HARVARD UNIVERSITY
DEPARTMENT OF ECONOMICS

General Examination in Microeconomic Theory

Spring 2012

You have **FOUR** hours. Answer all questions
Part A(Glaeser)
Part B (Game Theory)
Part C (Hart)
Part D (Green)

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Part A: Verbal Problem (Glaeser, 50 points)

For credit you must give a mathematical answer or justification.

The Springfield Nuclear Power Company has paid the fixed costs of building a power system for the city of Springfield. Springfield Nuclear is a natural monopolist owned by the draconian Charles Montgomery Burns, who has been known to bribe Springfield Mayor Joseph Quimby.

As the Burns Professor of Economics at Springfield University, the citizens of Springfield look to you to provide guidance on the form of government regulation that Springfield Nuclear should face. You have decided to quickly write down a formal model to sharpen your analysis.

1. Derive the conditions that describe the firm's choice of pricing assuming the consumers are homogeneous. (5 points)

2. Describe outcomes of several policies assuming that Mayor Quimby leads a benevolent government that maximizes equally-weighted social welfare. Will these policies differ in their social welfare implications? Briefly explain in each case. (10 points)
   a. The company is publicly owned (Mr. Burns leaves Springfield in disgust)
   b. The company is private faces price controls (a price ceiling)
   c. The company is private and water production is subsidized / taxed optimally

3. Assume that instead that Mayor Quimby has been bribed by Mr. Burns and does exactly Mr. Burns' bidding ("excellent!") if he has not been forced out by public ownership. Describe the outcomes in this case under the three different types of intervention (public ownership, price controls, subsidies). Briefly explain the economic intuition of if and why the policies differ. (7 points)

4. Now assume Mayor Quimby is Rawlsian and acts in the interest of poorer consumers who neither pay taxes nor own the company (if it is private). Describe outcomes in this case under the three different types of intervention. (You may assume that consumers have quasi-linear preferences $U = v(q) + Y - pq$ where q is power consumption at price p). Briefly explain the economic intuition of if and why the policies differ. (8 points)

5. Finally, assume that there is a fixed probability that Mayor Quimby can be bribed to do what is best for Mr. Burns and a fixed probability that he will act turn out to be Rawlsian and do what is best for the poor. Derive conditions on the optimality of the three types of intervention conditional upon the values of these probabilities. Define "optimal" with respect to the utilitarian sum of utilities, including that of Mr. Burns, even though this is not what Quimby optimizes in either case. Explain your answer. (20 points)

6. Bonus: Discuss how your answer to this question and specifically part 5 informs federal policy regarding the institutional design of Freddie Mac and Fannie Mae.

Game Theory Questions for generals
(25 points each)

B1)
Consider the game tree shown in the diagram and the following strategies indicated for players 1 and 2 who have three and one information sets respectively:
player 1: BCF
player 2 Y
(i) Do these strategies form a Nash equilbrium? (*6*)
(ii) Are these strategies, when combined with suitable beliefs at the four information sets in this game, part of a sequential equilibrium?(*10*)
(iii) If so, what are the beliefs? If not, what is a sequential equilbrium (strategies and beliefs)? (*9*)

B2)
Consider the two player game in normal form shown below

	l	c	r
T	$7,7$	$1,7$	$1,0$
M	$7,1$	$5,5$	$0,2$
B	$0,1$	$2,0$	$6,0$

a) What are all the pure strategy Nash equilibria of this game? (*5*)
b) What are all the pure strategy trembling hand perfect Nash equilibria of this game? (*5*)
c) What are the strategies that are not eliminated by iterative deletion of strictly dominated strategies? (*5*)
d) For the game remaining after such deletions are made, reconsider parts a) and b) above and explain the results obtained. (*10*)

