HARVARD UNIVERSITY
DEPARTMENT OF ECONOMICS

General Examination in Microeconomic Theory

Spring 2013

You have **FOUR** hours. Answer all questions
Part A(Glaeser): 55 minutes
Part B (Maskin): 55 minutes
Part C (Hart): 60 minutes
Part D (Green): 70 minutes

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Part A: Verbal Problem (Glaeser, 55 points)

For credit you must give a mathematical answer or justification.

Consider an economy with two goods: bread and magic beans. To produce magic beans, a vast factory must be built, requiring K units of labor. After the initial fixed cost, the beans can be produced for C units of labor indefinitely. Bread is produced individually using a constant returns to scale technology.

Answer the following questions by crafting a static equilibrium model of a closed economy. Aside from part 2, assume there is a single factory that acts as a monopolist. For parts 1-3 you may assume that there is a fixed time budget which can only be used at work. Part 4 asks you to extend the model to allow for a labor supply margin.

1. Characterize the market equilibrium price of magic beans and bread, the wage rates, profit levels, and welfare levels, assuming that the factory is owned by the public. Compare equilibrium prices and quantities with socially optimal prices and quantities.

2. Would a second factory have an incentive to enter, assuming the already-built first factory cannot commit to price and quantity before the second factory enters? Show how your answer differs depending on whether the factories compete by setting prices (Bertrand) or quantities (Cournot).

3. The government would like to intervene to improve welfare and has asked you to evaluate three different policies: (i) a price ceiling, (ii) a production subsidy, and (iii) public ownership. How do the welfare implications of the three policies differ, assuming that the government has access to lump sum taxes? Compare each policy to the social optimum.

4. How would your answer to part 3 change if the government finances its operations with a linear tax on bread production? Assume that the government also had to spend "G" on general operations, again funded with a linear tax on bread production. For this part, assume that individuals have a taste for leisure or equivalently a disutility of effort.

Generals Question: Game Theory

Suppose that a worker proposes a wage w to a potential employer, and the employer then accepts or rejects the proposal. If the proposal is accepted, the worker's payoff is w and the employer's payoff is $\theta - w$, where $\theta > 0$ (θ is the worker's contribution to the employer's profit). If the proposal is rejected, each player's payoff is 0.

(a) Assuming that the value of θ is common knowledge, find the set of Nash equilibrium payoff pairs. (Don't forget about possible mixed-strategy equilbiria, including those where both players randomize). Find the set of subgame-perfect equilibrium payoff pairs.

(b) Suppose that the above game is repeated infinitely often, that future repetitions are discounted by a factor δ $(0 < \delta < 1)$, and that players maximize the discounted sum of their payoffs from the repetitions. Find the set of subgame-perfect equilibrium payoff pairs. Briefly sketch the equilibrium strategies that sustain a typical payoff pair. (Make sure that the strategies form an equilibrium off the equilibrium path).

(c) Now modify the game so that the worker still knows the value of θ, but the employer knows only that $\theta = 1$ or $\theta = 2$ (each with probability 1/2). Find the set of perfect Bayesian equilibrium payoffs.

(d) Consider the game of part (c) but suppose that before she makes her proposal, the worker has the opportunity to acquire "education" at a cost to her of $2/\theta$. Suppose that the employer can observe whether or not the worker has acquired education. Find the set of separating perfect Bayesian equilibrium payoffs.

Economics 2010b
Final Exam
Spring 2013

1. (a) Consider an exchange economy with I consumers and L goods. The preferences of consumer i are given by $\succsim_i$ and consumer i's endowment is $\omega_i \in R^L_+$. Suppose $(p^\star, x^\star)$ is a Walrasian equilibrium of this economy, where $p^\star \in R^L$. Which of the following statements is true (and why)? If there are simple extra conditions required for the statement to be true, say what are they.

(i) $x_i \succ_i x_i^\star \Rightarrow p^\star x_i > p^\star \omega_i$

(ii) $x_i \succsim_i x_i^\star \Rightarrow p^\star x_i \geq p^\star \omega_i$

(iii) $x_i \succ_i x_i^\star$ for all $i \Rightarrow p^\star \sum\limits_i x_i > p^\star \sum\limits_i \omega_i$

(iv) $p^\star \sum\limits_i x_i > p^\star \sum\limits_i \omega_i \Rightarrow x_i \succ_i x_i^\star$ for all i

(v) $x_i \succ_i x_i^\star$ for all $i \Rightarrow \sum\limits_i x_i > \sum\limits_i \omega_i$

(b) Consider a two consumer, two good exchange economy. Consumer 1's utility function is $x_{11} + x_{21}$, while consumer 2's is $x_{12} + x_{22}$. Consumer 1's endowment is (1,0) and consumer 2's is (0,1). What is the Walrasian equilibrium?

