Harvard University
Department of Economics

Economics 2010b: Final Examination and General Examination in Microeconomic Theory

Spring Term 2002

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

You have **FOUR** hours.

2. Answer all questions

Part

Consider the following family of decision problems. There is a finite number of actions $a \in A$, a finite set of states of the world $\theta \in \Theta$, a prior probability p on Θ, and a (vN-M) utility function $u(a,\theta)$; the decision maker's objective is to maximize the expected value of u given the distribution p.. Let $W(p) = \max_{a \in A} E_p u(a,\theta)$, where E_p denotes the expectation with respect to probability distribution p.

a) Prove that if p and p' are probability distributions on Θ, and $q = \lambda p + (1-\lambda)p'$ for some λ between 0 and 1, (*add the distributions by adding the two vectors)* then $W(q) \leq \lambda W(p) + (1-\lambda)W(p')$. That is, W is weakly convex on the set of probability distributions over Θ.

b) When is it the case the $W(q) \quad \lambda W(p) + (1 \quad \lambda)W(p')$?

c) Suppose that the decision maker currently has beliefs p about θ, and that before choosing an action the decision maker has the option of observing a signal with two possible values, s and s'. What does the analysis above tell us about the desirability of observing the signal?

Now suppose that u is a vN-M utility function for money (what MWG call a "Bernoulli utility function) that is continuous, strictly increasing, and strictly risk averse. Suppose that the decision maker's choice set A consists of a finite number of maps from Θ to money payoffs, and let $W(p) = \max_{a \in A} E_p u(a(\theta))$, where we continue to denote the distribution over Θ by p.

d) Is the function u concave or convex? Give a brief defense of your answer.

e) Do the conclusions in parts (a) and (b) above still hold, or do they need to be modified? If so, how?

Part II

This question concerns two related versions of the following game. In the first version, which has complete information, there is a mechanic (M) and two car owners, C1 and C2; M and C1 play in period 1, and M and C2 play in period 2. In period 1, C1 chooses either In or Out. If C1 plays Out, then both M and C1 get 0 that period. If C1 plays In, then M observes a signal $z \in \{E, T\}$, where E indicate that the car needs a new engine, and T indicates that it needs a tune-up, and each signal has probability ½ . M then chooses either action e or action t. If the action chosen matches the state, then M and C1 both receive +1; if the action differs from the state, M and C1 both receive -2. At the beginning of period 2, player C2 observes the actions chosen in period 1, which will be one of the three events {(Out), (In, e), (In, t)}; this is all of the information that C2 observes. The structure of the game is the same as that in period 1, except that all of the payoffs are multiplied by 3, so that for example the payoff to In, e when the signal is E is 3. Player M's utility is the sum of the per-period payoffs.

a) Is there a Nash equilibrium where player M always chooses the action that matches the signal?

b) Are there any other Nash equilibria?

Now consider an incomplete-information version of this game, where there are two types of player M- an "friendly" type with the payoffs specified above, and an "engine-crazy" type who always chooses e regardless of the signal. The prior probability of the crazy type is p.

c) Show that there is a p^* strictly between 0 and 1 such that for $p \in [0, p^*]$ there is a perfect Bayesian equilibrium in which the friendly type of M always (that is, at every history) chooses the action that matches her signal. What is the value of p^*?

d) Find a perfect Bayesian equilibrium for the case where the prior p is larger than

p

Part III

Problem - General Equilibrium

This problem examines the aggregation of consumer demand when agents have homogeneous preferences and collinear endowments. More formally, consider a Walrasian economy with L goods ($L \geq 2$) and a continuum of agents. Consumers have a unique preference specified by a continuously differentiable Marshallian demand $\xi(p, I) \in \mathbb{R}^L_+$.

Individual endowments are collinear to a given vector $\omega \in \mathbb{R}^L_{++}$. The endowment of a consumer is of the form $y\omega$, where y is a real number contained in the interval $[0, 1]$. We assume for simplicity that the parameter y is uniformly distributed in the interval $[0, 1]$. The aggregate demand function is thus

$$X(p) \quad \int_0^1 \xi[p, (p'\omega)y]dy.$$

For any price $p \in \mathbb{R}^L_{++}$, we want to show that the Jacobian matrix

$$\frac{\partial X}{\partial p}(p) \quad \left[\frac{\partial X_i}{\partial p_j}(p)\right]_{1 \leq i,j \leq L}$$

is negative semi-definite on the hyperplane $H(\omega) = \{v \in \mathbb{R}^L : \omega' v = 0\}$.[1] As we will see, this property has important consequences for the monotonicity of $X(p)$ and the uniqueness of equilibrium.

a) Show that

$$\frac{\partial X}{\partial p}(p) \quad \int_0^1 \frac{\partial \xi}{\partial p}[p, (p'\omega)y]dy + A(p),$$

where $A(p) \quad \left\{\int_0^1 y \frac{\partial \xi}{\partial I}[p, (p'\omega)y]dy\right\}\omega'$

b) Show that

$$\frac{\partial X}{\partial p}(p) = A(p) + S(p) \quad B(p)$$

where $B(p) = \int_0^1 \frac{\partial \xi}{\partial I}[p, (p'\omega)y] \; \xi[p, (p'\omega)y]' \; dy$ is the matrix of income effects, and $S(p)$ is the average Slutsky substitution matrix.

c) Let $v \in H(\omega)$. Show that $A(p)v = 0$ and explain why $v'S(p)v \leq 0$. Conclude that the matrix $A(p) + S(p)$ is negative semi-definite.