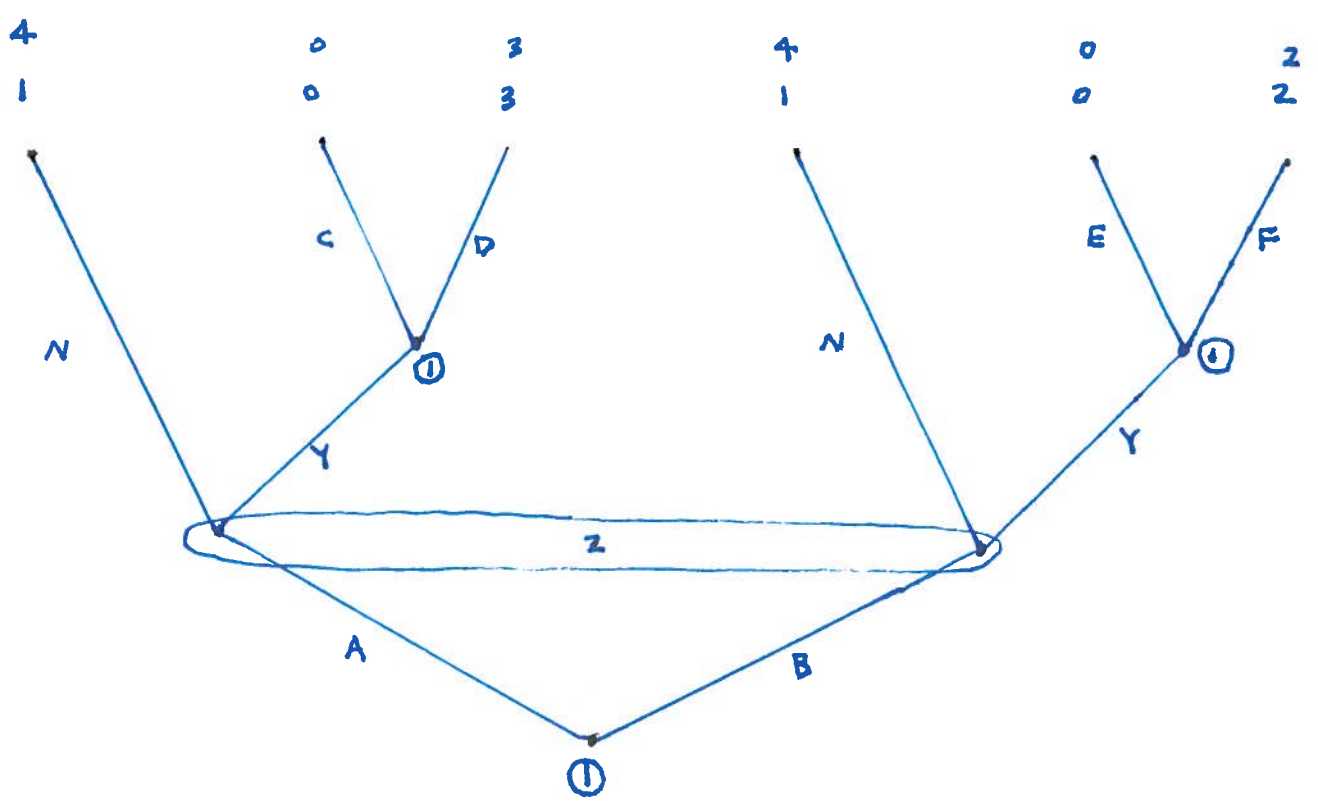

Diagram for Question B1

General Exam
Spring 2012
O. Hart

1. (a) Define the core of an exchange economy. Prove that a Walrasian equilibrium lies in the core.

