Economics Honors Exam 2008 Solutions Question 1

(a) **(2 points)** The steel firm's profit-maximization problem is

$$\max_{s,x} p_s s - c_s(s,x) = p_s s - \alpha s^2 + \beta x - \gamma x^2$$

Partial credit awards

0.5 points: for realizing that profit is revenue minus cost
0.5 points: for realizing that the two choice variables are s and x
1 point: for specifying the formula correctly

(b) **(2 points)** The fishery's profit-maximization problem is

$$\max_{f} p_f f - c_f(f,x) = p_f f - \delta f^2 - \theta x^2$$

Partial credit awards

0.5 points: for realizing that profit is revenue minus cost
0.5 points: for realizing that the *only* choice variable is f
1 point: for specifying the formula correctly

(c) **(6 points)** The first order conditions for the steel firm are derived by taking first order partial derivatives

$$FOC_s: \qquad p_s - 2\alpha s^* = 0 \quad \Rightarrow \quad s^* = \frac{p_s}{2\alpha}$$

$$FOC_x: \qquad \beta - 2\gamma x^* = 0 \quad \Rightarrow \quad x^* = \frac{\beta}{2\gamma}$$

The first order condition for the fishery is

$$FOC_f: \qquad p_f - 2\delta f^* = 0 \quad \Rightarrow \quad f^* = \frac{p_f}{2\delta}$$

Partial credit awards

1 point: for getting the FOC w.r.t. s correct
1 point: for solving s^* correctly
1 point: for getting the FOC w.r.t. x correct
1 point: for solving x^* correctly
1 point: for getting the FOC w.r.t. f correct
1 point: for solving f^* correctly

(d) **(2 points)** The profit-maximization problem of the combined firm is

$$\max_{s,x,f}\left[p_s s - \alpha s^2 + \beta x - \gamma x^2\right] + \left[p_f f - \delta f^2 - \theta x^2\right]$$

Partial credit awards

0.5 points: for combining the profit functions of the two firms
0.5 points: for realizing that the three choice variables are s, x and f
1 point: for specifying the formula correctly

(e) **(8 points)** The first order conditions for the combined firm are derived by taking first order partial derivatives

$$FOC_s: \quad p_s - 2\alpha s^* = 0 \quad \Rightarrow \quad s^* = \frac{p_s}{2\alpha}$$

$$FOC_x: \quad \beta - 2\gamma x^{**} - 2\theta x^{**} = 0 \quad \Rightarrow \quad x^{**} = \frac{\beta}{2(\gamma+\theta)}$$

$$FOC_f: \quad p_f - 2\delta f^* = 0 \quad \Rightarrow \quad f^* = \frac{p_f}{2\delta}$$

The optimal amounts of steel (s^*) and fish (f^*) remain the same for the combined firm, however, the optimal amount of pollution (x^*) is reduced because the new firm will take into account the benefit as well as the <u>full</u> cost of the pollution.

Proof:

$$x^{**} < x^* \quad iff \quad \frac{\beta}{2(\gamma+\theta)} < \frac{\beta}{2\gamma}$$
$$\Leftrightarrow \quad \theta > 0$$

Since we know that the fishery is adversely affected by the pollution, we can get that $\theta > 0$ using the structure of the cost function for the fishery.

Partial credit awards

1 point: for getting the FOC w.r.t. s correct
1 point: for solving s^* correctly
1 point: for getting the FOC w.r.t. x correct
1 point: for solving x^{**} correctly
1 point: for getting the FOC w.r.t. f correct
1 point: for solving f^* correctly
2 points: for realizing that $x^{**} < x^*$ (also depends on the quality of the student's reasoning)

(f) **(6 points)** Suppose the government imposes a unit tax t on the pollution generated by the steel firm.

