Harvard University
Department of Economics

General Examination in Microeconomic Theory

Spring 2009

1. You have **FOUR** hours.
2. Answer all questions

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Glaeser Question - Part A

Verbal question—Reminder, the question may be verbal but no points will be given for non-algebraic answers.

1. Individuals consume two goods—economics papers and hamburgers—and these goods are produced, with labor and capital, and traded in a closed economy. There is a fixed amount of capital, and the returns to this capital are spread evenly across society. Production of economics papers is more pleasant than production of hamburgers.
2. Assuming that everyone is identical, provide the first order conditions that determine the price of economics papers, hamburgers and the wage rates and the returns to capital. Derive conditions under which individuals will specialize in producing one good.
3. What impact will a proportional tax on labor income have on the prices of the two commodities?
4. What impact will a minimum wage have on prices? Can a minimum wage increase welfare in this setting?
5. Assume now that individuals differ in their ability to produce the two goods. What will this heterogeneity do to the level of specialization in society? Now, what impact will the proportional tax have on the prices of the two goods and the level of income inequality in society? What will a minimum wage do to the level of income inequality in society?

Rodenberg
Part B

1. Consider the following two-period game between a supervisor and a worker. In period $t = 1, 2$ the worker has type $\theta_t = 1$ with probability .9, and type $\theta_t = 0$ with probability .1; θ_1 and θ_2 are independent. Each period, the worker observes θ_t, and then decides whether to pay a cost $j > 0$ to prevent the supervisor from observing θ_t. If the worker pays the cost, the supervisor cannot make a report. If the worker does not pay the cost, the supervisor observes θ_t and then decides whether or not to report it to the firm. The supervisor is either a "normal type" or a "silent type" who is unable to report. The supervisor's type is the same in both periods; the prior probability that the supervisor is silent is s, where $s < 1 - j$. The normal type of supervisor and the worker both try to maximize the expected value of the sum of their period-by-period payoffs, which are given in the following table, where the row player is the worker and the column player is the supervisor.

	Report	Don't Report
pay	$\theta_t - j, 0$	$\theta_t - j, 0$
don't pay	$0, w_t$	$\theta_t, 0$

Assume $0 < w_1 < .9w_2$ and $(1 - j)^2 < s$.

a) Is there a pure-strategy perfect Bayesian equilibrium in which the worker never pays in the first period, and the normal supervisor reports the agent when the first-period type is $\theta_1 = 1$?

b) Is there a pure-strategy perfect Bayesian equilibrium in which the worker never pays in the first period and the normal supervisor never reports the agent in the first period?

c) Is there a perfect Bayesian equilibrium in which the worker never pays in the first period and the supervisor reports with some probability strictly between 0 and 1 (whenever he can make a report)? *Hint*: When there is such a PBE, what can you say about the the probability with which a $\theta_2 = 1$ worker pays in the second period following first period outcome (don't pay, don'treport)?

Part C

1. (a) Define an excess demand function for an economy. Discuss what properties an excess demand function has. Explain (informally!) what the "anything goes" theorem of Sonnenschein-Debreu-Mantel tells us about this issue.

(b) Consider a two consumer, two good exchange economy. Consumer 1's utility function is $x_{11}^{\frac{1}{2}} + x_{12}^{\frac{1}{2}}$ and consumer 2's is x_{21}, where x_{il} is consumer i's consumption of good *l*. The total endowment of each good is 1. What is the set of Pareto optima for this economy? Assume that the total endowment is split equally between the two consumers. What is the Walrasian equilibrium?

2. (a) A principal owns a firm, but needs an agent to operate it. The firm's production function is $q = a + \varepsilon$ where q is revenue, a is the agent's action, and ε is a random variable. The agent's action is unobservable to the principal. The agent is risk neutral and is not wealth constrained. Explain, using words and mathematics, why it is optimal for the principal to sell the firm to the agent.

(b) Consider the same situation as in (a), but now the agent has zero wealth. The agent can take two actions, a_L and a_H. There are two outcomes: q_1 and q_2, where $q_1 < q_2$. The cost of a_L is zero and of a_H is c. The probabilities of the outcomes are $\left(\frac{1}{2}, \frac{1}{2}\right)$ under a_L and $\left(\frac{1}{4}, \frac{3}{4}\right)$ under a_H. (The first component refers to q_1.) The agent's reservation wage (that is, utility) is zero. Suppose that the principal wants the agent to choose a_L. What is the cheapest way of implementing this? Suppose that the principal wants the agent to choose a_H. What is the cheapest way of implementing this? Use these findings to determine the optimal second-best outcome for the principal.

(c) Continue with (b), but suppose now that there is a third outcome q_3, which occurs only under a_H. The probabilities of the three outcomes are $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$ under a_L and $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right)$ under a_H. What is now the cheapest way of implementing a_H?

Part D

In answering the two questions in part D you can use any result in MWG or any result discussed in the lectures, provided you give a clear statement of that result.

Question 1 (37 points)

Three jurisdictions, which we will call "states" are part of a larger federal system which we will call the "country". From time to time the country passes a law which creates an unfunded mandate that applies to all the states. For example, the mandate may be that "all rivers must maintain a specified water quality". The states have no choice but to fulfill these mandates, even though they are costly.

