HARVARD UNIVERSITY
DEPARTMENT OF ECONOMICS

General Examination in Microeconomic Theory

SPRING 2011

You have **FOUR** hours. Answer all questions

Part A: 55 minutes
Part B: 55 minutes
Part C: 60 minutes
Part D: 70 minutes

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Part A: Verbal Problem (Glaeser)

For credit you must give a mathematical answer or justification.

A movie theater engages in three nonstandard pricing policies:

(1) People over the age of 65 are charged P-D to attend a movie, where P is the price of the ticket for people under 65.
(2) The marginal cost of producing and delivering the movie is C, but the theater charges consumers C+M.
(3) The theater runs an action movie followed by a romance movie. The price for each of these movies independently is P. The price for the two of them together is 2P–K.

a) Derive three complete, distinct, and connected models (similar preferences and assumptions but *separate* models) to explain each of these practices. Label each separate model as i, ii, and iii and state your common assumptions clearly. For each model, summarize in one or two sentences the economic intuition that the model captures as to why the movie theatre engages in each practice.

b) Derive comparative statics on the movie theatre's optimal choice of:

i) D in model 1

ii) M in model 2

iii) K in model 3

These comparative statics should illustrate how the most relevant exogenous parameters in your model affect the movie theatre's pricing policies. Note that you must determine what the most relevant exogenous parameters are in each of the three models (they may be different for each model). For each comparative static summarize in one or two sentences why the exogenous variable relevant for the particular pricing policy what economic intuition your comparative static captures.

c) Use your models to assess the welfare consequences of:

i) Banning price discounts (for practice 1)

ii) Mandating marginal cost pricing (for practice 2)

iii) Mandating that the movie theater must allow people to bring in their own popcorn (assume it can be bought outside the theater at a constant marginal cost C)

Question 2

Say whether the following statements are true or false, and explain why.
Assume throughout that the games in question are finite.

1. In a two-period multistage game with observed actions, subgame perfect equilibrium and sequential equilibrium are observationally equivalent.

2. If σ is a self-confirming equilibrium, and player i's strategy σ_i assigns strictly positive probability to pure strategies s_i and $\overline{s}_i$, then $u_i(s_i, \sigma_{-i}) = u_i(\overline{s}_i, \sigma_{-i})$. (Here as usual u_i denotes player *i's* utility function.)

3. Every Nash equilibrium is robust to small changes in payoffs in the following sense: Consider a family of strategic form games with the same set of players and the same strategy space S, and utility functions u_i indexed by a real-valued parameter λ, and assume that each u_i is continuous in λ. If σ is a Nash equilibrium for the utility functions corresponding to $\lambda = 0$, and $\lambda_n \to 0$, then there is a sequence of strategy profiles $\sigma^n \to \sigma$ such that each σ^n is a Nash equilibrium for the utility functions $u_i(\bullet, \lambda_n)$.

4. A player's Nash equilibrium payoff can decrease if the player gets more information.

5. In a cheap-talk game, there is always a pooling equilibrium.

6. In a one-shot simultaneous move game, player i's payoff in any Nash equilibrium is at least her minmax payoff.

General Questions

1. (i) An exchange economy consists of two consumers and two goods. Consumer 1 cares only about consumption of good 1 and consumer 2 cares only about consumption of good 2. Consumer 1's initial endowment is (1,2) and consumer 2's is (4,5).
What is the set of Pareto optima for this economy? Compute the Walrasian equilibrium.

   (ii) An economy consists of two consumers and two goods, x and y. Consumer 1 likes good x and can supply good y, while consumer 2 likes good y and can supply good x. The utility functions of the consumers are:

$$\text{Consumer 1:} \quad U_1 = X - \frac{1}{2} l_y{}^2 ,$$

$$\text{Consumer 2:} \quad U_2 = Y - \frac{1}{16} l_x{}^2 .$$

   Here X is consumer 1's consumption of good x, l_y is consumer 1's supply of good y, Y is consumer 2's consumption of good y, and l_x is consumer 2's supply of good x.
Compute the Walrasian equilibrium.

2. An entrepreneur has a date 0 investment project that yields a stochastic return $\widetilde{R_1}$ at date 1 and an expected return of R_2 at date 2 (if it is continued). Here $R_1 = 100$ with probability $\frac{1}{2}$ and 0 with probability $\frac{1}{2}$, and $R_2 = 300$. The project costs 250 and the entrepreneur's initial wealth is 190. The project can be liquidated for L=60 at date 1; its liquidation value at date 2 is zero. The entrepreneur can divert all cash flows from the project to himself. The entrepreneur is risk neutral and there is no discounting.

   The entrepreneur approaches a risk neutral investor for the extra 60 to finance the project. There is symmetric information and the investor will observe the realization of R_1 at date 1 (but this realization is not verifiable). The entrepreneur offers the investor a contract of the form: "I will pay you D or zero at date 1. If I pay you D, I will continue the project with probability 1. If I pay you zero, I will continue the project with probability π

(i.e., the project will be liquidated with probability 1-π). You, the investor will receive all the receipts from liquidation." Assume initially that this contract cannot be renegotiated.

(i) Analyze the entrepreneur's incentive compatibility constraint at date 1, i.e., the constraint that, given π, the entrepreneur will choose to pay D when $R_1 = 100$, and zero when $R_1 = 0$.

(ii) What is the highest value of π such that the investor breaks even? What is the corresponding value of D? Does the entrepreneur also break even from such a contract?

(iii) Suppose now that, when $R_1 = 100$, the entrepreneur can pay zero and can renegotiate the contract: specifically, he can make a take-it-or-leave-it offer to the investor (where the investor's threat point is to liquidate with probability 1-π).
How does your answer to (i) and (ii) change? Will the project still go ahead?

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Part D
Question D1) (38 points)

Four ranches are located at various distances from a water source and need to be connected to this source via an irrigation ditch. They lie along a straight line at distances of 40, 55, 75 and 100 miles from the source. They must share the cost of digging the ditch. This cost is \$1 per mile.

Consider the following proposal for sharing the total cost of \$100:

> Each ranch shares equally in the cost of the portion of the ditch that it uses, together with the other ranches that use the same segment. For example, the first ranch uses only the first 40 miles, and all four ranches use this segment, so the cost to ranch #1 is \$10. Ranch #1 does not use any of the other segments of the ditch and thus does not pay for them.

Now consider the following alternative proposal for cost sharing:

> The four ranches should construct a cooperative game on the principle that $v(S)$ is the cost that S would incur if it had to build an irrigation ditch to serve only its own members. Then the Shapley value of this game determines the cost allocation. (Note: the $v(S)$ are costs but are written as positive numbers. The Shapley value is computed as usual, but it has the meaning of the amounts of money that each player is required to pay toward the total cost $v(N)$).

(i) Show that these two cost sharing proposals are the same for the four ranches with the distances given above. (5)

(ii) Show that these two cost sharing proposals are the same for any number of ranches located at any distances from the source. (10)

(iii) What is the EANS cost allocation in this example? (5)

(iv) Give arguments, primarily in words rather than in mathematics, for and against the EANS and Shapley solutions to this problem. (10)

(v) Show that the core of the cooperative game defined above is non-empty in this example. (You may use without proof any result from the lectures or the problem sets.) (8)

Question D2) (22 points)

Consider the one–buyer, one–seller model with a single unit of an indivisible good initially owned by the seller, as in Lectures 10 and 11 and Problem Set 11. Specifically, consider the framework in which

- The players are given by $\mathbf{I} = \{1(\text{buyer}), 2(\text{seller})\}$, with valuations $\tilde{\theta}_1, \tilde{\theta}_2 \sim U[0,1]$.
- The outcomes are given by $k \in \{1, 0\}$, where $k = 1$ indicates that the buyer gets the good and $k = 0$ indicates that the seller keeps the good.
- Players have quasi–linear private–value utility functions, given by $u_i(k, \theta) = v_i(k, \theta_i) + t_i(\theta)$, where

$$\begin{cases} v_1(1, \theta_1) &= \theta_1, \\ v_1(0, \theta_1) &= 0, \\ v_2(1, \theta_2) &= 0, \\ v_2(0, \theta_2) &= \theta_2. \end{cases}$$

Suppose that the good can be exchanged between the seller and the buyer using the expected externality mechanism (EEM) to elicit the parameters θ_1 and θ_2. Under the EEM the exchange is efficient given the true parameter values and the monetary transfers sum to zero, requiring no external source or sink for funds.