Then the profit maximization problem for the fishery is unchanged

$$\max_f p_f f - c_f(f,x) = p_f f - \delta f^2 - \theta x^2$$

$$FOC_f: \qquad p_f - 2\delta f^* = 0 \quad \Rightarrow \quad f^* = \frac{p_f}{2\delta}$$

The profit maximization problem for the steel company is now

$$\max_{s,x} p_s s - c_s(s,x) - tx = p_s s - \alpha s^2 + \beta x - \gamma x^2 - tx$$

$$FOC_s: \qquad p_s - 2\alpha s^* = 0 \quad \Rightarrow \quad s^* = \frac{p_s}{2\alpha}$$

$$FOC_x: \qquad \beta - 2\gamma x^{***} - t = 0 \quad \Rightarrow \quad x^{***} = \frac{\beta - t}{2\gamma}$$

The purpose of the Pigouvian tax is to create a condition under which the Pareto efficient level of pollution will be reached. If we set our function for the optimal level of pollution under the tax equal to the Pareto optimal level ($x^{***} = x^{**}$), then we will be able to figure out the efficient Pigouvian tax

$$\frac{\beta - t}{2\gamma} = \frac{\beta}{2(\gamma + \theta)}$$

$$\Rightarrow \qquad t = \frac{\beta\theta}{\gamma + \theta}$$

The amounts of s, f, and x under this policy are all equal to the amounts that occur under the merged firm.

Implication: the government can achieve the Pareto optimal outcome by setting the efficient level of tax.

Partial credit awards

2 points: for specifying and solving the fishery's maximization problem correctly
2 points: for specifying and solving the steel firm's maximization problem correctly
2 points: for solving the efficient Pigouvian tax correctly and realizing that the amounts of s, f, and x are equal to those under the merged firm

(g) **(4 points)** The price p_x that exactly internalizes the externality for the steel firm should lead us back to the Pareto optimal amount of pollution (x**). The price will place an additional cost on the steel firm identical to the tax in (f) but also add an additional source of revenue for the fishery to counteract the cost of pollution it will incur. As a result, the steel firm will essentially maximize the same profit function as in (f). Since the fishery cannot choose the amount of pollution from the steel firm, it will only choose the optimal amount of fish production and take the additional revenue as given. Therefore the price will equal the tax found in (f). I.e.

$$p_x = t = \frac{\beta\theta}{\gamma + \theta}$$

Partial credit awards

0-3 points: depending on the quality of the student's reasoning (award full credits if the student writes out the maximization problem and solves it correctly)
2 points: for getting the number p_b correct

Economics Honors Exam 2008 Solutions Question 2

(a) **(8 points)** Neither party has a dominant strategy here. Therefore, there can be no dominant strategy equilibrium.

You can see from the best responses that the best response of a player depends on the action taken by the other player.

If Driver *B* was to swerve, the best response of *A* is not to swerve.
If Driver *B* was not to swerve, the best response of *A* is to swerve.

Partial credit awards

3 points: for realizing that there is no dominant strategy equilibrium
0-5 points: depending on the quality of the reasoning

(b) **(10 points)** Yes, there are <u>two</u> PSEs. They are:

{*swerve*, *don't swerve*} and {*don't swerve*, *swerve*}

You can find these by finding the best responses of each party to each of the other party's actions.

Partial credit awards

3 points: for getting that there are two PSEs
3.5 points: for getting the first PSE correct
3.5 points: for getting the second PSE correct

(c) **(12 points)** Yes, there is a mixed strategy equilibrium.

If Driver *B* plays *swerve* with probability p, then Driver *A* has the following payoffs:

Payoff (A plays *swerve*) = 2
Payoff (A plays *don't swerve*) = $6p$

In equilibrium the payoffs must be equal (otherwise Driver *A* would choose the action with the highest payoff)

$$6p = 2$$

Therefore, $p = \frac{1}{3}$

This implies that in equilibrium:

Driver *B* plays *swerve* with probability $\frac{1}{3}$
Driver *B* plays *don't swerve* with probability $\frac{2}{3}$

Driver *A* plays *swerve* with probability $\frac{1}{3}$
Driver *A* plays *don't swerve* with probability $\frac{2}{3}$

Partial credit awards

3 points: for getting that there is one mixed strategy equilibrium
5 points: for getting the methodology correct
4 points: for getting the answers correct (**1 point** for each probability value)

Economics Honors Exam 2008 Solutions Question 3

(a) (10 points) The production function $Q = L^{1/6}H^{1/3}K^{1/2}$ has decreasing marginal rates of substitution, and so the cost minimizing inputs are strictly positive for positive output Q. We have

$$dQ/Q = \frac{1}{6}dL/L + \frac{1}{3}dH/H + \frac{1}{2}dK/K \quad \text{and} \quad dC = w_L dL + w_H dH + r dK.$$

For the producer with fixed $K = \overline{K}$, we cannot vary dK. Applying the principle of variation to a variation (dL, dH), we conclude that at a cost minimizing input vector (L, H),

$$6w_L L = 3w_H H \text{ and the variable cost is } C_{var} = w_L L + w_H H.$$

Hence at the optimum,

$$L/H = 3w_H/6w_L,\ w_L L = \frac{2}{6}C_{var} \text{ and } w_H H = \frac{2}{3}C_{var}.$$

Furthermore

$$\begin{aligned} Q &= L^{1/6}H^{1/3}\overline{K}^{1/2} \\ &= \overline{K}^{1/2}(2C_{var}/6w_L)^{1/6}(2C_{var}/3w_H)^{1/3} \\ &= (2\overline{K})^{1/2}C_{var}^{1/2}(6w_L)^{-1/6}(3w_H)^{-1/3}. \end{aligned}$$

Hence

$$C_{var} = \frac{1}{2\overline{K}}Q^2(6w_L)^{1/3}(3w_H)^{2/3},$$

$$\begin{aligned} C(Q; w_L, w_H, r) &= r\overline{K} + \tfrac{1}{2\overline{K}}Q^2(6w_L)^{1/3}(3w_H)^{2/3} && \text{if } Q > 0, \\ &= 0 && \text{if } Q = 0. \end{aligned}$$

For $\overline{K} = 1$ and the input prices $w_L = 1$, $w_H = 2$, $r = 27$, we have

$$\begin{aligned} C(Q) &= 27 + 3Q^2 && \text{if } Q > 0, \\ &= 0 && \text{if } Q = 0. \end{aligned}$$

Partial credit awards

2 points: for getting the total differential equations correct
1 point: for realizing that K is fixed
2 points: depending on the quality of the solution procedure
1 point: for getting the cost minimizing ratio $L(Q; w_L, w_H, r)/H(Q; w_L, w_H, r)$ correct

1 point: for getting the expression of the general cost function $C(Q; w_L, w_H, r)$ for $Q > 0$ correct

1 point: for realizing that cost is zero when $Q = 0$

2 points: for specializing the cost function to the given input prices correctly

(b) (7 points) Note that the cost function is convex, so the producer has decreasing returns to scale. The producer maximizes profit $\Pi = pQ - C(Q)$. In the range $Q > 0$, the maximum occurs where $p = C'(Q) = 6Q$, or $Q = p/6$. At this Q, $\Pi = p^2/6 - 27 - 3p^2/36 = p^2/12 - 27$.If $\Pi < 0$ the optimal output is $Q = 0$. Therefore

$$\begin{aligned} Q_{opt}(p) &= p/6 \quad \text{if } p \geq 18, \\ &= 0 \quad\quad \text{if } p < 18. \end{aligned}$$

$$\begin{aligned} \Pi_{opt}(p) &= p^2/12 - 27 \quad \text{if } p \geq 18, \\ &= 0 \quad\quad\quad\quad\quad \text{if } p < 18. \end{aligned}$$

When $p = 18$, the producer is indifferent between not producing and producing the quantity $18/6 = 3$.

Partial credit awards

1 point: for realizing that profit is revenue minus cost

1 point: for realizing that profit is maximized where marginal revenue equals marginal cost

1 point: for getting the profit function correct

1 point: for realizing that the optimal output is zero when the maximized profit (at positive output) is negative

1 point: for getting the optimal output function correct

1 point: for getting the optimal profit function correct

1 point: for getting the output price p at which the producer will be indifferent between producing and not producing correct (0.5 points for getting the idea correct)

(c) (3 points) The long-run equilibrium consists of producers described above, and a price p which reduces these producers to zero economic profit. Hence, by part (b), $p = 18$ and those producer that remains in the market each produces the output $Q = 3$. In the equilibrium, we also have Supply=Demand. If the demand curve is $p + Q_{demand} = 1050$ and there are N producers, then $18 + 3N = 1050$. So $N = 344$.

