Harvard University
Department of Economics

General Examination in Microeconomic Theory

Spring 2007

1. You have **FOUR** hours.

2. Answer all questions

3. Points for each part:
   I. 50 points
   II. 50 points
   III. 70 points
   IV. 70 points

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Part 1
Glaeser

People receive unobservable shocks to their cost of working, but are otherwise identical. Put most of your work into the first two parts of the question and only turn to the third if you have abundant time-- if you do attempt the third, certainly feel free to choose one functional form for utility. For the first part of the model, you probabily want to assume that utility has the form u(cash)-effort costs.

(1) Derive the conditions for an optimal one-period disability system that insures people against these shocks (10 points).
(2) Derive comparative statics on the determinants of the generosity of the system? (5 points)
(3) Extend the model to an infinite horizon where individuals all have costs of working that increase over time. Individuals only differ (unobservably) in the intercept of their costs of working. Limit your system to a linear tax on income coupled with a single retirement benefit and derive the optimal system (10 points).

Part 2
Wilson

Game Theory Question

A three-player committee must decide whether to implement a new policy, or to remain at the status quo.

1. Suppose first that there are two possible alternatives, A and B, to the status quo. Preferences are as follows:

$$\text{player 1:} \quad A > S > B$$
$$\text{player 2:} \quad B > A > S$$
$$\text{player 3:} \quad S > B > A$$

where S denotes the status quo option. The decision will be made by a simultaneous majority vote. If one of the three options receives at least two votes, it is chosen. If there is no majority winner, then player 1 decides the outcome.

Determine all outcomes that survive iterated deletion of weakly dominated strategies.

2. Now assume that there is only one alternative, A, to the status quo S.

Modify the decision rule as follows: first, player 1 votes. If player 1 votes for S, then player 2 is consulted: if he also votes for S, the game ends and S is implemented; if he votes for A, then player 3 is asked to decide. If player 1 instead votes A, then player 3 is asked to decide (and player 2 is never consulted).

Payoffs to players 1,2,3 are as follows:

$(1,1,1)$ if players 1,2, both vote for S

$(0,0,1)$ if player 3 decides, and chooses S

$(4,4,0)$ if player 3 decides, chooses A, and player 2 was consulted

$(3,2,3)$ if player 3 decides, chooses A, and player 2 was not consulted

Draw an extensive form game tree describing this situation, assuming that all votes are observed. Find all Nash and subgame perfect equilbria of the game.

3. Modify the game in part (b) as follows: if player 3 is asked to decide, he is informed only that there was at least one vote for A (that is, he does not know whether or not player 2 was consulted). Again draw the extensive form game tree, and find two pure strategy Nash equilibria.

4. For each of your equilibria in (c), determine whether or not it can be supported as a sequential equilibrium strategy profile.

Part 3

2010b (Postlewaite) Question for general exam

Consider an economy with one private good and one public good, where the public good can be produced from the private good according to the production function $y = f(x) = x$, where y is the quantity of public good that can be produced using x units of the private good. There are n agents and agent i's initial endowment is $(w_i, 0)$. (That is, no agent has an initial endowment of the public good. Agent i's utility function is $u_i : R^2 \to R$, and is concave, strictly increasing and twice differentiable.

a. Define a Lindahl equilibrium for this economy.

b. State and prove the analog of the first welfare theorem for Lindahl equilibria.

Suppose $u_i(x_i, y) = x_i + \theta_i \ln y$, $i = 1, ..., n$ for the remaining questions.

c. Calculate the Lindahl equilibrium taxes and public good level for the n-person economy, assuming that the Lindahl outcome is interior.

d. What are the conditions under which agent 1's allocation is not interior at a Lindahl equilibrium?

e. Suppose agent 1's allocation is not interior at a Lindahl equilibrium but all other agents' allocations are. How does the fact that agent 1's allocation is not interior affect the Lindahl taxes for the agents whose allocations are interior?

f. Suppose $n = 3$ and $w_i = 1$, $\theta_i = 1$, $i = 1, 2, 3$. What is the Nash equilibrium of the voluntary provision game? What can you say about the efficiency of the public good level?

g. Suppose $n = 3$, $w_i = 1$, $i = 1, 2, 3$ and $\theta_1 > \theta_2 > \theta_3$. Suppose that there is a majority rule vote about the level of public good y, where each person will pay 1/3 of the cost of providing that level of public good. What can you say about this level as compared to the Lindahl equilibrium level of public good?

Part 4
Green

Part D has two questions. Answer both. Question D1 is worth 40, Question D2 is worth 30. Numbers following each part are the maximum points for that part and represent the approximate amount of time (in minutes) allocated to it.

Question D1

Alice (A), Bob (B) and Charlie (C) are friends. They are such good friends that they are contemplating marriage. Specifically, Alice and Bob could get married (x), or Alice and Charlie could get married (y), or they could all remain single (z).

Alice prefers marrying Bob to marrying Charlie, and prefers marrying Charlie to remaining single.

Bob prefers that everyone remain single (he wants to continue dating Alice), but he prefers marrying Alice himself to seeing Alice marry Charlie.