A given mandate is described, for our purposes, by the most efficient costs of meeting it: $c(\{i\})$ is the cost if i fulfills the mandate alone, $c(\{i,j\})$ if i and j cooperate in fulfilling it, and $c(\{1,2,3\})$ if they all work together to fulfill it.

Any coalition can decide to do things separately instead of collaboratively. Thus it is natural to assume that

$$\begin{aligned} c(\{i,j\}) &\leqq c(\{i\}) + c(\{j\}) \text{ for all pairs } i,j \\ c(\{1,2,3\}) &\leqq c(\{i,j\}) + c(\{k\}) \text{ for all partitions } (\{i,j\},\{k\}) \text{ of } \{1,2,3\} \end{aligned}$$

At the same time, it is always more costly to fulfill the unfunded mandate over a larger set of states than a smaller set. Therefore,

$$c(S) \leqq c(T) \text{ for all } S \subseteq T$$

The domain of cost functions that this problem considers are those satisfying the conditions above.

We will be considering a specific numerical example that lies within this domain.

$$\begin{aligned} c(\{1\}) &= 9,\ c(\{2\}) = 5,\ c(\{3\}) = 3 \\ c(\{1,2\}) &= 12\ , c(\{1,3\}) = 10,\ c(\{2,3\}) = 6 \end{aligned}$$

The value of $c(\{1,2,3\})$ will be treated as a parameter of the model. The domain given above implies that $c(\{1,2,3\}) \in [12,15]$.

a) For which values of $c(\{1,2,3\})$ is the core non-empty? *(10)*

When there is some i that is not affected at all by a certain mandate this country uses a particular method for allocating the costs among those who are affected, j and k . First j and k pay their individual costs, $c(\{j\})$ and $c(\{k\})$; then they each receive a rebate of half of the excess revenue that has been collected, $c(\{j\}) + c(\{k\}) - c(\{j,k\})$, so that the total revenue collected from them is exactly $c(\{j,k\})$.

The three states want to extend this method to the general case. They want to find a cost allocation x with the property that for any pair (i, j) the cost shares they are assigned, (x_i, x_j) are equal to what this two-state method would have produced in the problem where only i and j are affected and their minimum costs is $c(\{1,2,3\}) - x_k$ instead of $c(\{i,j\})$.

b) Take any value of $c(\{1,2,3\})$ that you found in part a). Apply this extension principle. What cost allocation do you get? (12)

c) Define the nucleolus for cost allocation problems such as this. Show that the above principle for extending the two-person method to the three person case produces the nucleolus for the value of $c(\{1,2,3\})$ that you have used in part b) above. *(10)*

d) What is the Shapley value for this value of $c(\{1,2,3\})$? Comment on anything that you observe. (5)

Question 2 (38 points)

There are two people $i = 1, 2$ and two alternatives $k = a, b$. One alternative must be chosen and it will affect both people. Each individual's preference is characterized by a parmeter θ_i that is uniformly distributed on the interval $[-1, 1]$. The θ_i are statistically independent of each other. Thus the density of (θ_1, θ_2) is $\frac{1}{4}$ everywhere on $[-1, +1] \times [-1, +1]$.

In addition to selecting the alternative the two individuals may receive (or pay, if negative) a monetary transfer t_i. An outcome is a triple (k, t_1, t_2). The Bernoulli utilties for the outcomes are given by

$$\begin{aligned} u_i(k, t_i, \theta_i) &= \theta_i + t_i \text{ if } x = a \\ &= -\theta_i + t_i \text{ if } x = b \end{aligned}$$

In addition to these consumers there is a government that can serve as a sink for any net negative transfers that might be collected.

Each θ_i is i's private information.

a) What is the Pivotal (or Clarke) mechanism in this example? What outcome does it select as a function of (θ_1, θ_2)? Does it implement this outcome in dominant strategies, or in Bayesian optimal strategies? Why? (5)

b) Comment on the efficiency of Pivotal mechanism. (3)

c) Conditional on observing θ_i, what is the expected utility that i would have from playing this mechanism? (5)

d) Suppose that after learning θ_i an individual had the option to cancel the mechanism unilaterally, in which case the government would have no information, would select the alternative at random (50-50 probability), and would set both $t_i = 0$. Show that there is no value of θ_i at which the people would prefer to cancel the mechanism in this way instead of going through with it? (3)

Now consider a simple Voting Mechanism as follows. After observing θ_i each person announces one of the two alternatives, a or b. If they announce the same alternative, it is implemented. If their announcements disagree, the government chooses an alternative at random (50-50 probability). No monetary transfers are ever made.

e) What outcome (or randomized outcome) does it select as a function of (θ_1, θ_2)? (2)

f) Compute the interim expected utilities of the two players in this mechanism. (5)

g) Which mechanism is more efficient from an ex ante viewpoint? (5)

h) Is there a better mechanism than either of these? What considerations other than the expected utilities produced are relevant when you decide whether or not to use a particular alternative mechanism? (Discuss...no computations required.) (10)