Honors General Exam
Part 1: Microeconomics
Solutions

Harvard University

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Question 1. (12 points)

A rare baseball card investor is deciding when to sell the novice card of a recent superstar. The more time that passes, the more valuable the card will get due to scarcity. In particular, the investor estimates that consumer willingness to pay for the card after t years is $P(t) = 30 + 10t$.

(a) **(4 pts)** Suppose that it costs \$5 a year to maintain the card in its pristine condition, and money appreciates at an interest rate of 5%. When will the investor sell his card? At what price will he sell?

Solution: When the investor sells the card, he can put the money in a bank where it will apprecaite according to the interst rate. Thus, he will sell the card when the return to saving $1 + r = 1.05$ is equal to the return on investment. The return on investment is $(P(t+1) - c)/P(t)$, the ratio of the price in year $t+1$ less the storage cost, c, to the price in year t. Thus, the investor will sell the card at t s.t. $1.05 = (30 + 10(t+1) - 5)/(30 + 10t)$. Sovling this, we obtain $t = 7$. The investor would thus sell the card at $P(7) = 100$.

(b) **(4 pts)** Suppose the interest rate is instead 10%. What happens to the values of t and $P(t)$? Explain.

Solution: Solving $1.1 = (30 + 10(t+1) - 5)/(30 + 10t)$, we find that $t = 2$ and $P(t) = 50$. Intuitively, at a higher interest rate, the appreciation of the value of holding the card is lower, and so the collector sells it sooner.

(c) **(4 pts)** Suppose instead that the interest rate remains at 5% but the cost of storage is \$4 per year? Explain intuitively whether the investor would hold the card for a longer or shorter amount of time compared to part (a). Solving for the exact impact is unnecessary.

Solution: If storage becomes cheaper, the return on investment rises. Thus, keeping the card longer becomes more lucrative and the collector will do so. Indeed, if the interest rate remained 5%, the collector would keep the card for $t = 9$ years.

Question 2. (12 points)

You have been tasked to consult on investing in a new startup. Preliminary research shows you that the startup's long-run cost function will be $C(Q) = 5Q^2 - 2Q + 125$.

(a) **(8 pts)** What is the smallest price at which the startup could supply a positive amount of output in the long run? State the condition(s) under which the firm will operate in the long run as part of your answer.

Solution: The startup will suppply a positive amount in the long run if the minimum average cost is lower than the price, so that the startup can make a profit. The average long run cost for this startup is given by:

$$AC = \frac{C(Q)}{Q} = \frac{5Q^2 - 2Q + 125}{Q} = 5Q - 2 + \frac{125}{Q}.$$

To find the minimum price that can be sustained, we need to find the minimum average cost. This is a convex function, so we can find the minimum by looking at the FOC:

$$\frac{\partial AC}{\partial Q} = 5 - \frac{125}{Q^2} = 0$$

Solving this, we obtain that the quantity that minimizes average cost can be profitably produced is $Q = \sqrt{25} = 5$.

Thus, the minimum average cost, and hence the lowest price under which the startup will continue to produce, is $AC(Q = 5) = 5(5) - 2 + 5 = 18$.

(b) **(4 pts)** Suppose that in the short run, the startup's cost curve for the good is $\frac{1}{8}Q^2 + 50Q$. If the market price for the good is \$75 per unit, how much should the startup produce?

Solution: To maximize profits, the firm will set $MR = MC$, e.g. Ouput Price = Marginal Cost.

To find the marginal cost curve, we differentiate:

$$MC = \frac{\partial C(Q)}{\partial Q} = 50 + \frac{Q}{4}.$$

Thus, the firm will choose Q to satisfy:

$$75 = 50 + \frac{Q}{4}.$$

Solving, yields $Q^* = 100$.

Question 3. (12 points)

In the aftermath of the death of Justice Antonin Scalia, President Obama (POTUS) and Congress found themselves at an awkward standoff. The president would like to nominate a liberal to shift the leaning of the court left, but he'd really like his nominee to pass confirmation with Congress. Republican-controlled Congress, on the other hand, faces pressure to delay confirmation in hopes of a conservative candidate winning the 2016 presidential election. On the other hand, the congressional Republicans can get away with confirming a moderate at a small political loss since if a Democrat wins again, (s)he will surely get a liberal on the court.

The payoff matrix for this game is as follows:

		Congress	
		Confirm	*Don't Confirm*
President Obama	*Liberal*	12,0	3,3
	Moderate	4,4	7, 3
	Conservative	0,10	0,0

(a) **(4 pts)** Does the game have any pure strategy Nash Equilibria? If so, what are they?

Solution: There are no PSNE. First note that *Conservative* is strictly dominated for POTUS, so it will never be played in equilibrium. Furthermore, we can see by inspection that in each pair of strategies, at least one player prefers to devaiate.

(b) **(8 pts)** Does the game have any mixed strategy Nash Equilibria? If so, what are they?

Solution: Yes, there is a mixed strategy equilibrium.

Nominating a *Conservative* is strictly dominated for POTUS, so it will never be done in equilibrium.

If POTUS nominates a *liberal* with probability p, then Congress has the following payoffs:

- Payoff (*Confirm*) $= 4(1-p)$
- Payoff (*Don't Confirm*) $= 3p + 3(1-p) = 3$

In equilibrium the payoffs must be equal (otherwise Congress would choose the action with the higher payoff): $4(1-p) = 3$.

Therefore, $p = 1/4$.

This implies that in equilibrium: POTUS nominates a *liberal* with probability 1/4 and nominates a *moderate* with probability 3/4.

Analogously, if Congress confirms the nominee with probability q, then POTUS has the following payoffs:

- Payoff $(\textit{Liberal}) = 12q + 3(1-q) = 9q + 3$
- Payoff $(\textit{Moderate}) = 4q + 7(1-q) = 7 - 3q$

Equating these, we find that $q = 1/3$. Thus, in equilibrium, Congress confirms POTUS's nominee with probability 1/3, and rejects the nominee with probability 2/3.

Question 4. (12 points)

Megyn gets her utility from a combination of commodities x and y. Her utility from consuming x and y amounts of each commodity is given by $U_M(x, y) = (x + 4)(2y - 6)$.

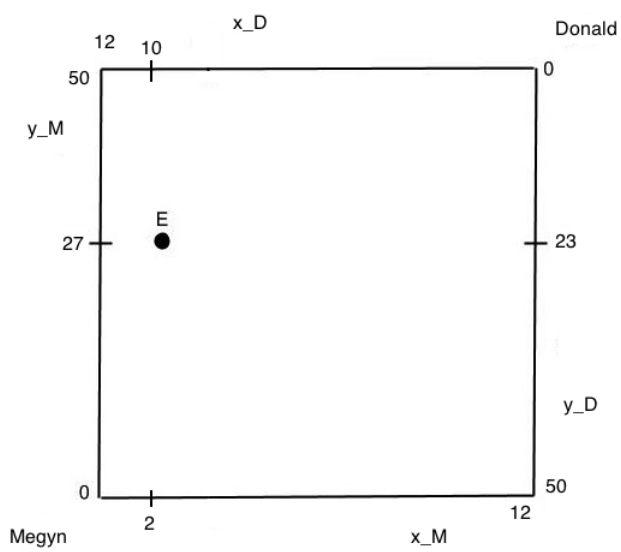
(a) **(4 pts)** When we first observe her, Megyn employs a marginal rate of substitution of -5. How many units of y is she consuming if she consumes 2 unis of x?

Solution: Megyn's utility function is $U(x, y) = (x + 4)(2y - 6) = 2xy - 6x + 8y - 12$. We know that $MRS = -MU_x/MU_y$. Computing these with partial derivatives, we find $MU_x = 2y - 6$ and $MU_y = 2x + 8$. Plugging in $MRS = -5$ and $x = 2$, we obtain that $-5 = -(2y - 6)/(12)$ so that $2y - 6 = 60$ and $y = 33$.

(b) **(4 pts)** Megyn lives in a closed economy with Donald, who has a utility function of $U(x, y) = 4x + 6y^{1/18}$. When we find him, Donald is seen consuming $x = 10$ and $y = 23$.

Draw an Edgeworth box for this economy. Be sure to label everything clearly (including the endowment point at the time we first see Megyn and Donald).

Solution: See the figure below.



(c) **(4 pts)** For an allocation in this economy to be Pareto efficient, it must maximize the utility of consumer A given the utility of consumer B. Is the initial endowment Pareto efficient? Why or why not?

Solution: The initial endowment point is <u>not</u> Pareto efficient. A Pareto efficient point equates the marginal rates of substitution of the two parties. At point E, the MRS are not equal. At point E:

$$MRS_D = \frac{-4}{(1/3)y} = \frac{-12}{23}.$$

We know that Megyn's MRS at E is -5, which is not the same, so E cannot be Pareto efficient.

Question 5. (12 points)

In the aftermath of their recent spat, Kanye and Taylor Swift receive utility shocks in proportion to the bombast that they display. If Kanye exerts x units of bombast, he receives a utility shock of $55x - (2.5)x^2$. TSwift also profits from her bombast, but because Kanye has made demeaning comments about her, she suffers from the extra media attention that she and Kanye get. Thus, if TSwift exerts y units of bombast, she receives a utility shock of $40y - 2y^2 - (0.5)xy$.

(a) **(6 pts)** Suppose that TSwift's fans pressure Kanye into compensating Taylor for the cost that his bombast imposes on her. What levels of bombast do Kanye and Taylor choose in equilibrium? What utilities do they obtain?

Solution: If TSwift is compensated from her losses from Kanye's bombast, she receives a total utility of $40y - 2y^2 - (0.5)xy + (0.5)xy$. This is concave in y, so we can take the FOC: $40 - 4y = 0$ so that Taylor's optimal choice of bombast is $y^* = 40/4 = 10$. Kanye's utility, on the other hand, is $55x - (2.5)x^2 - (0.5)xy$. This too is concave, and it is maximized at $55 - (0.5)y^* - 5x = 0$, so that $x^* = \frac{55-5}{5} = 10$. Taylor's utility shock would therefore be 200 and Kanye's would be 250.

(b) **(3 pts)** Suppose that Kanye and TSwift's managers get together over drinks and figure out a plan to collude and jointly set both of their clients' bombast. What levels of bombast do they choose? Feel free to approximate.

Solution: The managers would maximize the aggregate utility: $U(x, y) = 55x - (2.5)x^2 + 40y - 2y^2 - (0.5)xy$. We maximize this by looking at the FOCs in each dimension: $\frac{\partial U}{\partial x} = 55 - 5x - .5y = 0$ and $\frac{\partial U}{\partial y} = 40 - 4y - .5x = 0$. Solving these two equations, we obtain $x_C^* = \frac{400}{39.5} \approx 10.13$ and $y_C^* = 110 - 10\left(\frac{400}{39.5}\right) \approx 8.73$.

(c) **(3 pts)** What is the change in aggregate welfare from the managers' collusion? Is Kanye better off relative to part (a)? How about Taylor? Show all your welfare calculations.

Solution: TSwift will obtain a utility of 196.77, while Kanye will obtain a utility of 256.39 under the managers' collusion. Aggregate utility increases by 3.17, but Taylor will lose out! Approximating to the nearest integer, we would have $x_C = 10$ and $y_C = 9$ yielding a utility of 198 for TSwift and a utility of 255 so that aggregate utility rises by 3, and Kanye benefits while TSwift loses out.