We can interpret this as a model of labor contracting in which θ_1 is the value of the worker's labor to firm profitability, and θ_2 is the worker's reservation wage. We assume that only full time employment is possible – justifying the idea that in the model there is a single unit of an indivisible good that might be traded. Note here that the firm plays the role of the "buyer" (one wishing to purcahse labor) and the worker plays the role of the "seller" (one wishing to sell labor).

In this example EEM transfer function is given by

$$\begin{aligned} t_1(\hat{\theta}_1, \hat{\theta}_2) &= \tfrac{1}{2}\hat{\theta}_2^2 - \tfrac{1}{2}\hat{\theta}_1^2 \\ t_2(\hat{\theta}_1, \hat{\theta}_2) &= \tfrac{1}{2}\hat{\theta}_1^2 - \tfrac{1}{2}\hat{\theta}_2^2, \end{aligned}$$

where the $\hat{\theta}_i$ is the valuation announced by player i.

(i) Verify that truthful strategies form a Bayesian Nash equilibrium for the EEM. (5)

The rest of this question concerns the participation contraints for this problem.

Note that in a truth–telling equilibrim of the EEM the ex post realized utilities are given by

$$u_1(\theta_1, \theta_2) = \begin{cases} \theta_1 + \frac{1}{2}\theta_2^2 - \frac{1}{2}\theta_1^2, & \text{if } \theta_1 > \theta_2 \\ \frac{1}{2}\theta_2^2 - \frac{1}{2}\theta_1^2, & \text{if } \theta_1 < \theta_2 \end{cases}$$

and

$$u_2(\theta_1, \theta_2) = \begin{cases} \frac{1}{2}\theta_1^2 - \frac{1}{2}\theta_2^2, & \text{if } \theta_1 > \theta_2 \\ \theta_2 + \frac{1}{2}\theta_1^2 - \frac{1}{2}\theta_2^2, & \text{if } \theta_1 < \theta_2. \end{cases}$$

According to US labor law, only the buyer of labor (the firm in our case) can make an irrevocable promise about employment and compensation to the seller (the worker). It is typically illegal, and therefore unenforceable, for the worker to make an irrevocable promise to work for the firm.

(ii) Will the EEM as specified above result in the efficient exchange of labor with probability one when the participation constraints are imposed reflecting the US labor law? (6)

(iii) Suppose that the parties can modify the EEM by subtracting a fixed payment from the buyer's transfer and giving it to the seller as a lump sum, independently of the announcements made. Can such a modification of the transfer function in the EEM guarantee universal participation given the US labor law? (6)

(iv) Compare your answer in (iii) to what we know about Bayesian incentive compatible mechanisms when participation contraints are imposed for both sides of the market at the interim stage. (5)

D3) (10 points)

In the United States today the Democrats have only one Presidential candidate, Barack Obama, whereas the Republicans have a large number of possible candidates, Sarah Palin, Donald Trump,....Each voter has preferences as follows: Either Obama is the top candidate or Obama is their bottom candidate. In all other respects individual preferences are unrestricted.

True or **False**. Provide a short explanation, stating without proof any results from the course that you need for your argument:

(i) The Uncovered Set of the majority relation for these preferences is necessarily the same as the Top Cycle. (5)

(ii) Suppose that one of the Republicans is the Plurality winner. Then one of the other Republicans drops out of the race and the result is recomputed. The Plurality winner may change but it will still be a Republican. (5)