   (b) Consider a competitive economy with one (type of) consumer and two (types of) firms. There are two dates 0 and 1, two states of the world, and all uncertainty is resolved at date 1. There is one good at each date. The consumer cares only about consumption at date 1: her utility function is $\pi \log x_1 + (1-\pi)\log x_2$, where x_i is date 1 consumption in state i and π is the probability of state 1. The consumer has an endowment of one unit of the good at date 0. The two firms have constant returns to scale technologies. Firm 1 can transform 1 unit of the date 0 good into $(2,1)$ (i.e., 2 units of the good at date 1 in state 1 and 1 unit of the good at date 1 in state 2) and firm 2 can transform 1 unit of the date 0 good into $(1,2)$.

   Compute the Arrow-Debreu equilibrium for this economy under the assumption that both firms operate. Show that this is indeed the equilibrium for π in some range. Characterize the equilibrium also when π is outside this range.

2. A buyer B and a seller S meet at date 0 and plan to trade a widget at date 1. At date $\frac{1}{2}$ the buyer will make an investment i, which costs i (borne by B). B's value for the widget at date 1 is given by $v(i)$, where $v' > 0$, $v'' < 0$, $v'(0) = \infty, v'(\infty) = 0$. S's costs are zero and S makes no investment. Both parties are risk neutral, do not face wealth constraints, and there is no discounting. Assume that B and S have equal bargaining power at date O and also at date 1 (if bargaining occurs at date 1).

   (a) Characterize the first best level of i.

   (b) Suppose that B and S can contract on the date 1 widget type and price p ex ante at date 0. What are the equilibrium values of i and p?

   (c) Suppose that B and S cannot contract on the date 1 widget type or price ex ante but can contract on i. The widget price p will be determined by bargaining at date 1. What are the equilibrium values of i, p, and T where T is the lump sum payment made from S to B?

   (d) Suppose that B and S cannot contract on either i or p ex ante. What are the equilibrium values of i, p, and T?

   (e) Briefly discuss possible solutions to the inefficiency in (d) (beyond the approaches described in (b) and (c)).

Question D1
IIA, Condorcet, and the French Presidential Election
(20 points, 4 for each part)

This question is about voting rules that are "order-valued" – that is, for any profile of individual's preferences, they produce an ordering of all the alternatives. You can consider only strict orders, both in the domain and in the range, and disregard the possibility of ties in voting (due to the usual large-numbers considerations).

a) Give an example of a voting system that satisfies Arrow's IIA (Pairwise independence, the term used in MWG) but not Condorcet consistency.

b) Give an example of a voting system that satisfies Condorcet Consistency but not Arrow's IIA.

c) Give an example of a voting system that satisfies neither Condorcet Consistency nor Arrow's IIA but is still reasonably well known and is in use. Why do you think this voting system is used? Does it have some other property that you think is valuable?

d) On Sunday France will hold an election for President – the second stage of a two-stage procedure known as Plurality with a Runoff. This system is describe by a first stage in which everyone votes for their favorite candidate and a second stage in which the two highest vote getters from the first stage compete in a majority rule election. For the purposes of this question assume that everyone votes honestly in both stages.

Plurality with a Runoff is designed to select a winner, not to produce an ordering. In this question we consider only the three-candidate case (whereas in France there were actually many candidates in the first stage.). In the three-candidate case Plurality with a Runoff determines an ordering as follows: The candidate eliminated in the first stage is deemed to be last in the ordering and two surviving candidates are ordered in accordance with the outcome of the second-stage vote.

Does this procedure satisfy Arrow's IIA? How about Condorcet Consistency?

e) Assume that in France the original candidates are Hollande (left), Sarkozy (center) and many right-wing candidates (Le Pen,...). (I know very little about French politics so this is purely hypothetical, for the purposes of this question.) Assume that the right-wing candidates are a set of clones, as defined in Lecture 3. What is the definition of "Independence of Clones" presented in that Lecture? Does the system of Plurality with a Runoff satisfy this definition?

Question D2
Treasure Hunters
(25 points)

A group of treasure hunters has found a map showing the location of buried treasure on a remote island. They believe that it is 100% accurate and the treasure, worth T, will be recoverable by the first group to travel to the island. Unfortunately there are some pirates near this island. A group that is not sufficiently strong will be repelled by these pirates and will have to return home empty-handed. That outcome has the same value as never having set forth to find the treasure at all.

