Harvard University
Department of Economics

Economics 2010b: Final Examination and General Examination in Microeconomic Theory

Spring Term 2004

1. You have **FOUR** hours.

2. Answer all questions

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

2010 Generals exam
Part A

(1) Parents invest in their children's preferences.

(a) What would you expect the impact of parental wages, income, patience and altruism to be on the level of their child's patience? Show these results formally?

(b) Under what conditions will patience within a family dynasty display convergence to the mean?

(c) What is going to determine the degree to which parents invest in their children's altruism to grandchildren vs. the degree to which parents invest in their children's altruism towards themselves?

(d) What is going to determine the investment in the risk-aversion?

(e) How will parents' invest differently in male and female children?

(f) What would you expect divorce or singe parenthood to do the level of these investments?

(g) What is the appropriate government policy towards these forms of investment?

Part B (Answer all three questions in the part.)

B1) (*15 points*)

Much attention in game theory is devoted to the issue of whether all equilibria are equally good predictors of the likely outcomes of a game. With reference to the game tree below, discuss:

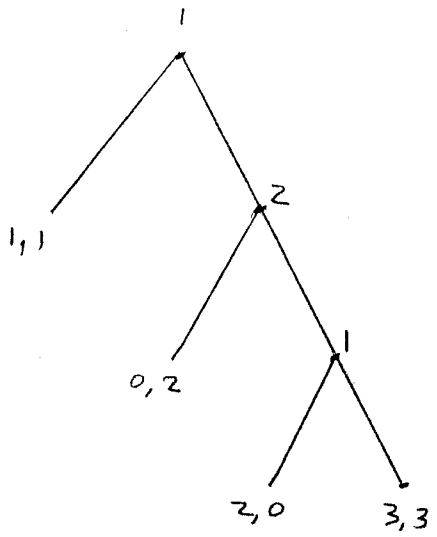
(i) What are the Nash equilibria?

(ii) How does game theory differentiate among them?

(iii) What is the normal form of this game?

(iv) How does game theory differentiate among the equilibria of this game in normal form?

(v) Does game theory make the same predictions about a game whether it is presented in its extensive form or only in its normal form? Discuss.



B2) (*10 points*)

(i) Define the concept of a correlated equilibrium of a normal form game.

(ii) Give an example of a game in which there exists a correlated equilibrium that is different from any of the Nash equilibria.

B3) (*25 points*)

Two people ($i = 1,2$) have the opportunity to establish a partnership that will share in a random amount of money. The total to be shared, x, might be positive or negative. It is normally distributed with mean $\mu > 0$ and variance σ^2. The players' Bernoulli utility functions are given by

$$u_i(x_i) = -e^{-\alpha_i x_i}$$

The players know each other's Bernoulli utilities and can make any contingent contracts they like that specify the shares $x_1(x)$, $x_2(x)$ as a function of the total that is available ($x_1(x) + x_2(x) \equiv x$). The players decide to use the Nash Bargaining Solution to determine the optimal sharing rule. The disagreement point is that they do not form a partnership at all, and they both receive $x_i = 0$.

(i) What sharing rule does the Nash Bargaing Solution recommend. How does it depend on μ, σ^2, α_1, and α_2. Give an economic interpretation of your results.

(ii) Consider the simultaneous move game in which the players announce their risk aversion to a referee and they can lie about it at will, announcing any non-negative number for α_i. The referee computes the contingent contract that would be the Nash Bargaining Solution for these two risk aversion parameters (i.e. your answer in (i) above, using the two announcements). The expected payoffs of the players is their expected utility of this contract, evaluated using their true utility functions. What is the Nash equilibrium of this game?

Laurent E. Calvet
Spring 2004

PROBLEM - Aggregation

This problem illustrates that the aggregation of consumer demand can generate economic properties that need not hold at the individual level.

Consider an exchange economy with L goods and a continuum of consumers $h \in [0,1]$. All agents have the same utility u, which is continuous, strongly monotonic and strictly quasi-concave. The corresponding individual Marshallian demand $\xi(p,w)$ is assumed to be differentiable on $\mathbb{R}^L_{++} \times \mathbb{R}_+$.

Agents have collinear endowments:

$$e^h = \alpha(h)\bar{e},$$

where $\alpha(h) \in \mathbb{R}_+$ and $\bar{e} \in \mathbb{R}^L_{++}$. We assume that across consumers, the scalars $\alpha(h)$ are uniformly distributed on $[0,2]$. These hypotheses fully specify the income distribution in the population.[1]

a) Define the mean excess demand function $Z(p)$ and show that it satisfies

$$Z(p) \equiv \frac{1}{2}\int_0^2 \xi(p, \alpha p.\bar{e})d\alpha - \bar{e},$$

for every price vector $p \in \mathbb{R}^L_{++}$.

b) Show that the Jacobian matrix $DZ(p) = [\partial Z_i/\partial p_j(p)]_{1\leq i,j\leq L}$ can be decomposed as the sum of three $L \times L$ matrices:

$$DZ(p) = S(p) - A(p) + B(p), \tag{0.1}$$

where $S(p)$ is negative semi-definite, $A(p) = [A_{i,j}(p)]_{1\leq i,j\leq L}$ has components

$$A_{i,j}(p) = \frac{1}{2}\int_0^2 \xi_j(p, \alpha p.\bar{e})\frac{\partial \xi_i}{\partial w}(p, \alpha p.\bar{e})d\alpha,$$

[1]For any $A \in [0,2]$, the fraction of agents s.t. $\alpha(h) \leq A$ is thus $A/2$.

and $B(p) = [B_{i,j}(p)]_{1 \leq i,j \leq L}$ has components

$$B_{i,j}(p) = \frac{1}{2} \int_0^2 \alpha \bar{e}_j \frac{\partial \xi_i}{\partial w}(p, \alpha\, p.\bar{e}) d\alpha.$$

Hint: Use the Slutsky equation.

c) Let $v \in \mathbb{R}^L$ denote a vector orthogonal to the mean endowment: $v.\bar{e} = 0$. Prove that $v^\top B(p)v = 0$ and $v^\top A(p)v \geq 0$. Conclude that $v^\top DZ(p)v \leq 0$.

d) Consider two vectors $p, q \in \mathbb{R}^L_{++}$ assigning the same dollar value to the aggregate endowment: $p.\bar{e} = q.\bar{e}$. Show that

$$(p - q).(Z(p) - Z(q)) \leq 0.$$

This property is called the *law of demand.* Hint: Consider the auxiliary function $\varphi(t) = v.[Z(p + tv) - Z(p)]$, where $t \in [0, 1]$ and $v = q - p$.

Note: We did not assume that the law of demand holds at the individual level.

e) Microeconomists often consider the strong law of demand:

$$(p - q).(Z(p) - Z(q)) < 0$$

for all p, $q \in \mathbb{R}^L_{++}$, $p.\bar{e} = q.\bar{e}$, $p \neq q$. Show that under the strong law of demand, there exists a unique Walrasian equilibrium.

Part D (Answer all four questions in this part)

D1) (*5 points*)

Suppose you have a mechanism that induces truthful responses in a Bayesian setting. Assume that the outcome for a player that will result if mechanism is not played has the same value no matter what the individuals private information may be. If the mechanism induces participation by all individuals based on their expected utilities before they receive their private information, will it induce participation of all individuals after they learn their own private information?

D2) (*15 points*)
There are three alternatives a, b, c and three voters $1, 2, 3$. The decision will always be made as follows:

step 1: Majority rule determines whether b or c is eliminated from further consideration.
step 2: Majority rule determines the winner between a and the alternative that has not been eliminated at step 1.

This procedure is used no matter what the preferences of the voters are. All preferences are always strict. At the time the votes take place, all the voters know each other's preferences – indeed these preferences are common knowledge. They all know that each other is rational and play the resulting game in accordance with game theory as studied in 2010a.

(i) Describe in detail the social choice function that is implemented by this procedure. What game theoretic solution concept are you using and why have you chosen it?
(ii) Define what it means for a social choice function to be Paretian.
(iii) Does this procedure implement a Paretian social choice function?
(iv) Show that this procedure selects a Condorcet winner whenever there is one.

D3) (*25 points*)

There is a monopolist who is the textbook business. This monopolist owns the rights to a large number of textbooks of varying difficulty for elementary school students. The cost of printing and distributing these textbooks is zero. The monopolist sells these books to local school boards, which use them in their communities. All the students in a given community must use the same book (because that is the book that the teacher will use in class).

The level of difficulty of a texbook is q. The monopolist can set a schedule for textbook prices $p(q)$ which means that textbooks of level q will be supplied to a community if the school board pays the monopolist $p(q)$. We will assume that the monopolist sets a linear price schedule

$$p(q) = pq$$

Every community consists of three families. Each family has a preference relation for the level of book q that its children use and for the amount of money t_i that it has to pay to the school board (in taxes that finance the school board's purchases). The preference of family i is given by

$$U_i(q, t_i, \theta_i) = \theta_i q - \frac{1}{2}q^2 - t_i$$

The law requires that the families share the cost of the books equally – that is $t_i = \frac{1}{3}p(q)$.

Given the schedule $p(q)$, the preferences of the three families, and the equal sharing of the costs, each school board determines the level of book to purchase based on a vote of the three families. The selected level is the value of q which can defeat any other q in a pairwise majority vote.

(i) For any $p \geqq 0$ show that there exists a unique q that will win this vote? Explain this result using the terminology of social choice theory.

(ii) Show that your answer to part (i) of this question is independent of the knowledge that each family has about the preferences of the other two families in the community.

(iii) Now suppose that θ is distributed independently and identically in the population and that there are a large number of school districts. In particular, the distribution of each θ_i is uniform on the interval $[1, 2]$. Assume that the monopolist knows this distribution and the rules by which school boards make their decisions. Which value of p describes the revenue maximizing linear pricing strategy of the monopolist?

D4) (*30 points*)

Two siblings, a sister and a brother, have been left an inheritance consisting of one indivisible house. Only one of them can own it. They have sufficient financial resources to make payments to each other, and their preferences are quasi-linear in these payments.

Their valuations are the private information. Their valuations are independently and identically distributed according to a distribution function F that has a positive density function over an interval $[y,z]$.

Before learning their own valuations, they hire a consultant (who knows F and all of the assumptions just mentioned, but not the actual valuations) to design a mechanism to decide who gets the house and how much one should pay the other. They require that the mechanism have the following properties:

(a) The house goes to the sibling who values it more.

(b) Any monetary transfers are between the two of them and do not involve any outside parties as a source or sink for funds.

(i) Find a mechanism that the consultant could recommend that meets these requirements.

(ii) Prove that this mechanism has the desired properties.

(iii) Now suppose that the house has a market value of $\$x$ (which is known in advance) and that either person, after learning his or her own valuation, could decide not to play the mechanism you found in part (i). If anyone elects not to play the mechanism the house will be sold to outsiders and both parties get $\$x/2$. How can you determine the x which is sufficiently low that this possibility does not arise?

(iv) If x is such that the default option will be selected by someone, with positive probability, will it be someone with a relatively high valuation (close to z) or a low valuation (close to y)?