

Review Section Questions

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1 Questions and Answers received since the review section

Question: *What if the rate of growth of E was g instead of 0? How would we have solved part a for the steady state income per worker as a function of the exogenous parameters?*

Answer: Consider the production function:

$$Y = K^\alpha (EL)^{1-\alpha}$$

and continue to let the growth rate of L be given by n (i.e. $\dot{L}/L = n$), but change the growth rate of E to be g . Define $A \equiv E^{1-\alpha}$ and rewrite the production function as:

$$Y = AK^\alpha L^{1-\alpha}$$

where we find:

$$\frac{\dot{A}}{A} = \frac{\frac{\partial E^{1-\alpha}}{\partial t}}{E^{1-\alpha}} = \frac{(1-\alpha) E^{-\alpha} \dot{E}}{E^{1-\alpha}} = (1-\alpha)g$$

such that if E grows at the rate g , A will grow at a rate $(1-\alpha)g$.

When we did the capital accumulation function at the review section we normalized capital by labor. To find a steady state level in a world with exogenous technological change we need to normalize by AL . Hence, define $k \equiv \frac{K}{AL}$ and find the capital accumulation function as:

$$\dot{k} = \frac{\partial \left(\frac{K}{AL} \right)}{\partial t} = \frac{\dot{K}}{AL} - \frac{K}{(AL)^2} (\dot{A}L + \dot{L}A)$$

where we use the quotient rule first and the product rule second. We have the accumulation function for aggregate capital:

$$\dot{K} = sAK^\alpha L^{1-\alpha} - \delta K$$

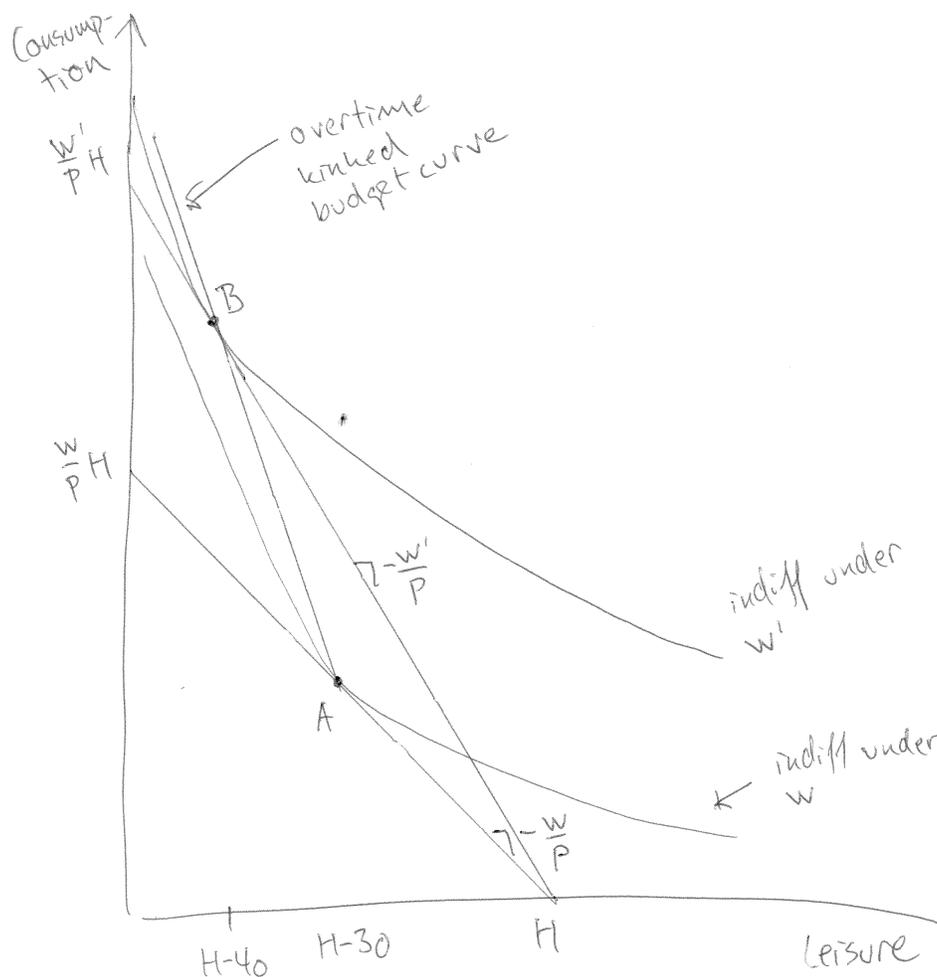
and inserting this you can simplify and find:

$$\frac{\dot{k}}{k} = sAk^\alpha - (\delta + (1-\alpha)g + n)k$$

where the additional term $(1 - \alpha)g$ comes from the fact that we have technological growth.