The set of treasure hunters is N. The members of N denoted $i = 1, ..., n$ are each characterized by two numbers a strength level y_i and a cost c_i. Let $S \subseteq N$ be a set of treasure hunters. The aggregate strength level needed to overcome the pirates is Y. If $\sum_{i \in S} y_i \geqq Y$ the trip will succeed and the treasure will be recovered; if not the pirates will repel S. If S makes a trip they come home with a net value $T - \sum_{i \in S} c_i$ if $\sum_{i \in S} y_i \geqq Y$ and zero otherwise. Assume that the trip is worthwhile if undertaken by everyone, although that may not be the efficient way to do it: $T - \sum_{i \in N} c_i > 0$ and $\sum_{i \in N} y_i \geqq Y$.

The treasure hunters hold a meeting to plan the trip. They decide to use the Shapley value of the cooperative game that represents this situation to determine who should make the trip and how the net value should be divided among all of N. They can make any monetary transfers they desire.

a) Suppose that everyone is required or else the trip will not succeed. That is, $\sum_{i \in N} y_i \geqq Y$ but $\sum_{i \in S} y_i < Y$ for all $S \subsetneq N$. What is the Shapley value? How does it depend on T, Y and the individuals' parameters $(y_i, c_i)_{i \in N}$. *(6)*

For the remainder of this question, assume that the strengths y_i are all the same, y. Also assume that there are only two values of c_i in the population: n_L people have c_L and $n - n_L$ people have c_H, with $c_H > c_L$. Suppose that k treasure hunters are required to overcome the pirates: $(k-1)y < Y < ky$ and that the c_L people alone cannot succeed: $n_L < k$.

b) Give a formula for the Shapley value in this problem. Which treasure hunters should actually go on the trip? Are the people not going on the trip receiving any payments from those who do go? Explain. *(12)*

c) Is the core of this game empty or non-empty? Explain in detail. *(7)*

Question D3
Distributing the Interim Payoff in an Auction
(25 points)

There are three bidders whose valuation for a single indivisible object are θ_i. The θ_i are uniformly and independently distributed on $[0, 1]$. As usual, assume that the bidders are risk-neutral and have utility functions that are quasi-linear in money.

A mechanism is constructed to allocate the object efficiently at a Bayesian Nash equilibrium. That is, the highest evaluator always gets the good. This mechanism is symmetric across the bidders and returns all the revenue collected to the bidders, on average. That is the net expected revenue to the seller is zero when the mechanism is played in equilibrium, although it may not be zero for every realization of the θ_i.

a) What is the interim utility $W(\theta_i)$ achieved for each bidder by this mechanism. (You may make reference to any result covered in class or in MWG. You should offer a full explanation of it.) *(10)*

b) What is $W(\frac{1}{2})$ for this mechanism? *(3)*

c) The remainder of this question concerns mechanisms that might not always achieve the efficient allocation, but which might be useful in getting to more equitable allocations of expected welfare for different θ.

The mechanism designer, who psychologically identifies himself with the median evaluation of $\theta = \frac{1}{2}$, is unhappy with the result of part b) above and consults you about whether there might be a more equitable way to arrange for interim welfare outcomes, by which he means a way that has a higher value of $W(\frac{1}{2})$. He asks you about the following interim welfare functions. For each one, state whether there is any Bayesian mechanism that can achieve it without an external infusion of money on average. *(6 each)*

(i) $W(\theta_i) = \frac{1}{3}\theta_i$ (which produces $W(\frac{1}{2}) = \frac{1}{6}$)

(ii) $W(\theta_i) = \min(\frac{1}{2}\theta_i, 0)$ (which produces $W(\frac{1}{2}) = \frac{1}{4}$)