General Examination in Microeconomic Theory

Fall 2005

1. You have FOUR hours.

2. Answer all questions

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.
News stories are supplied by firms that invest in acquiring information and then decide how to present that information. News stories are consumed by readers who pay for those stories. News stories can be presented in an unbiased way or a way that is biased.

In this problem, you are going to craft a model of the determinants of bias. No points will be given for any verbal answers without algebraic derivations, except for sub-question (7).

1. First, begin by defining news stories in a way that permits them to be biased. Produce a natural way of capturing the amount of bias in a new story presented by the firm.

2. Write down the consumer's problem. Incorporate a demand for news and the role that bias might play in that demand. You might want to think about the differences involved in news as consumption vs. news as knowledge.

3. Produce something like a demand curve that will translate the firms choices about information and sales into revenues.

4. Write down the producer's problem. You might want to start with a monopolist. Consider the potential biases of people who fund and/or work for news firms and connect this intellectually with the Becker discrimination model.

5. Extend the producers problem to a multi-firm setting and define a market equilibrium.

6. Generate key comparative statics on the degree of bias in the market. Consider the role of changes in cost technologies, changes in the preferences of consumers and/or suppliers, changes in the size of the market. Your ability to produce coherent comparative statics will effectively determine most of your grade for this question.

7. Briefly (no model is necessary), discuss the welfare consequences of your comparative statics. Do the welfare consequences depend on the factors that drive bias?
Consider the following signalling game. Nature moves first, and chooses player 1’s type \( \theta \), which is either \( \theta^\prime \) (with probability \( p \)) or \( \theta'' \) (with probability \( 1-p \)). Player 1 observed Nature’s move, and chooses an action \( a_i \in \{U,D\} \).

Player 2 sees 1’s move but not 1’s type, and chooses \( a_2 \in \{L,R\} \).

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(a) Show that this game does not have a separating Perfect Bayesian Equilibrium (PBE).

(b) For what values of \( p \) does the game have a pooling PBE?

(c) For what values of \( p \) does the game have a PBE in which type \( \theta'' \) plays \( U \) with probability 1 and type \( \theta' \) assigns strictly positive probability to both actions?
Micro General Exam Fall 2005 – Part C

There is one question in this part of the exam; answer all four parts of the question. The
parts will be weighted equally. You may use any results proved in class, section, problem
sets, notes, or texts, with appropriate citations, provided the question is not directly asking
you to prove this result.

C1. Consider an exchange economy with $L$ goods and $m$ consumers in which each consumer
has a preference order $\succeq_i$ that is continuous, strongly monotone, and strictly convex,
and an initial endowment $\omega_i \in \mathbb{R}^L_+$. Assume that the aggregate endowment $\omega$ is strictly
positive ($\omega \gg 0$). A social planner implements a progressive tax scheme in which agents
receive transfers proportional to the difference between their wealth and the average
wealth in the population of the following form:

$$T_i(p) = \alpha \left( \frac{p \cdot \omega}{m} - p \cdot \omega_i \right)$$

where $\alpha \in [0, 1]$ is a fixed constant.

An equilibrium with progressive taxes is a feasible allocation $(x_1^*, \ldots, x_m^*)$ and a price
vector $p^*$ such that each $x_i^*$ is optimal for $\succeq_i$ over the budget set
$B_i(p^*) = \{ x_i \in \mathbb{R}^L_+ : p^* \cdot x_i \leq p^* \cdot \omega_i + T_i(p^*) \}$.

a. Show that for every $\alpha \in [0, 1]$, an equilibrium with progressive taxes exists in this
model.

b. Show that every equilibrium allocation with progressive taxes is Pareto optimal.

c. Can every interior Pareto optimal allocation be supported as an equilibrium with
progressive taxes for some $\alpha \in [0, 1]$?
If not, characterize as completely as possible the set of allocations that can be
supported as an equilibrium with progressive taxes for some $\alpha \in [0, 1]$.

question continues on next page
d. Maintaining the assumptions above, now consider an alternative tax system in which agents pay taxes based on the sales of their endowments, but not on consumption of their endowments. For an allocation $x = (x_1, \ldots, x_m)$ and price $p$, set

$$T_i^s(p, x) = \alpha \left( \frac{1}{m} \sum_{j=1}^{m} p \cdot (\omega_j - x_j)_+ - p \cdot (\omega_i - x_i)_+ \right)$$

where for any vector $y \in \mathbb{R}^L$, $y_+ = (\max\{y_1, 0\}, \ldots, \max\{y_L, 0\})$, and $\alpha \in [0, 1]$ is fixed.