Question: in the first micro question, when the firm tries to get her to work 40 hours by paying overtime, I got the kink part, but I don't get why her indifference curve shifts up. Also, if her indifference curve shifts up, how will the firm get her back to the 40 hours point, not the 50+ point?

Answer: The argument is the following: Consider first the situation with the old wage, 20 dollars an hour, where the worker chooses to work 30 hours a week. This is denoted by point A in the graph below. The first question is how we can induce her to work more by increasing the overall wage rate for all hours worked to, say, w' . If we do the budget line we see that this implies a steeper budget curve, which intersects the x -axis at H (just as the budget curve for w did) and the y -axis at $\frac{w'}{p}H$. This shifts up her chosen point to B (This is not a theoretical necessity, but will happen for these indifference curves), which implies that she will consume less leisure (work more) and consume more of the good. Since we are paying her more she moves to a higher indifference curve.



For concreteness let's say that she was originally working 30 hours at \$20 making \$600 a week, and that

the required amount necessary for her to work 40 hours is \$40 dollars implying that she would be making \$1600 a week. Note that by tangency at point B she values one hour of leisure at exactly 40 dollars. Had she valued it at more she would have worked less, had she valued it at less she would have worked more.

Now, to demonstrate that the firm can do better by just paying overtime consider if the firm were to pay an overtime rate that would cut through point B . In such a case they would have to pay \$100 per overtime hour in which case she would be making $30 \times 20 + 10 \times 100 = 1600$. Since the budgetline passes through the point B the worker could have chosen this point. But she will not! Why is this? We just argued that at point B she will value leisure at \$40 an hour. If she is being paid \$100 an hour for overtime she will definitely want to work more. Hence, she will choose a point to the northwest of B (not drawn in the graph). This can be seen from the fact that the kinked budget line intersects the indifference curve in point B . Hence there is a higher indifference curve to the northwest in which there is a tangency point.

Since the firm only wanted her to work 40 hours it can reduce the overtime rate to, say, 60 (or even lower than 40 depending on preferences) and get her to work 40 hours. This will only cost $30 \times 20 + 10 \times 60 = 1200$ and will therefore be cheaper.

The reason for letting the kinked budget curve pass through B is therefore not because I claim that this will be optimal, but because by contradiction I can show that something lower than this will be, and hence it will be cheaper to use a kinked budget curve, that is overtime.

Question Hi - I have a question from the 2007 honors exam that I can't figure out. It is problem number 5, which is the following:

There are 2 classes of people, capitalists and workers, with N of each. All workers supply 1 unit of labor but capitalists do not, they make money off capital income. There is a discount factor B , the production function is $Y=K^a L^{1-a}$, there is no depreciation, all factors are paid their marginal product, and the interest rate and wage are w and r . Capital income is taxed at t .

a. What will be the steady state capital in terms of parameters and the tax rate?

The solution starts by saying that capitalists maximize (I put subscripts in parenthesis)

$$(1) \sum \beta^{t+1} * u(C(t))$$

Then it says that

$$(2) C_t = (1-t)r*k(t-1) + k(t-1) - k(t)$$

Here is what I'm stuck on. They then say the first order conditions are:

$$(3) (1-t)r*u'(c(t)) = B*u'(c(t+1))$$

My question is what happened to the second term in equation 2? Why aren't the FOC:

$$u'(c(t)) = B*u'(c(t+1))*[(1-t)r+1]$$

Answer: You are right. Consider the constrained maximization problem:

$$\max_{\{k_t\}_{t=0}^{\infty}} \sum \beta^t u(c_t)$$

subject to:

$$c_t = (1 - \tau) r k_{t-1} + k_{t-1} - k_t$$

for all t . differentiate wrt to k_t and get:

$$-\beta^t u'(c_t) + \beta^{t+1} u'(c_{t+1}) [(1 - \tau) r + 1] = 0 \Leftrightarrow$$

$$u'(c_t) = \beta u'(c_{t+1}) [(1 - \tau) r + 1] = 0$$

which is what you get.