General Examination in Microeconomic Theory

SPRING 2014

You have **FOUR** hours. Answer all questions
Those taking the FINAL have **THREE** hours

Part A (Glaeser): 55 minutes
Part B (Maskin): 55 minutes
  Part C (Hart): 60 minutes
  Part D (Green): 70 minutes

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.
Glaeser (Part A)

In Country A, richer families are observed to have more children. In country B, richer families are observed to have fewer children.

(1) Produce a simple model that connects family income with fertility levels. Derive the comparative static connecting fertility with earnings. Discuss the result.
(2) Compare the impact on fertility of unearned income, male earnings and female earnings.
(3) How is the fertility decision, and its connection with income, impacted by the quality and availability of free public schooling?
(4) How is the fertility decision, and its connection with income, impacted by expected lifespan of the children?
(5) List four hypotheses that can potentially explain the differences between Country A and Country B. Discuss what data would be useful in determining the validity of the hypotheses.
General Exam Question- May 2014

Consider a two-player game (plus nature) in which (i) nature first chooses $U$ or $D$ with corresponding probabilities $\frac{3}{4}$ and $\frac{1}{4}$; then (ii) player 1 sends a “signal” $u$ or $d$ to player 2; and finally (iii) players play a normal form game in which the payoffs are

- **Table 1**

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<td>T</td>
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<td>B</td>
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if nature chose $U$

- **Table 2**

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if nature chose $D$

(A) Suppose that nature’s choice is common knowledge between the players. Find all Nash equilibria (including those in mixed strategies) of this game.

(B) Continue to suppose that nature’s choice is common knowledge. But now assume that, after nature has made its choice and player 1 has sent her signal, either the game of Table 1 (if nature chose $U$) or the game of Table 2 (if nature chose $D$) is repeated infinitely many times and that players maximize the sum of their discounted payoffs. Find the set $V^*$ of payoff pairs such that, for any $(v_1, v_2) \in V^*$, there exists a discount factor $\delta > 0$
and a sub-game perfect equilibrium of the repeated game for which the discounted average payoffs are \((v_1, v_2)\) when players discount using \(\delta\). Sketch the equilibrium strategies that attain a typical point \((v_1, v_2) \in V^*\).

(C) Let us revert to assuming that the games of Table 1 and Table 2 are played just once after nature moves and player 1 sends her signal. But now suppose that only player 1 gets to observe nature’s choice (player 2 knows just the probabilities ¾ and ¼), and that this information structure is common knowledge. Find all pure-strategy perfect Bayesian equilibria of this modified game. (Hint: there are 4 such equilibria)
1. (A) Consider a two consumer, two good exchange economy. Consider 1’s utility function is $x_{11} + x_{21}$, while consumer 2’s is $- (x_{21} - 1)^2 - (x_{22} - 1)^2$. There are two units of each good. Using a diagram or otherwise characterize the set of Pareto optima for this economy. Can all these Pareto optima be decentralized as Walrasian equilibria with transfers?

(B) Consider a two date, one good, two consumer exchange economy with consumption only at the second date. There are two states of the world. Consumer 1 believes that state one occurs with probability $\frac{3}{4}$, while consumer 2 believes that it occurs with probability $\frac{1}{2}$. Each consumer’s (von Neumann-Morgenstern) utility function is given by $\log x$. Each consumer has an initial endowment of one unit of the good in each state of the world. Compute the Arrow-Debreu equilibrium. Is there a sense in which this equilibrium is Pareto optimal? Is there a sense in which it is not?

2. A seller $S$ can provide a good to a buyer $B$ at date 1. The parties meet and contact at date 0 and $S$ invests $e$ at date $\frac{1}{2}$ at personal cost $\frac{1}{2} e^2$. The value of the good to $B$ is $v + e$. (So $S$’s investment is quality-improving.) $S$’s cost at date 1 is zero. There is no discounting and there are no wealth constraints.

(A) What is the first-best level of $e$?

(B) Suppose now that $S$’s choice of $e$ is observable but not verifiable. $B$ and $S$ bargain over the price of the good at date 1, splitting the gains from trade 50:50. (The gains from trade equal the value of trade at date 1 minus the sum of the parties’ outside options. Sunk costs are, of course, ignored.) Assume that the parties’ outside options are zero at date 1. What is the second-best level of $e$?

(C) Suppose now that an asset can be allocated to $B$ or $S$. If $B$ owns the asset his outside option is $v$ (and $S$’s is zero). If $S$ owns the asset her outside option is $(1/2)e$ (and $B$’s is zero). Compare total surplus under $B$ and $S$ ownership? Which is higher? Explain.

(D) Consider the same situation as in (C), but suppose now that the date 1 bargaining process is costly: a fraction of the gains from trade are lost. Can you see any reason why it might be optimal for $B$ to own the asset? (This part of the question is verbal.)
Question D1

Voting on Incomplete Sets of Pairs

15 points

There is a set of $n$ alternatives $X$. The central authority polls individuals on their preferences regarding pairs of alternatives in $X$. However, because $n$ is large there is not time to ask people for their preferences on all $\frac{n(n-1)}{2}$ pairs. The authority selects a family $Y$ of pairs $(x, x') \in Y \subset X \times X$, with $x \neq x'$ and records the result of a majority election between each such pair.

Assume that the votes on pairs in $Y$ do not reveal any cycles of majority rule. That is, there is no sequence $x_1, \ldots, x_k$ where $x_i$ defeats $x_{i+1}$, $x_k = x_1$ and $(x_i, x_{i+1}) \in Y$ for $i = 1, \ldots, k-1$.

a) Show that there is at least one complete strict order (anti-symmetric, complete, and transitive) on all the pairs in $X \times X$ that agrees with the observed majority preferences on $Y$. [6]

b) Now let us look at the situation facing the central authority before the results of the pairwise votes on $Y$ are known, but after $Y \subset X \times X$ has been selected. Suppose that $Y$ cannot reveal any voting cycle because there is no sequence $x_1, \ldots, x_k$ with $(x_i, x_{i+1}) \in Y$ for $i = 1, \ldots, k-1$, and $x_k = x_1$. That is, because of the structure of $Y$, no cycles could be observed no matter what the preferences in the population might be. Use your result from part a) to show that for any voting results on pairs in $Y$ you can define a "social welfare functional" $f$ based on the observed votes on pairs in $Y$:

$f : \{-1, +1\}^Y \rightarrow P_X$

where $\{-1, +1\}^Y$ is the set of possible voting outcomes on pairs in $Y$ $[+1$ means $x_i$ defeats $x_{i+1}$ and $-1$ means $x_{i+1}$ defeats $x_i]$, and $P_X$ is the set of complete strict orders on $X$. [3]

c) How would you state a Pareto-like axiom that this social welfare functional should satisfy on its domain? Will the $f$ you have defined in part b) satisfy it? [2]

d) How would you define an axiom in the spirit of IIA in this limited information setting? Would your $f$ satisfy it? [4]
A law firm consisting of $n$ lawyers, $i = 1, \ldots, n$, has a number of clients, indexed by $k$. Each client requires the services of a subset of the firm’s lawyers, $S_k$. Moreover, in order to serve the needs of the client all the lawyers in $S_k$ must work the same amount of time, $\beta_k$ (hours per year). If the needs of the client are met, the firm is paid an amount $y_k$ by the client. No client $k$ needs more hours than $\beta_k$ from any lawyer who works for it, and if any lawyer does not work that amount of time the client is dissatisfied and does not pay at all. Each lawyer can work for only one client at a time.

All the lawyers have the same amount of time available each year – a fixed number of hours. It so happens that every member of the firm is fully employed, exactly using all of their hours as they fulfill the needs of all of the clients that require their services. No one has any extra time. Thus the law firm has \( \sum_k y_k \) to distribute to its lawyers as their total annual compensation.

a) Write the cooperative game that best models this situation. What is the worth of coalitions composed of a subset $S$ of lawyers who are not one of the subsets $S_k$ that any particular client requires? [4]

b) The firm decides to use the Shapley value of this game to determine the individual compensation of its lawyers. What are these compensation levels? [6]

c) Someone suggests that the nucleolus be used instead of the Shapley value. What is it? [6]

d) Someone else suggests that the core be used. Is the core empty? Is the nucleolus in the core? Are there other points in the core? [4]
Question D3

Bargaining and Risk Aversion

20 points

Two people have the opportunity to share one dollar. There are three non-stochastic sharing options that they can agree upon:

- \( a \) – person 1 gets the entire dollar
- \( b \) – person 2 gets the entire dollar
- \( c \) – person 1 gets 0.2 and person 2 gets 0.8

There are no other non-stochastic ways to share the dollar that are available to them – dividing the dollar in any proportions other than 100%, 20% or 0% for person 1 is not possible. However, the two people can make an agreement to randomize among the three options \( a \), \( b \), and \( c \) using any probabilities.

If they do not make any agreement at all, both players get zero. They are each expected utility maximizers. Their Bernoulli utility functions are \( u \) and \( v \) respectively.

a) For this part of the problem, assume that \( u \) and \( v \) are both very slightly risk-averse, but almost risk neutral. This means that, for player 1, \( c \) is as good as a randomization between \( a \) and \( b \) in which \( a \) gets a probability weight of just slightly over .2 and, for player 2, \( c \) is equivalent to a randomization between \( a \) and \( b \) in which \( a \) gets a probability weight of just slightly under .2. What outcomes, non-stochastic or randomized, will these two people agree upon when they choose the Relative Utilitarian solution? [2]

b) What is the Nash Bargaining solution at the situation described in part a)? Again, express your answer both as a combination of utilities (approximately) and in real terms, as a non-stochastic choice from \( \{a, b, c\} \), or a probabilistic mixture of \( a \), \( b \), and \( c \). [3]

c) Now suppose that player 1 becomes significantly more risk averse while player 2’s utility remains only slightly risk averse as in part a). For player 1, option \( c \) is now indifferent to a mixture of \( a \) and \( b \) with probabilities .4 and .6 respectively. What are the Relative Utilitarian and Nash Bargaining Solutions now? [3]

d) Show that as player 1 becomes increasingly more risk averse there will come a point at which the Relative Utilitarian and Nash solutions coincide. [3]

e) What happens when 1 is even more risk averse than the level found in part d)? [2]

f) Suppose that player 1’s utility is CRRA: \( u = \frac{1}{1-\rho}x^{1-\rho} \) with \( 0 < \rho < 1 \), and \( x \) is the realized level of payoff in dollars. What is the value of the risk aversion coefficient (approximately) at which these two solutions coincide? [2]

g) How would these bargaining solutions deal with a player 1 whose CRRA utility is described by the same formula but with \( \rho > 1 \)? [2]

h) Explain the behavior of these two bargaining solutions in terms of the logical foundations of what they are trying to achieve. [3]
Question D4

Incentives in Dominant and Bayesian Implementations

15 points

Consider an incentive problem with $n$ agents who must take a collective decision $k \in \mathbb{R}_+$ and may make a vector of monetary transfers $t = (t_1, ..., t_n)$. We will not place any restrictions at all on $t$ as far as feasibility is concerned – for example, there may be a net transfer to or from the mechanism designer. Every agent has a utility of the form

$$u_i(k, \theta_i) + t_i = \theta_i v(k) + t_i$$

where $\theta_i$ is a privately observed parameter taking values in $[\theta_{\text{min}}, \theta_{\text{max}}]$ and $v_i(k)$ is a positive, increasing, concave function of $k$. We will not make any assumptions at all on the participation decision of these agents – effectively, participation can be compelled.

**True or False** (You may use any results from the course or MWG if you state them clearly and explain as necessary):

a) If the decision function $k(\theta)$ is Bayesian implementable then $k(\theta)$ is also implementable in dominant strategies, although perhaps by a mechanism using a different system of monetary transfers. [6]

b) The interim utilities $U_i(\theta_i)$ arising from any mechanism that implements $(k(\theta), t(\theta))$ in Bayesian strategies can be an increasing and strictly concave function of $\theta_i$. [5]

c) For this part of the question use the restriction that $\sum_i t_i(\theta)$ must be non-negative at every realization of the mechanism in use.

The value of $\sum_i EU_i(\theta_i)$ for any dominant strategy mechanism must be strictly lower than the realized value of $\sum_i EU_i(\theta_i)$ in the Bayesian mechanism that achieves the maximum possible ex ante utility. [4]