An equilibrium with sales taxes is a feasible allocation $(x_1^*, \ldots, x_m^*)$ and a price vector $p^*$ such that each $x_i^*$ is optimal for $\succeq_i$ over the budget set

$$B_i(p^*, x^*) = \{x_i \in \mathbb{R}^L_+ : p^* \cdot x_i \leq p^* \cdot \omega_i + T_i^s(p^*, (x_i, x_{-i}^*)) \}.$$ 

Is every equilibrium allocation with sales taxes Pareto optimal? Either prove that this is true, or provide as rigorous an argument as you can that this is not true.
Part D - Question 1

There are seven people who need to get from the airport to downtown. All seven have the same willingness to pay for the trip: $7. (That is, if they go they are willing to pay $7, and if they don’t go, that is worth $0.) There are three taxis at the airport (and there are no other taxis that will arrive later). Each taxi can hold up to three people. The cost for the taxi to take the trip is $6, irrespective of how many people are in the taxi.

All ten people, the seven passengers and the three taxi drivers, can freely negotiate the fees and transportation arrangements (i.e. who rides in which taxi and how much they each pay).

a) What is the characteristic function for this cooperative game?

b) Define the concept of blocking for a game in characteristic function form.

c) Show that any set of fees and transportation arrangements can be blocked by some coalition.

d) Explain, without computing it, how to get the the Shapley value of this cooperative game. Do all the taxi’s get the same payoff in the Shapley value?

e) Give an intuitive argument to show that if there were a fourth taxi available (with the same capacity and cost) then the Shapley value of taxis would decline and the Shapley value of passengers would increase. (hint: think about the random order approach).
Part D - Question 2

Two people share an asset equally, in that both can use it and both have an equal share of its value when it is sold. The use of the asset is non-rivalrous in that each one's use does not reduce utility that the other receives from using it. The asset, if sold on the market, has a capital value, \( x \). The asset does not depreciate and the interest rate is zero. Thus if it continues to be used and owned by these players their total willingness to pay for it is the sum of their valuation of its use and its capital value.

Circumstances change such that one of the partners \((i = 1)\) can no longer use it (i.e. his willingness to pay for the right to use it is zero) and he must liquidate his share of its capital value. The capital value \( x \) is unknown and is distributed according to \( F \). Both parties are risk neutral. The expected value \( x \) is \( E_F \). \( x \) is a non-negative random variable distributed on the interval \([x_{\min}, x_{\max}]\).

The other partner \((i = 2)\) has a valuation \( v \) for using the asset, which is his private information. From the point of view of player 1, \( v \) has the distribution \( G \), and \( v \) is sure to be non-negative.

There is no statistical relationship between the realization of \( v \) and \( x \).

The parties must decide whether the asset should be sold on the market, or transferred to player 2. (Player 2 would then have the sole right to use the asset now and sell it for its capital value later.) (Ignore the possibility that someone else might buy only player 1's share or any other fractional share.) If it is sold, they have to decide how the proceeds should be divided. If it is transferred to player 2 they must decide how much player 2 should pay for player 1's share.

The point of this question is to evaluate the expected utilities produced by each of the following procedures, and to recommend which one of them the partners should agree to.

(i) Sell the asset for the random price \( x \) and divide the proceeds equally.

(ii) Get a benevolent, neutral outside party to choose a price \( p \) (with the knowledge of \( F \) and \( G \), but not \( v \) or \( x \)). Then player 2 has the right to buy the asset from player 1 at the price \( p \) or to decline to buy it, in which case the asset is sold for \( x \) and the proceeds are divided equally.

(iii) Get a benevolent, neutral outside party to choose a price \( p \) (with the knowledge of \( F \) and \( G \), but not \( v \) or \( x \)). Then determine the market value \( x \) (by obtaining a costless appraisal), thus removing any uncertainty about \( x \). Continue as in method (ii) with player 2 having the right to buy the asset from player 1 for a price of \( p \) or to decline to buy it, in which case the players receive \( x/2 \) each.
a) What interim expected utilities (i.e. the utilities generated by the mechanism given the private information available but before knowing the values of $x$) can be achieved by each of these three methods? (That is, for each method, what is the set of pairs of interim expected utilities that can be achieved.) For methods (ii) and (iii), how do the interim expected utilities depend on the choice of $p$?

b) Which method do you recommend? In particular, comparing methods of types (ii) and (iii), is it useful to or harmful to learn the value $x$ in advance? If you recommend (ii) or (iii), what value of $p$ should be used. Comment, using the language and concepts of mechanism design theory.