Partial credit awards

1 point: for getting the long-run equilibrium price correct

1 point: for getting the long-run equilibrium output per shop correct

1 point: for getting the long-run equilibrium number of shops correct

Note: 0.5 points for getting the ideas right for each point above

(d) (10 points) The production function $Q = L^{1/6}H^{1/3}K^{1/2}$ has decreasing marginal rates of substitution, and so the cost minimizing inputs are strictly positive for positive output Q. We have

$$dQ/Q = \frac{1}{6}dL/L + \frac{1}{3}dH/H + \frac{1}{2}dK/K \quad \text{and} \quad dC = w_L dL + w_H dH + r dK.$$

Applying the principle of variation, we conclude that at the cost minimizing inputs (L, H, K)

$$6w_L L = 3w_H H = 2rK \quad \text{and} \quad w_L L + 3w_H H + 2rK = C$$

Hence

$$w_L L = \frac{1}{6}C, w_H H = \frac{1}{3}C, rK = \frac{1}{2}C. \quad \text{and} \quad Q = L^{1/6}H^{1/3}K^{1/2}.$$

We deduce

$$(L : H : K) = (\tfrac{1}{6w_L} : \tfrac{1}{3w_H} : \tfrac{1}{2r}),$$
$$C_{LR}(Q; w_L, w_H, r) = Q(6w_L)^{1/6}(3w_H)^{1/3}(2r)^{1/2}.$$

In the long-run equilibrium, economic profit is zero. Hence

$$\begin{aligned} pQ &= C_{LR}(Q; w_L, w_H, r) \quad \text{and} \\ p_{\text{LR Equilibrium}} &= (6w_L)^{1/6}(3w_H)^{1/3}(2r)^{1/2}. \end{aligned}$$

For the input prices $w_L = 1$, $w_H = 2$, $r = 27$, we have $p = (6)^{1/6}(3.2)^{1/3}(2.27)^{1/2} = (6)^{1/6}(6)^{1/3}(6)^{1/2}(9)^{1/2} = 6.3 = 18$. With the market demand curve $p + Q_{demand} = 1050$, the equilibrium quantity is $Q = 1050 - 18 = 1032$. But we cannot determine what each producer produces. The long-run technology (when all three inputs are variable) has constant return-to-scale, so the same output can be produced by one shop using input (L, H, K) or N shops each using $(L/N, H/N, K/N)$. Furthermore, in the long run equilibrium a producer with constant return-to-scale is indifferent to what quantity it produces - whatever it is, the profit is zero.

Partial credit awards

1 point: for getting the total differential equations correct
1 point: for applying the principle of variation to solve the total differential equations
1 point: for getting the long-run cost function $C_{LR}(Q; w_L, w_H, r)$ correct
1 point: for getting the cost-minimizing input ratio $(L : H : K)$ correct
1 point: for using the $pQ = C_{LR}$ condition to solve for long-run equilibrium price
1 point: for getting the expression for the long-run equilibrium price p correct
1 point: for getting the number for the long-run equilibrium price p correct

1 point: for getting the long-run equilibrium quantity Q correct (0.5 points for getting the idea right)

1 point: for pointing out that the long-run equilibrium output per producer is undetermined

1 point: depending on the quality of reasoning

Economics Honors Exam 2008 Solutions Question 4

(a) (10 points) Atlantis has demand curve

$$\begin{aligned} q_A = & \quad \sqrt{(100/p_A)} - 1 \quad & \text{if } p_A \leq 100, \\ q_A = & \quad 0 & \text{if } p_A > 100. \end{aligned}$$

For $0 \leq p_A \leq 100$, we have $p_A(q_A+1)^2 = 100$. Since cost is zero, profit Π =revenue= $p_A q_A = 100 q_A (q_A+1)^{-2}$. Profit is maximized when $d\Pi/dq_A = 100(q_A+1)^{-3}\{(q_A+1)-2\} = 0$. So All-Tel's optimal output is $q_A = 1$ million and the optimal price is $p_A = 100(q_A+1)^{-2} = 25$. Its profit is then $\Pi = p_A q_A = 25$ million.

Total Surplus in this market =
$\int_0^1 p_A dq_A = \int_0^1 100(q_A+1)^{-2} dq_A = 100\left[-(q_A+1)^{-1}\right]_0^1 = 50$ million.

Partial credit awards

1 point: for realizing that profit is equal to revenue
1 point: for expressing profit in terms of quantity correctly
1 point: for realizing that profit is maximized when $d\Pi/dq_A = 0$
1 point: for getting the expression for $d\Pi/dq_A$ correct
1 point: for getting the optimal output correct
1 point: for getting the optimal price correct
1 point: for getting the profit correct
2 points: for solving for total surplus with the correct formula (1 point for noting that the higher limit of the integral is 1 instead of, say, infinity)
1 point: for getting the total surplus value correct

(b) (5 points) Bestland has the demand curve

$$\begin{aligned} q_B = & \quad 120 \quad & \text{if } p_B \leq 1, \\ q_B = & \quad 0 & \text{if } p_B > 1. \end{aligned}$$

Profit is maximized by setting $p_B = 1$ and $q_B = 120$ million. Profit = Revenue = Total Surplus = $p_B q_B = 120$ million.

Partial credit awards

1 point: for realizing that profit is maximized at the maximum price that can be charged without demand falling to zero
1 point: for getting the optimal price correct
1 point: for getting the optimal quantity correct
1 point: for getting the profit correct
1 point: for getting the total surplus correct

(c) (8 points) If $p > 1$, demand and hence profit from market B would be 0. Since profit from market A cannot exceed its total surplus 100 and market B

alone can generate profit 120, $p > 1$ cannot be optimal for the joint monopoly. On the other hand, suppose $p < 1$. Note that $\Pi_A = p_A q_A = p_A(\sqrt{(100/p_A)} - 1)$. Hence $d\Pi_A/dp_A = (\sqrt{(100/p_A)} - 1) - \frac{1}{2}10p_A(p_A)^{-3/2} = 5(p_A)^{-1/2} - 1 < 0$, for all $p_A < 25$; so Π_A is decreasing in p_A in that range. Hence $\Pi_A(p) < \Pi_A(1)$ and $\Pi_A(p) < \Pi_A(1)$ for $p < 1$. So $p < 1$ cannot be optimal either. Thus $p = 1$ is the optimal price for the joint market. In that case $q_A = \sqrt{(100/p)} - 1 = 10 - 1 = 9$ million and $\Pi_A = 9 \cdot 1 = 9$ million. For $p_B = p = 1$, $q_B = 120$ million and $\Pi_B = 120$ million. The combined profit is $\Pi = \Pi_A + \Pi_B = 9 + 120 = 129$ million.

Partial credit awards

1 point: for realizing that optimal price cannot be smaller than 1
0-2 points: depending on the quality of the reasoning
1 point: for realizing that optimal price cannot be greater than 1
0-2 points: depending on the quality of the reasoning
1 point: for getting the combined output correct
1 point: for getting the combined profit correct

(d) (7 points)

How much (at most) would BuzzCom bid for All-Tel?

Once All-Tel is merged with BuzzCom, the annual proft of the joint monopoly would be 129. But BuzzCom by itself can generate profit 120. So the economic profit of the merger for BuzzCom is only $129 - 120 = 9$. Hence BuzzCom would bid at most the present value of $9 = 9(\frac{1}{(1.05)} + \frac{1}{(1.05)^2} + ...) = 9/(0.05) = 9 \times 20 =$ 180 million.

How much (at most) would CapItal bid for All-Tel?

If CapItal buys All-Tel, it will continue to generate the profit 25 every year. The present value of that is $25(\frac{1}{(1.05)} + \frac{1}{(1.05)^2} + ...) = 25/(0.05) = 25 \times 20 = 500$ million. So CapItal would be willing to bid up to that value and be the winner.

Partial credit awards

1 point: for realizing that the annual addition to BuzzCom's profit is $129 - 120 = 9$
1 point: for getting the DPV formula correct
1 point: for getting the number (180 million) correct
1 point: for realizing that the annual addition to CapItal's profit is 25
1 point: for getting the DPV formula correct
1 point: for getting the number (500 million) correct
1 point: for identifying CapItal as the winner