Harvard University
Department of Economics

General Examination in Microeconomic Theory

Fall 2002

You have **FOUR** hours.

Solve all questions.

The Exam has 5 parts. Each part has it's own sheet. Please spend the following time on each part.

I.	60 minutes
II.	60 minutes
III.	60 minutes
IV.	60 minutes

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Micro General Part I (60 minutes)

Question 1

A household consumes x_1 and x_2. x_1 is purchased on a market with a price p_1. x_2 cannot be purchased directly; it is made by the household out of two inputs y and z which the household can buy at prices p_y and p_z respectively. It takes two units of y and 3 units of z to make one unit of x_2. The household's initial wealth is w.

a. Define the household's indirect utility function as it depends on wealth and on the prices of the goods it can purchase (x_1, y, and z).
b. For a given utility level u*, define the household's expenditure function as it depends on these prices.
c. Show that y and z must be complements.
d. Show that x_1 and y must be substitutes.

Question 2

A consumer has the von Neumann-Morgenstern utility function:

$$u(x) = x^{\gamma} \qquad 0 < \gamma <$$

The consumer has an initial wealth $w > 0$. w can be invested in "bonds," which return R (> 1) per dollar invested, or in "stocks" which have a random return z per dollar invested, with distribution function F (z).

Assume:

$$E_F z > R.$$

a) Set up the consumer's investment problem and show the first-order condition for the optimal portfolio allocation.
b) Show that the fraction of wealth invested in "stocks" is independent of w.
c) Based on the result in b, which one of the following properties characterized the consumer's preferences for "bonds and "stocks"

☐ quasi-linearity

☐ homotheticity

☐ linear, parallel Engle curves.

Micro Question
Part II - 60 minutes

1 Consider the following signaling game. An entrepreneur, player E, currently owns all of the stock in her company. She wants to add a new project, but to do so she must get an investment of I from player V, Victor the Venture Capitalist, and the only way she can do that is by selling him an equity stake in the company.

The profitability π of her company can be either 1 or 0; this is private information for E. V's (common knowledge) prior beliefs are that $\pi = 1$ with probability p and $\pi = 0$ with probability $1-p$. Player E offers Victor an equity share s of the company, where $0 \leq s \leq 1$. Victor observes s but not π, and then either accepts or rejects the offer. If V rejects, he gets 0 and player E gets π. If V accepts, he gets $s(\pi + N) - I$ and E gets $(1-s)(\pi + N)$, where $N > I$ is the value of the new project.

a) For what values of p is there a pure-strategy perfect Bayesian equilibrium (PBE) in which both types of E offer $s = 0$ and V rejects?

b) Suppose that $pN < I - (N^2 - IN)$. Show that there are no pure-strategy PBE in which both types of E offer the same $s > 0$ and V accepts.

c) Find a sufficient condition for there to be a pure-strategy PBE in which both types of E offer the same $s > 0$ and V accepts. (Your answer should explicitly give both the strategies and the V's posterior belief function, and check that these satisfy the definition of PBE.) For extra credit, find a necessary and sufficient condition.

d) Are there pure-strategy PBE in which the two types of E make different offers and at least one of the offers is accepted?

e) Compare the PBE here to what would happen if π were public information.

Part III - 60 minutes

Fall 2002

MICRO GENERALS

PROBLEM

Consider an Arrow-Debreu exchange economy with H agents ($h = 1, .., H$), two periods ($t = 0, 1$) and S states of nature ($s = 1, .., S$). There is a unique good in every date-event. Each agent has endowment $\omega^h = (\omega_0^h, \omega_1^h, .., \omega_S^h)$ and utility

$$U^h(x_0, x_1, .., x_S) = x_0 + \beta \sum_{s=1}^{S} \gamma_s \left(x_s - \frac{1}{2}\alpha^h x_s^2 \right)$$

We normalize to 1 the price of the good at the initial date ($p_0 \quad 1$), and denote the aggregate endowment by $\omega = (\omega_0, \widetilde{\omega}) = \sum_{h=1}^{H} \omega^h$.

(a) Calculate the state prices.

(b) Calculate the equilibrium allocations.

(c) How can a social planner reallocate resources to make all agents strictly better off?

(d) Consider a stock delivering $\widetilde{x}$ units of the good at date $t = 0$. Define the gross return $\widetilde{r}_x$, the risk coefficient β_x, and the stock's risk premium. Show that there exists a linear relation between β_x and the premium.

(e) We now assume that there are 3 equiprobable states ($\gamma_s = 1/3$ for all s) and that the aggregate endowment satisfies $\widetilde{\omega} = (8, 10, 11)$. Consider the stocks $\widetilde{x} = (0, 0, 1)$ and $\widetilde{y} = (1, 0, 0)$. Which stock is the most valuable? Why?

Micro General

Part IV – 60 minutes

Consider a principal agent problem. There are N risk neutral agents and one risk neutral principal. The payoff of the principal is equal to revenues (denoted by r) minus wages paid to the agents. The principal's payoff is r- $w_1 \ldots$- w_N Revenues (r) is a random variable, the probability distribution of revenues is denoted by $f(r|e_1, e_2 \ldots e_N)$ where e_i denotes the effort of agent i. We assume that an agent's effort is unobservable to the principal. Also assume that expected value of revenues is increasing in effort level of every agent. The principal makes a take it or leave it offer of wage schedules, where the wage of agent i is a function of total revenues r, we denote wage schedules by $w_i(r)$ (obviously, wage can not depend directly on agent effort because the principal does not observe effort). The principal's objective is to maximize his expected payoff. If an agent refuses the principal's offer his payoff is zero, otherwise the payoff of agent i is his wage minus the cost of effort, w_i-$g_i(e_i)$ where $g_i(e_i)$ is an increasing function, it denotes the cost of effort.

a. Is the above problem example of moral hazard, adverse selection or both at the same time?
b. Suppose N=1, (we continue to assume that agents and the principal are risk neutral). Compare the expected payoff of the principal in the world where he can observe agent's effort to the expected payoff of the principal in the above set up where effort is unobservable. Which one is bigger? Does your answer change if the principal is risk averse?
c. Now suppose that N>1, (we continue to assume that agents and the principal are risk neutral). Describe the wage schedule that the principal would offer to agents (assuming that effort is not observable). Now suppose that effort is observable how would it change expected payoff of the principal and effort levels of agents? Would your answer change if the principal where risk averse?
d. Formulate the Gibbard-Satterthwaite theorem. Is it applicable to the principal-agent problem described above? Why or why not?