho_j|\overline{\sigma}^i, \overline{\mu}^i)$ is the probability of a report in ρ_j under strategy $(\overline{\sigma}^i, \overline{\mu}^i)$ and $x_j \equiv g(\rho_j|\sigma_0, m_j)$ is the probability of a report in ρ_j under strategy (σ_0, m_j) for each $m_j \in \Omega^J$.

The wage vector is driven by the ratio of the probability of a report in ρ_j under high effort and the probability of a report in ρ_j under shirking and sending message m_j. Note that w_j only depends on $\mathbf{q}$ through the dependence of p_j and x_j on $\mathbf{q}$.

Proof. Fix an arbitrary $\boldsymbol{\rho} = (\rho_1, ..., \rho_k)$. Let $p_j = g(\rho_j|\overline{\sigma}^i, \overline{\mu}^i)$ be the probability of a report in set ρ_j under strategy $(\overline{\sigma}^i, \overline{\mu}^i)$ and let $x_j = g(\rho_j|\sigma_0, m_j)$ be the probability of a report in set ρ_j under the strategy (σ_0,m_j). Note that $g(\rho_j|\sigma_0, m_l) = 0$ for all $l \neq j$. Let $\mathbf{p} = (p_1, ..., p_k)$ and $\mathbf{x} = (x_1, ..., x_k)$ be the corresponding vectors. The incentive constraint to prevent deviating to strategy (σ_0,m_j) is

$$\mathbf{p} \cdot \mathbf{w}^T - J \geq x_j w_j$$

for each $j = 1, ..., k$. Define

$$\Delta = \begin{bmatrix} p_1 - x_1 & p_2 & \dots & p_k \\ p_1 & p_2 - x_2 & & p_k \\ \dots & & & \dots \\ p_1 & p_2 & \dots & p_k - x_k \end{bmatrix}$$

and $\mathbf{J} = (J, ..., J)$. Note the diagonal entries are negative, $p_j - x_j \leq 0$. Rewrite the set of incentive constraints as

$$\Delta \cdot \mathbf{w}^T \geq \mathbf{J}$$

and satisfy it with equality by setting $\mathbf{w}^T = \Delta^{-1} \cdot \mathbf{J}$, where Δ^{-1} is

$$\frac{1}{d} \begin{bmatrix} \prod_{j \neq 1} x_j - \sum_{j \neq 1} p_j \prod_{l \neq 1,j} x_l & p_2 \prod_{j \neq 2,1} x_j & \dots & p_k \prod_{j \neq k,1} x_j \\ p_1 \prod_{j \neq 1,2} x_j & \prod_{j \neq 2} x_j - \sum_{j \neq 2} p_j \prod_{l \neq 2,j} x_l & & p_k \prod_{j \neq k,2} x_j \\ \dots & & & \dots \\ p_1 \prod_{j \neq 1,k} x_j & p_2 \prod_{j \neq 2,k} x_j & \dots & \prod_{j \neq k} x_j - \sum_{j \neq k} p_j \prod_{l \neq k,,j} x_l \end{bmatrix}$$

and

$$d = \sum_{j=1}^{k} p_j \prod_{l \neq j} x_l - \prod_{j=1}^{k} x_j = \prod_{j=1}^{k} x_j \left(\sum_{j=1}^{k} \frac{p_j}{x_j} - 1 \right).$$

Summing each row and multiplying by J/d yields:

$$\mathbf{w} \quad = \quad \frac{J}{d}\prod_{j=1}^{k} x_j \cdot (1/x_1, ..., 1/x_k) = \frac{J}{\left(\sum_{j=1}^{k} \frac{p_j}{x_j} - 1\right)} \cdot (1/x_1, ..., 1/x_k)$$

For a given $\boldsymbol{\rho}$, this characterizes the vector of wages that satisfies the set of incentive constraints for deviations to σ_0 and sending a message $m \in \Omega^J$. Obviously, for any σ^i, sending a message $m \notin \Omega^J$ is never an optimal deviation, since $W(r) = 0$ for any report generated by $m \notin \Omega^J$. Therefore, $\mathbf{w}$ satisfies the set of incentive constraints for all deviations to strategies $\{(\sigma_0, m)\}$.

□

Lemma 8. *Fix a monitoring rate* $\mathbf{q}$ *and job size* J*. Given the simple wage vector* $\mathbf{w}(\boldsymbol{\rho})$ *characterized in Lemma 7, the optimal set of reports* $\boldsymbol{\rho}^*$ *to deter deviations to strategies* $\{(\sigma_0,m_j)\}_{j=1}^{k}$ *is:*

1. *Full Commitment:* $\boldsymbol{\rho}^* = (R_J(m_1), ..., R_J(m_k))$
2. *Partial Commitment:* $\boldsymbol{\rho}^* = (R(m_1), ..., R(m_k))$.

Proof. The optimal $\boldsymbol{\rho}$ is the set of reports that minimize the expected wage bill for an agent. The wage vector $\mathbf{w}$ and vectors of probabilities $\mathbf{p}$ and $\mathbf{x}$ depend on $\boldsymbol{\rho}$; write $\mathbf{w}(\boldsymbol{\rho})$, $\mathbf{p}(\boldsymbol{\rho})$ and $\mathbf{x}(\boldsymbol{\rho})$ to capture this dependence. Given $\boldsymbol{\rho}$, the expected wage bill for a single agent is

$$\mathbf{p}(\boldsymbol{\rho}) \cdot \mathbf{w}(\boldsymbol{\rho})^T = \sum_{j=1}^{k} p_j(\rho_j) w_j(\rho_j) = J \frac{\left(\sum_{j=1}^{k} \frac{p_j(\rho_j)}{x_j(\rho_j)}\right)}{\left(\sum_{j=1}^{k} \frac{p_j(\rho_j)}{x_j(\rho_j)}\right) - 1},$$

where $p_j \leq x_j$. Minimizing the expected wage bill is equivalent to maximizing

$$\max_{\boldsymbol{\rho}} \sum_{j=1}^{k} \frac{p_j(\rho_j)}{x_j(\rho_j)},$$

where $1 \leq \sum_{j=1}^{k} \frac{p_j}{x_j} \leq k$.[13]

[13]The lower bound is derived from only including reports with no monitoring in $\boldsymbol{\rho}$, which yields $\sum_{j=1}^{k} \frac{p_j}{x_j} = \sum_{j=1}^{k} \binom{k}{j} \pi^{k-j} (1-\pi)^j = 1$, and upper bound from achieving $p_j = x_j$.

Full commitment. Suppose $\rho_j = R_J(m_j)$, the report where an agent is monitored and matches on all tasks. If an agent plays strategy $(\overline{\sigma}^i, \overline{\mu}^i)$, then the probability of a report in ρ_j is the probability of message m_j, $\Pi_J(m_j)$, times the probability of being monitored on all tasks, $\overline{q} = \prod_{t=1}^{J} q_t$, times the probability of matching on all tasks conditional on playing m_j, which is 1. Therefore, $p_j(\rho_j) = \overline{q}\Pi_J(m_j)$. If an agent deviates to strategy (σ_0, m_j), then the probability of a report in ρ_j is the probability of message m_j, which is 1, times the probability of being monitored on all tasks, $\overline{q}$, times the probability of matching on all tasks conditional on playing m_j, $\Pi_J(m_j)$. Therefore, $x_j = \overline{q}\Pi_J(m_j)$. Hence, $p_j = x_j$. For all $r \in R(m_j)$ such that $r \notin R_J(m_j)$, $g(r|\overline{\sigma}^i, \overline{\mu}^i) < g(r|\sigma_0, m_j)$. Therefore, $p_j = x_j$ uniquely holds for $\rho_j = R_J(m_j)$ and $\frac{p_j(\rho_j)}{x_j(\rho_j)}$ is maximized at $\rho_j = R_J(m_j)$. Therefore, the optimal structure is $\boldsymbol{\rho}^* = (R_J(m_1), ..., R_J(m_k))$.

Partial commitment. Partial commitment requires that for all $r \in \rho_j$, if $r' \subset r$, then $r' \in \rho_j$. There must be at least one report $r \in \rho_j$, otherwise it is not possible to incentivize an agent to play message m_j. Let $r \in R_0(m_j)$. Then $r \subseteq r'$ for all $r' \in R(m_j)$. Given $\rho_j \neq \emptyset$, partial commitment requires $R_0(m_j) \in \rho_j$.

Suppose $\rho_j = R_0(m_j)$, the minimal set in which it is possible to satisfy partial commitment. Suppose $r_1 \in R_1(m_j)$. Let m_{j1} be the element of m_j which is monitored in report r_1. Then

$$\frac{p_j}{x_j} = \frac{g(R_0(m_j)|\overline{\sigma}, \overline{\mu})}{g(R_0(m_j)|\sigma_0, m_j)} = \frac{\Pi_J(m_j)}{1} < \frac{g(r_1|\overline{\sigma}, \overline{\mu})}{g(r_1|\sigma_0, m_j)} = \frac{\Pi_J(m_j)}{\Pi_1(m_{j1})}$$

Therefore,

$$\frac{p_j}{x_j} < \frac{g(R_0 \cup r_1|\overline{\sigma}, \overline{\mu})}{g(R_0 \cup r_1|\sigma_0, m_j)},$$

and adding r_1 to ρ_j would increase p_j/x_j without violating partial commitment, given r_1 has a unique subset $r \in R_0(m_j)$, which is already in ρ_j.[14] Therefore $\rho'_j = R_0(m_j) \cup \{r_1\}$ is strictly preferred to ρ_j.

Let $R_2(m_j)|_{r_1} = \{r_2 \in R_2(m_j) : r_1 \subset r_2\}$. This is the set of reports that it is

[14] This inequality follows from if $a_1/b_1 < a_2/b_2$, then $a_1/b_1 < (a_1 + a_2)\,/\,(b_1 + b_2)$.

possible to add to ρ'_j without violating partial commitment. To simplify notation, suppose $\mathbf{q}$ is symmetric with task monitoring rate q. Comparing ρ'_j and $\rho''_j = R_0 \cup R_1 \cup R_2|_{r_1}$,

$$\begin{aligned} \frac{g(\rho'_j|\overline{\sigma},\overline{\mu})}{g(\rho'_j|\sigma_0,m_j)} &= \frac{\left[(1-q)^J + q\,(1-q)^{J-1}\right] \mathit{\Pi}_J(m_j)}{(1-q)^J + q\,(1-q)^{J-1}\,\mathit{\Pi}_1(m_{j1})} \\ \frac{g(\rho''_j|\overline{\sigma},\overline{\mu})}{g(\rho''_j|\sigma_0,m_j)} &= \frac{\left[(1-q)^J + Jq\,(1-q)^{J-1} + (J-1)\,q^2\,(1-q)^{J-2}\right] \mathit{\Pi}_J(m_j)}{\left((1-q)^J + q\,(1-q)^{J-1}\,\mathit{\Pi}_1(m_{j1})\right)\left(1+\frac{q}{1-q}\sum_{t=2}^J \mathit{\Pi}_1(m_{jt})\right)} \end{aligned}$$

Since $\sum_{t=2}^J \mathit{\Pi}_1(m_{jt}) \leq (J-1)$,

$$\frac{g(\rho'_j|\overline{\sigma},\overline{\mu})}{g(\rho'_j|\sigma_0,m_j)} < \frac{g(\rho''_j|\overline{\sigma},\overline{\mu})}{g(\rho''_j|\sigma_0,m_j)}$$

and adding $R_1 \setminus \{r_1\} \cup R_2|_{r_1}$ to ρ'_j would increase p_j/x_j and still satisfy partial commitment. Therefore ρ''_j is strictly preferred to ρ'_j. Analogous calculations establish the same property for asymmetric $\mathbf{q}$.

Similar logic establishes $\rho^{l+1}_j = \cup^l_{n=0} R_n \cup R_{l+1}|_{r_1}$ is strictly preferred to $\rho^l_j = \cup^{l-1}_{n=0} R_n \cup R_l|_{r_1}$ for all $l < J$ and satisfies partial commitment, where $R_l(m_j)|_{r_1} = \{r \in R_l(m_j) : r_1 \subset r\}$. Note $R_J(m_j)|_{r_1} = R_J(m_j)$ is a singleton. Therefore, $\rho^J_j = \cup^{J-1}_{n=0} R_n \cup R_J|_{r_1} = R(m_j)$ is preferred to any other set of reports containing r_1. This holds for all $r_1 \in R_1(m_j)$. Therefore, the optimal set of reports for message m_j is $\rho^*_j = R(m_j)$, and the optimal structure is $\boldsymbol{\rho}^* = (R(m_1), ..., R(m_k))$.[15]

[15] It is more straight forward to establish that

$$\frac{g(R_0|\overline{\sigma},\overline{\mu})}{g(R_0|\sigma_0,m_j)} < \frac{g(R_0 \cup R_1|\overline{\sigma},\overline{\mu})}{g(R_0 \cup R_1|\sigma_0,m_j)} < ... < \frac{g(\cup^l_{n=0} R_n|\overline{\sigma},\overline{\mu})}{g(\cup^l_{n=0} R_n|\sigma_0,m_j)} < ... < \frac{g(\cup^J_{n=0} R_n|\overline{\sigma},\overline{\mu})}{g(\cup^J_{n=0} R_n|\sigma_0,m_j)}.$$

Therefore, $\rho'_j = \cup^{l+1}_{n=0} R_n$ is strictly preferred to $\rho_j = \cup^l_{n=0} R_n$ for all $l < J$. Adding $r \in R_{l+1}$ to ρ_j will not violate partial commitment, as for any $r' \subset r$, $r' \in \rho_j$. However, under skewed priors, it is possible that adding a single report from R_{l+1} is better than adding the whole set,

$$\frac{g(\rho_j \cup \{r\}\,|\overline{\sigma},\overline{\mu})}{g(\rho_j \cup \{r\}\,|\sigma_0,m_j)} > \frac{g(\rho'_j|\overline{\sigma},\overline{\mu})}{g(\rho'_j|\sigma_0,m_j)}$$

for some $r \in R_{l+1}$, so this doesn't establish the optimality of $\rho_j = \cup^J_{n=0} R_n$ across all sets of

$\square$

Lemma 9. *Fix a monitoring rate* $\mathbf{q}$ *and job size* J*. The optimal simple wage* $(\boldsymbol{\rho}^*, \mathbf{w}^*)$ *to deter deviations to strategies* $\{(\sigma_0, m_j)\}_{j=1}^k$ *also deters deviations to any strategy* $(\sigma^i, \mu^i) \in \Sigma^i$*. Therefore,* $(\boldsymbol{\rho}^*, \mathbf{w}^*)$ *is the optimal simple wage structure.*

Proof. Let (σ_n, m) denote the strategy where player i deviates to exerting effort and reporting truthfully on n tasks and shirking and reporting message $m \in \Omega^{J-n}$ on $J - n$ tasks. Without loss of generality, assume the agent deviates on the first $J - n$ tasks. Given a contract with the simple wage $(\boldsymbol{\rho}^*, \mathbf{w}^*)$, the incentive constraint to deter a deviation to (σ_n, m) is

$$\sum_{j=1}^{k} w_j^* \left[p_j(\rho_j^*) - g(\rho_j^*|\sigma_n, m)\right] \geq J - n.$$

Let $m_j|_x$ denote the first x messages in a message profile.

Full Commitment. Under full commitment, $p_j(\rho_j^*) = \overline{q}\Pi_J(m_j)$ for state $m_j \in \Omega^J$ and $\sum_{j=1}^k w_j^* p_j(\rho_j^*) = J\left(\frac{|\Omega|^J}{|\Omega|^J - 1}\right)$. When the agent deviates, the probability of a report in ρ_j^* is $g(\rho_j^*|\sigma_n, m) = \overline{q}\Pi_J(m_j)$ for state $m_j \in \Omega^J$ if the first $J - n$ states match m, $m_j|_{J-n} = m$, and $g(\rho_j^*|\sigma_n, m_j) = 0$ for state $m_j \in \Omega^J$ if the first $J - n$ states do not match m, $m_j|_{J-n} \neq m$. Therefore, the incentive constraint for a deviation to (σ_n, m) simplifies to

$$\left(\frac{J|\Omega|^J}{|\Omega|^J - 1}\right) - \sum_{\left\{m_j \in \Omega^J \middle| m_j|_{J-n} = m\right\}} \frac{J}{|\Omega|^J - 1} \geq J - n.$$

When an agent reports truthfully on n tasks and shirks and plays message $m \in \Omega^{J-n}$ on the remaining tasks, there are $|\Omega|^n$ states in which the agent's message matches the state, $\left|\left\{m_j \in \Omega^J \middle| m_j|_{J-n} = m\right\}\right| = |\Omega|^n$. The incentive constraint simplifies to

$$\frac{J}{|\Omega|^J - 1}(|\Omega|^J - |\Omega|^n) \qquad \geq J - n \tag{6}$$

reports that satisfy partial commitment.

The RHS of(6) is decreasing linearly in n, while the LHS of (6) is decreasing and concave in n. At $n = 0$ and $n = J$, (6) is satisfied with equality. Therefore, (6) holds for all $n = 1, ..., J-1$. Intuitively, the cost (in lost wages) of deviating to shirking on x tasks is concave in x, while the gain (in saved effort) is linear. Therefore, if it is profitable to shirk on one task, it is profitable to shirk on them all.

Partial Commitment. Under partial commitment, $p_j(\rho_j^*) = \Pi_J(m_j)$ for state $m_j \in \Omega^J$ and $w_j = J/(\sum_{j=1}^k \Pi_J(m_j)/x_j - 1)x_j$. When the agent deviates, the probability of a report in ρ_j^* is $g(\rho_j^*|\sigma_n, m) = \Pi_J(m_j)$ for state $m_j \in \Omega^J$ if the first $J-n$ states match m, $m_j|_{J-n} = m$, and $g(\rho_j^*|\sigma_n, m_j) = 0$ for state $m_j \in \Omega^J$ if the first $J-n$ states do not match m, $m_j|_{J-n} \neq m$. When an agent reports truthfully on n tasks and shirks and plays message $m \in \Omega^{J-n}$ on the remaining tasks, there are $|\Omega|^n$ states in which the agent's message matches the state, $\left|\left\{m_j \in \Omega^J \middle| m_j|_{J-n} = m\right\}\right| = |\Omega|^n$. The incentive constraint for a deviation to (σ_n, m) simplifies to

$$\left(\frac{J}{\sum_{j=1}^k \frac{\Pi_J(m_j)}{x_j} - 1}\right)\left(\sum_{j=1}^k \frac{\Pi_J(m_j)}{x_j} - \sum_{\left\{m_j \in \Omega^J \middle| m_j|_{J-n}=m\right\}} \frac{\Pi_J(m_j)}{x_j}\right) \geq J-n. \quad (7)$$

where

$$\frac{\Pi_J(m_j)}{x_j} = \prod_{t=1}^J \frac{\pi(m_{jt})}{(1-q_t+q_t\pi(m_{jt}))}.$$

The RHS of(7) is decreasing linearly in n, while the LHS of (7) is decreasing and concave in n. At $n = 0$ and $n = J$, (7) is satisfied with equality. Therefore, (7) holds for all $n = 1, ..., J-1$.

This establishes that $(\boldsymbol{\rho}^*, \mathbf{w}^*)$ deters deviations to any pure strategy (σ^i, m^i). Therefore, $(\boldsymbol{\rho}^*, \mathbf{w}^*)$ deters deviations to any strategy $(\sigma^i, \mu^i) \in \Sigma^i$. Given that any wage structure must deter deviations to $\{(\sigma_0, m_j)\}_{j=1}^k$, and the optimal way to deter deviations to $\{(\sigma_0, m_j)\}_{j=1}^k$ also deters deviations to any strategy $(\sigma^i, \mu^i) \in \Sigma^i$, $(\boldsymbol{\rho}^*, \mathbf{w}^*)$ is the optimal simple wage structure. $\square$

Lemma 10. *The simple wage* $(\boldsymbol{\rho}^*, \mathbf{w}^*)$ *is optimal in the class of all wage structures that satisfy limited liability.*

Proof. Consider a wage structure where the Principal does not pay the same amount on all reports with a positive wage in the set $R(m_j)$.

Full Commitment. It follows immediately from the optimal $\boldsymbol{\rho}^*$ that paying a positive wage on any reports $r \in R(m_j) \setminus R_J(m_j)$ will weaken the incentive constraint for deviations to strategies $\{(\sigma_0, m_j)\}_{j=1}^k$ and is therefore not optimal.

Partial Commitment. The Principal would like to pay less on reports in $R(m_j) \setminus R_J(m_j)$ but the partial commitment constraint binds. Therefore, it is never optimal to pay more on such reports. □

Lemma 11 (Monitor Symmetrically Across Tasks)**.** *For any monitoring technology* $\mathbf{q} = (q_1, ..., q_J)$ *with* $q_j \neq q_k$ *for some* j, k*, there exists another monitoring technology* $\tilde{\mathbf{q}} = (\tilde{q}, ..., \tilde{q})$ *that enforces high effort and truthful reporting for the same expected wage per agent, but results in hiring a lower expected number of agents.*

Proof. Fix $\mathbf{q} = (q_1, ..., q_J)$ and consider the deviation to always shirking and sending message $m_j = (m_{j1}, ..., m_{jJ}) \in \Omega^J$. Then

$$x_j = \prod_{t=1}^{J} (1 - q_t + q_t \pi(m_{jt})).$$

For any monitoring rate $\mathbf{q} = (q_1, ..., q_J)$, the Principal can set a uniform monitoring rate $\tilde{\mathbf{q}} = (\tilde{q}, ..., \tilde{q})$ such that x_j is the same. In other words, there exists $\tilde{q}$ such that

$$\prod_{t=1}^{J} (1 - \tilde{q} + \tilde{q}\pi(m_{jt})) = x_j.$$

Since the monitoring rate only influences the expected wage per agent through x_j, any two monitoring rates with the same x_j result in the same expected wage per agent.

Under monitoring consistency, the expected number of agents hired for monitoring rate $\mathbf{q}$ is $n(\mathbf{q}) = 2/\left(2 - \frac{1}{J}\sum_{j=1}^{J} q_j\right)$ and the expected number of agents hired for monitoring rate $\tilde{\mathbf{q}}$ is $n(\mathbf{q}) = 2/(2 - \tilde{q})$. By the arithmetic-geometric mean inequality, the expected number of agents hired is lower under the symmetric monitoring rate. The Principal cares about the expected wage bill. Therefore, within the set of contracts that lead to a given expected wage per agent, the Principal picks the contract with the lowest expected number of agents hired – the symmetric monitoring contract. $\square$