Charlie's favorite alternative is to marry Alice himself. Because he values others' happiness he would rather see Bob marry Alice, his second place outcome, than have everyone remain single.

a) Alice, Bob and Charlie decide to use majority rule to determine the outcome. Is there a Condorcet winner in the set $\{x, y, z\}$? (3)

b) What is the top cycle of a majority relation? What is the top cycle in this example? (3)

Alice, Bob and Charlie have utility functions that represent the intensity of the ordinal preferences described above. Moreover they can agree to make zero-sum monetary transfers among themselves and their preferences are quasi-linear in money.

Their cardinal preferences for the alternatives in the absence of any monetary transfers are given in the following table:

	x	y	z
A	10	8	0
B	5	4	10
C	6	10	0

c) Alice, Bob and Charlie consider the intensities of their preference as described by these utilities. They realize that the efficient outcome is y – Alice should marry Charlie. Moreover, Bob objects to the outcome of part a) because he says that everyone should enter marriage voluntarily. He proposes that they use the Nash Bargaining Solution to determine both the marriage outcome and a zero-sum monetary transfer among them, with the status quo point being outcome z (and zero monetary transfers). What would the resulting outcome be? (5)

d) Now assume that these players decide to use Cooperative Game Theory to compute the outcome. They agree to use the characteristic function calculated as follows: For every coalition S, $v(S)$ is the largest value that S

can guarantee – that is it is the largest value that S can achieve given that the members of S can make a binding commitment to marry or not to marry, but recognizing that the complementary coalition can take any action (whether or not it is rational for them), and therefore that $v(S)$ is set at the lowest possible value that this action can induce. Using this characteristic function any coalition S of which Alice is a member can agree to a marriage between Alice and any other member of the coalition, or it can agree that all members of the coalition remain single. The coalition of Bob and Charlie can only achieve z since they do not have the right to assume that Alice will marry either of them (even though she will). Any coalition S consisting of a single player must assume that the complementary coalition will do whatever is worst for S. What is the core of this cooperative game? If you were Bob, how would you argue against implementing one of the points in the core? Comment on the construction of the characteristic function as given above. What would you get if you used a different construction? (12)

e) Since things are not going well in this threesome's marriage discussions, they decide to compute the Shapley Value of the game described above. At least that will be a single well-defined point. What is it? (9)

f) By now they are very frustrated with the conflicting advice given by all these normative solution concepts. So they decide to ignore the fact that their preferences are common knowledge. They agree to consult a mechanism designer, and play the mechanism she suggests, as if they knew nothing about each other's preferences. The mechanism designer proposes the pivotal (or VCG) mechanism. What is the pivotal (or VCG) mechanism? What outcome is implemented when Alice, Bob and Charlie play it? (8)

Question D2

The Federal government finally decides to do something about the health care situation in America. There are two types of people, differing in their needs for health care according to a parameter $\theta \in \{\theta_L, \theta_H\}$ with $0 < \theta_L < \theta_H$. Half of the population has each value of the parameter. Everyone's utility depends on the health care allocated to them q and the monetary transfer that they pay to the government t, according to the function

$$u(q, t, \theta) = \theta v(q) - t$$

Assume that $v > 0$, $v' > 0$ and $v'' < 0$.

Also assume that the health care is produced at constant marginal cost and thus the system which gives rise to the allocation $q(\theta_L), q(\theta_H)$ costs $c(q(\theta_L) + q(\theta_H))$ where $c > 0$.

a) The current Republican administration wants to find a way to implement the efficient pattern $q(\theta)$. Moreover Republicans believe in individual responsibility and individual freedom to choose: This means that (i) the payments $t(\theta)$ should be such that everyone pays for exactly what he or she uses, and (ii) everyone chooses their own (q, t) from a common menu. Is such an implementation possible? (5)

b) The Republican system is implemented and leads to great disparities in utility and in health care utilization. Thus when the Democrats get elected the first thing they do is to mandate an implementation of a pattern $q(\theta)$ that maximizes the average utility in the population. The Democrats also believe in individual freedom to choose. However they believe in fiscal responsibility (balanced budget) over all, but not necessarily that everyone should pay the exact cost of the health care that he or she selects. What allocation $(q(\theta_L), q(\theta_H))$ is implemented under the Democratic administration? (7)

c) Now the Socialists come to power. They are worried about the welfare of the worst-off members of society. Which type is worse off? Socialists also believe that the system must break even and give all people the same menu of choice. Their objective is the same as the Democrats, to maximize the welfare of the average person, but subject to the principle that the worst off people must get a certain minimum level of utility. The Socialists gradually raise the welfare of the worst off type of people by changing the menu of health care that is offered. Describe what happens to the optimal menu as this minimum level of utility is increased above what it was under the Democrats – starting at that level and raising it until it cannot be increased any more. (10)

d) Finally the Communists come to power. Their political philosophy is the same as the Socialists' as far as freedom of choice and the welfare objectives of government. They also face the same budget constraint. However they understand that the utility functions are not $\theta v(q) - t$, but rather are $\theta v(q) - t + \phi(\theta)$, where $\phi(\theta)$ represents the utility of type θ if it were to receive no health care and no transfer. Moreover, through careful empirical research the

Communists have ascertained that $\phi(\theta_L)$ exceeds $\phi(\theta_H)$ by a very large amount – thus the worst off people in the society are not those who do not need health care very much (L's), but rather those who would benefit the most from health care (H's). What policy do the Communists implement? (8)