2. Consider a principal-agent problem with four states of nature, $\theta_1, \theta_2, \theta_3$, and θ_4. The agent can choose one of two effort levels, a_L and a_H. The principal is risk neutral, while the agent has utility $\sqrt{w} - e$. Effort cost is 0 for a_L and .2 for a_H. The agent's reservation expected utility is 1.6. Revenue can take one of two values, 20 or 4, depending on both θ and a. The distribution of revenue is:

	θ_1	θ_2	θ_3	θ_4
Probability	.3	.3	.2	.2
Revenue under a_H	20	20	20	4
Revenue under a_L	20	20	4	4

(a) Derive the first-best outcome.

(b) Derive the second-best contract when only revenue is verifiable. (Assume the principal has all the bargaining power.)

(c) Assume that the principal can buy for a price c an information system that allows the parties to verify whether or not the state of nature θ_3 happened. For what values of c will the principal buy this system?

Microeconomics General Examination Spring 2013
Part D

D1) *30 Points*

This entire question concerns voting problems in which there are three alternatives x, y, z.

Suppose the binary votes between pairs of alternatives (the fraction of the population voting for the alternative in the row over the alternative in the column) are given by the following table:

	x	y	z
x	$-$	α	β
y	$1-\alpha$	$-$	γ
z	$1-\beta$	$1-\gamma$	$-$

Assume that $\alpha > \gamma > 1 - \beta > \frac{1}{2}$

a) Prove that the Kemeny Method, the Method of Ranked Pairs, and the Schulze Method (also called the beat-path method) all lead to the same ranking of the alternatives. (**15**)

b) Show that there are three Slater rankings, and that three of the six strict orders are not Slater rankings. (**5**)

c) Give an example of a population of voters – a distribution of preferences over the six possible strict orders for three alternatives – that induces a binary vote table satisfying the assumption above and for which the ranking produced by Plurality with a Runoff differs from the common recommendation of the Kemeny, Ranked Pairs and Schulze Methods. (By the ranking produced by Plurality with a Runoff, I mean the order that puts the alternative with the fewest first-place votes last and ranks the remaining two alternatives first and second, respectively, in the same way as the majority vote between them.) (**5**)

d) Is it necessarily the case that Plurality with a Runoff produces one of the three Slater rankings? Provide a proof or a counterexample. (**5**)

D2) *40 Points*

This problem concerns the very same example considered in Lectures 9, 10 and 11. We will not be referring to any of the particular calculations made in those Lectures. The intention is to explore some other aspects of mechanism design in a familiar context.

Consider a binary decision between two alternatives a and b by two people, $i = 1, 2$. Their preferences are determined by θ_1 and θ_2 according to the utilities:

$$\begin{aligned} u_i(k, t_i, \theta_i) &= v_i(k, \theta_i) + t_i = \theta_i + t_i \text{ if } k = a \\ u_i(k, t_i, \theta_i) &= v_i(k, \theta_i) + t_i = -\theta_i + t_i \text{ if } k = b \end{aligned}$$

The parameters θ_i are distributed independently on the interval $[-1, +1]$, with a positive density throughout this interval, and the distribution function $F(\theta_i)$ of each θ_i, is symmetric around 0:

$$F(z) = 1 - F(-z) \text{ for all } z \in [-1, 1].$$

The decision is made via the use of the Expected Externality Mechanism (EEM), played in a truthful Bayesian equilibrium. This equilibrium implements the efficient decision rule: $k = a$ if $\theta_1 + \theta_2 > 0$, $k = b$ if $\theta_1 + \theta_2 > 0$, and a random decision if $\theta_1 + \theta_2 = 0$.

a) What is the interim utility obtained by player i in the EEM as a function of the type θ_i? (You should express your answer as it depends on θ_i and F. The question is not asking for a numerical answer or a closed form solution, as no specific functional form of the distribution function F has been given. You should simply exhibit a formula for the interim utility as a function of θ_i and F. that could be calculated once F is known.) (**15**)

b) Which types θ_i have the highest and lowest interim utilities? Suppose that there is a type-independent participation constraint such that anyone whose interim utility falls short of the average per capita ex ante utility available to this population can quit and obtain that amount of utility without participating in this EEM. Describe the set of types that would exercise this option? (**5**)

c) Suppose that F were to change to another symmetric distribution G which can produce the same total expected payoff as F at the efficient decision rule. