Question 5

May 1, 2007

a. (4 points) Capitalists have to solve the following maximization problem:

\[
\text{Max } \sum_{t=1}^{\infty} \beta^{t+1} u(c_t),
\]

where \( c_t = (1 - \tau)r k_{t-1} + k_{t-1} - k_t \) (\( k_t \) is the capital owned by a capitalist at time \( t \)).

The first order condition for this maximization problem is:

\[
(1 - \tau)ru'(c_t) = \beta u'(c_{t+1})
\]

that is,

\[
\frac{u'(c_t)}{u'(c_{t+1})} = \frac{\beta}{(1 - \tau)r}
\]

In the steady state, the right-hand side of the equation will be equal to 1. Therefore we have that \( r = \frac{\beta}{1 - \tau} \).

We also know that capital will be paid its marginal product, hence

\[
r = MPL = \alpha \left( \frac{L}{K} \right)^{1-\alpha},
\]

where \( K \) is total capital.

We have \( N \) workers who inelastically supply one unit labor; therefore \( L = N \).

Solving for \( K \) we get

\[
K = N \left( \frac{\alpha(1 - \tau)}{\beta} \right)^{\frac{1}{1-\alpha}}.
\]

b. (1 point) The wage is equal to the marginal product of labor:

\[
w = MPL = (1 - \alpha) \left( \frac{K}{L} \right)^{\alpha} = (1 - \alpha) \left( \frac{\alpha(1 - \tau)}{\beta} \right)^{\frac{\alpha}{1-\alpha}}.
\]

c. (1 point) The total amount raised is

\[
T = \tau r K = \tau \frac{\beta}{1 - \tau} N \left( \frac{\alpha(1 - \tau)}{\beta} \right)^{\frac{1}{1-\alpha}}.
\]
Maximizing this amount with respect to the tax rate \( \tau \) gives us the following first order condition:

\[
(1 - \tau)^{\frac{\alpha}{1-\alpha}} \left( 1 - \frac{\alpha \tau}{(1 - \alpha)(1 - \tau)} \right) = 0.
\]

The tax rate that generates the highest revenue is therefore \( \tau = 1 - \alpha \).

d. (2 points) A tax rate equal to 100% would drive capital - and hence tax revenues - to 0.

e. (2 points) Wages plus transfers are equal to

\[
wN + T = (1 - \alpha) N \left( \frac{\alpha(1 - \tau)}{\beta} \right)^{\frac{\alpha}{1-\alpha}} + \tau \frac{\beta}{1 - \tau} N \left( \frac{\alpha(1 - \tau)}{\beta} \right)^{\frac{\alpha}{1-\alpha}}
\]

\[= N \left( \frac{\alpha}{\beta} \right)^{\frac{\alpha}{1-\alpha}} (1 - \tau)^{\frac{\alpha}{1-\alpha}} (1 - \alpha + \alpha \tau)
\]

\[= N \left( \frac{\alpha}{\beta} \right)^{\frac{\alpha}{1-\alpha}} (1 - \tau)^{\frac{\alpha}{1-\alpha}} (1 - \alpha + \alpha \tau).
\]

The total amount is decreasing in \( \tau \). Let \( F(\tau) = N \left( \frac{\alpha}{\beta} \right)^{\frac{\alpha}{1-\alpha}} (1 - \tau)^{\frac{\alpha}{1-\alpha}} (1 - \alpha + \alpha \tau) \).

We have \( F'(\tau) = \alpha (1 - \tau)^{\frac{\alpha}{1-\alpha}} \left( 1 - \frac{1 - \alpha + \alpha \tau}{(1 - \alpha)(1 - \tau)} \right) < 0 \) for any \( \tau \in (0, 1) \). Workers are actually better off without taxation (\( \tau = 0 \)).