[1] That is, $v' \frac{\partial X}{\partial p}(p)v = \sum_{1 \leq i,j \leq L} v_i v_j \frac{\partial X_i}{\partial p_j}(p) \leq 0$ for all $v \in H(\omega)$.

Part III cont.

d) Consider a vector $v \in \mathbb{R}^L$. Show that

$$v'B(p)v = (p'\omega)^{-1} \int_0^1 g(y)\frac{dg}{dy}(y)dy,$$

where $g(y) = v'\xi[p, (p'\omega)y]$

e) Infer that $v'B(p)v \geq 0$. Conclude that the Jacobian matrix $\frac{\partial X}{\partial p}(p)$ is negative semi-definite on the hyperplane $H(\omega)$.

f) Show the Law of Demand:

$$(q-p)'[X(q) - X(p)] \leq 0 \quad \forall p, q \in \mathbb{R}^L_{++},\ p'\omega = q'\omega.$$

Hint: consider the function $g(t) = (q-p)'[X(r(t)) - X(p)]$, where $r(t) = p + t(q-p)$ and $t \in [0, 1]$.

Further Results (Not for Credit). It is then easy to prove that equilibrium is generically unique.

Part IV

1. Consider a market with one seller and two buyers. The seller's type is his private information. We denote the seller type by θ and assume that the seller's value for the object is $r(\theta) = 0.8 \cdot \theta$. Assume that all buyers value the object at θ. (thus for any value of θ the seller's value for the object is lower or equal to that of the buyers). The buyers do not observe θ, the buyers believe that θ is uniformly distributed on the interval [0,2], this information is common knowledge (as well as structure of the game). Both buyers simultaneously announce the prices that they are willing to pay for the object. The seller observes the prices and decides if he wants to keep the object or to sell it to one of the buyers. The payoff of the seller is $r(\theta)$ if he keeps the object and p if he sells it. The payoff of a buyer is zero if he does not buy the object and $\theta - p$ if he buys it.

   (a) Would you describe this market as an example of moral hazard or/and adverse selection? What is the equilibrium price offered by the buyers?

   (b) Now let us consider a slightly more complicated game. Suppose before the buyers make their offers the seller has an opportunity to take a test that costs $c = 0.02$ and truthfully reveals his type θ to the buyers. The seller does not have to take the test. What is the range of types of the seller such that the seller chooses to take the test?

2. Consider an independent private value environment with three bidders and one object. Assume that the values of bidders are independently drawn from a uniform distribution with support on the interval [0,1]. The seller's valuation of the object is zero. This information structure is common knowledge.

   (a) Does conducting an efficient auction maximize the expected revenues of the seller? (By efficient auction we mean an auction that allocates the object to the buyer with the highest value.) If your answer is yes prove it otherwise give an example of an auction mechanism that generates higher expected revenues.

   (b) Suppose the seller decides to use a two stage auction procedure. Round 1: All bidders submit sealed bids. Two bidders with the highest value advance to the second round but only the highest bidder pays his bid. (The bidder with the second highest bid pays nothing for a privilege to advance to the second round.) Round 2 is a standard second price auction with two bidders. Both buyers who advanced to the second round submit sealed bids, the highest bidder gets the object and pays the second highest bid. (As usual, ties are broken by lottery. The total payments of a bidder in this mechanism is the sum of his payments in the first and second round). Does this mechanism yield an efficient allocation of the object? (Give two answers, one using revenue equivalence theorem and the other not using it).

   (c) If the mechanism described in part b has multiple equilibria find the one with the lowest expected revenues for the seller (the equilibrium concept is subgame perfect Nash Equilibrium).

3. Consider standard social choice setting with N individuals and I alternatives.

   (a) Formulate Arrow's impossibility theorem.

   (b) Let $r_n(x)$ denote the rank that individual n gives to alternative x (suppose individuals have strict preferences, that is they are never indifferent between two alternatives). Assign a score to each alternative equal to the worst rank that it received, more formally $U(x) = -\max[r_1(x), ..., r_N(x)]$. Consider a SWF that orders alternatives according to scores. What are the assumptions of the Arrow's impossibility theorem violated by this SWF?

   (c) Prove Arrow's impossibility theorem (if you do not remember the proof try to describe the idea behind the proof).