HARVARD UNIVERSITY
DEPARTMENT OF ECONOMICS

General Examination in Microeconomic Theory

FALL 2011

You have **FOUR** hours. Answer all questions

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Part A: Verbal Problem (Glaeser)

For credit you ***must*** give a mathematical answer or justification. In other words, you are expected to write down a formal mathematical model of the problem and explain your results, making whatever assumptions you see reasonable and necessary. Credit and partial credit is given for each part, so if you get stuck move on and do your best to answer parts 1-5.

Individuals live at location A and work in location B. Assume that location is entirely fixed. There is a technology that transports any number of individuals at a fixed cash cost K and fixed time cost T from A to B.

There is a second technology (roads) that allow people to travel from A to B for a time and cash cost. The time cost is a decreasing function of the investment in roads and an increasing function of the number of people using the roads. You are asked to consider several different conditions for the cash cost below (you may assume everyone owns a car).

(1) Derive and interpret a condition for socially optimal investment in and pricing of roads.

(2) Assume roads are competitively provided and derive the competitive price and investment. Will the roads require a tax subsidy to reach the social optimum of part (1)? Explain intuitively.

(3) Now assume the government mandates roads are free. Derive the optimal investment condition in roads. Explain intuitively what has changed.

(4) Derive the pricing and investment condition if there is a single road provided by a private monopolist. Explain intuitively what has changed.

(5) Under what conditions will (3) or (4) yield socially preferable outcomes? Under what conditions will (3) or (4) yield more investment?

(6) Extra Credit: How will your answer to (2) change during a recessions.

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Part B

Question 1

a) Consider a two-player non-zero sum game in normal form. Player1 gets a new strategy that can be played, that is, we add a new row to the payoff matrix. Give an example to show that player 1.s equilibrium payoff can decrease as a result of this increased .fexibility in strategic choice.
b) Consider a two-player zero sum game. Again assume that Player 1 gets a new strategy that can be played. What can happen to player 1'.s equilibrium payoff? Can it decrease? Why or why not?

Question 2

There are two players, 1 and 2, who simultaneously choose either Heads (H) or Tails (T). The payoffs resulting from these moves are not completely known to them in advance. There are two equally likely possibilities for the payoffs.The payoffs will either be as in Matching Pennies, in which case player 1 wants to match and player 2 wants to mismatch, with the winner receiving +1 from the loser; or they will be as in a Coordination Game in which both players receive +1 if they both play Heads and 0 if either one or both play Tails.
Before making their choice between H and T the two players receive private information about which game is actually being played. This information takes the form of one of two observations, "C" or "M". These two observations are statistically related to the true game being played as follows:
If the Coordination Game governs the payoffs, both players observe C with probability 1.
If Matching Pennies governs the payoffs, then the observations they receive are a pair of jointly distributed random variables. With probability 1/3 they receive each of the observations (C,M), (M,C), or (M,M) respectively -- the first symbol being the private observation of player 1 and the second being the observation of player 2.

a) This is a Bayesian Game. Using the structure and payoffs as specified in the description above, define the strategy sets from which the players choose and the requirements for strategies to form a Bayesian Nash equilibrium.

b) Show that the strategies in which both players play H when they see the C signal and T when they see the M signal is a Bayesian Nash equilibrium of this game.

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1. (i) Show that a Walrasian equilibrium is weakly Pareto optimal: it is impossible to make everyone strictly better off. Give an example of a Walrasian equilibrium that is weakly Pareto optimal but not strongly Pareto optimal: it is possible to make some people strictly better off without making others worse off.

   (ii) Consider an exchange economy with two consumers and two goods. Consumer 1 has utility function $x_1^{\frac{1}{4}} \quad x_2^{\frac{3}{4}}$ and consumer 2 has utility function $x_1^{\frac{1}{8}} \quad x_2^{\frac{7}{8}}$. Consumer 1's endowment is (α,α) and consumer 2's is $(1-\alpha,1-\alpha)$. Compute the Walrasian equilibrium. Show that the price of good 2 relative to good 1 is decreasing in α. Interpret the result.

2. A risk neutral principal can hire a risk neutral agent to carry out a task. The agent can exert effort or not. With effort the task succeeds with probability $\frac{3}{4}$ and fails with probability $\frac{1}{4}$. Without effort the task succeeds with probability $\frac{1}{2}$ and fails with probability $\frac{1}{2}$. The principal's revenues $=\Sigma$ if the project succeeds, and zero if it fails. Effort causes the agent a disutility equal to c. (No effort is costless.) The agent's consumption cannot fall below zero in any state of the world.

   (i) Suppose the agent has zero wealth. Write down the conditions for an incentive scheme to elicit effort from the agent. What is the lowest cost incentive scheme for the principal that satisfies these conditions?

   (ii) For what values of Σ and c is it worthwhile for the principal to elicit effort from the agent? Compare this to the first best outcome.

   (iii) Suppose that the agent has positive wealth A. How do your answers to (i) and (ii) change?

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Part D

Question 1

There are n individuals and m alternatives, with m > n > 2. The domain for preferences is the set of all possible n strict orders over the m alternatives. Consider the following method of constructing a social welfare function,
Player 1's favorite alternative is declared to be the socially best alternative. This alternative is then deleted from further consideration, leaving m 1 others. Next, Player 2's favorite alternative from among these m - 1 is declared to be the second-best socially ranked alternative. It is then deleted, leaving m-2 others. In each step the next player in numbered order determines the next alternative in the social order. When all n players have had a chance and n alternatives have been ranked, the choice reverts to Player 1, whose best alternative among those remaining becomes the $(n + 1)^{st}$ socially ranked alternative, etc.

(a) Does this procedure satisfy the Paretian axiom of Arrow's Theorem? Explain.

(b) Does it satisfy the Pairwise Independence (or, Independence of Irrelevant Alternatives) axiom of Arrow's Theorem? Explain, giving a proof if you think it does or a counterexample if you think it does not.

Question 2

There are seven people who need to get from the airport to downtown. There are exactly three taxis at the airport. Each taxi can hold up to three people. Each person has a willingness to pay of $7 for a taxi ride. The cost to the taxi driver of providing a ride is $6, irrespective of how many people are in the taxi. All ten people (the seven passengers and the three drivers) can freely negotiate the transportation arrangements and the monetary transfers (i.e. who rides in which taxi and how much each pays).
Now imagine that a fourth taxi arrives. The fourth taxi is superfluous in the sense that it is efficient for no one to ride in it.

Without computing the Shapley values of the two games, show that the arrival of the fourth taxi lowers the Shapley value of each taxi driver and raises the Shapley value of each passenger.