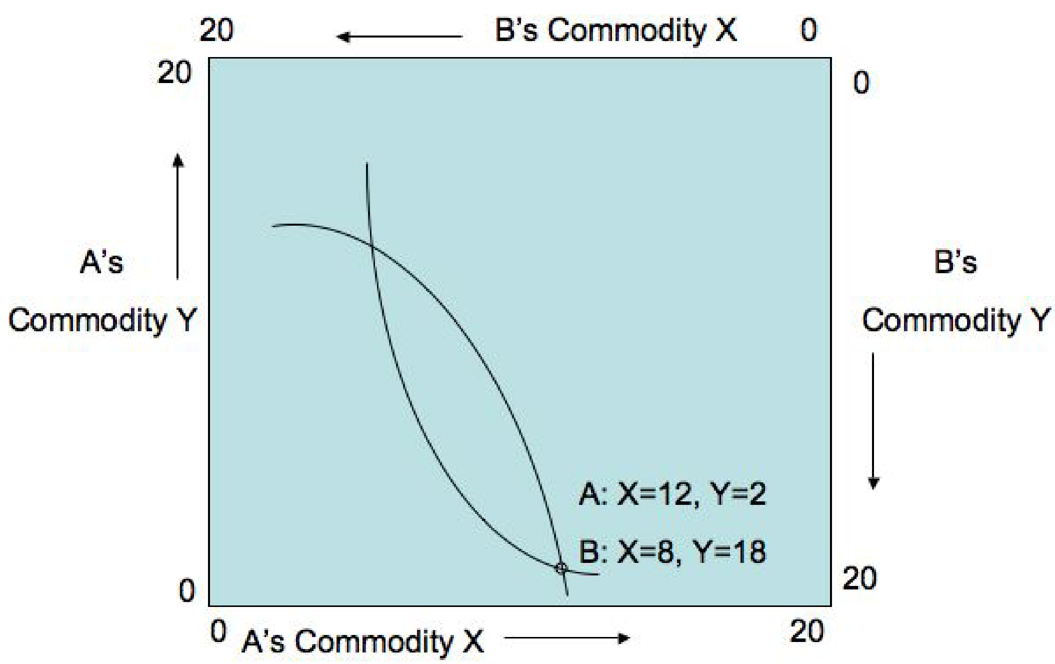
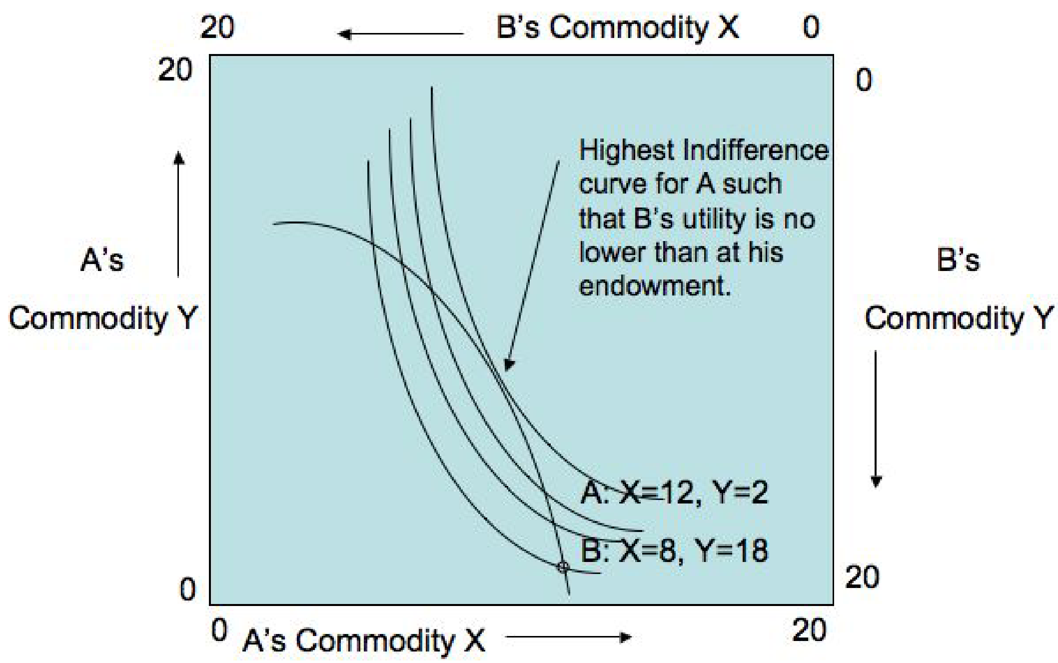
Economics Honors Exam 2007 Solutions Question 3

(a) (8 points) The Edgeworth box is a rectangular diagram with consumer A's origin on one corner and consumer B's origin on the opposite corner. The width of the box is the total amount of good x, and the height is the total amount of the good y. Thus, every possible division of the goods between the two people can be represented as a point in the box. The initial endowment is marked at the coordinates 12,2.



(b) (5 points) The efficient lens, bounded by the consumer A and B's indifference curves through the point of endowment (shown above), contains all feasible endowments that are weakly pareto superior to the initial endowment. In other words, it contains all feasible endowments such that A and B's utilities are no lower than at their endowment point.

(c) (5 points) No, the initial endowment is not Pareto Efficient. This can be shown by comparing the marginal rate of substitution between goods x and y for consumer A (1/6) and consumer B (9/4). Because they are not equal, pareto improving trades exist.
(d) (5 points)



(e) (7 points) A chooses a feasible x_A, y_A, x_B, y_B to maximize utility subject to $x_B y_B \geq 8 \cdot 18 = 144$. A solves the following constrained maximization problem:

$$\max_{x_A, y_A, x_B, y_B} x_A y_A$$

s.t.

$$x_A, x_B, y_A, y_B \geq 0$$
$$x_A + x_B \leq 20$$
$$y_A + y_B \leq 20$$
$$x_B y_B \geq 144$$

Assuming for now that the latter three constraints will hold with equality, we can simplify the constraints:

$$\max_{x_A, y_A} x_A y_A$$

s.t.

$$x_A, x_B, y_A, y_B \geq 0$$
$$x_A + \frac{144}{20 - y_A} = 20$$

Substituting the remaining constraint yields:

$$\max_{x_A, y_A} \left(20 - \frac{144}{20 - y_A}\right) y_A$$

s.t.

$$x_A, x_B, y_A, y_B \geq 0$$

The f.o.c. is:

$$20 - \frac{144 \cdot 20}{(20 - y_A)^2} = 0$$
$$(20 - y_A)^2 = 144$$

Since we require $y_A \geq 0$ this yields the unique solution $y_A = 8$. Thus, the pareto efficient allocation under the assumptions of (d) is $x_A = y_A = 8$ and $x_B = y_B = 12$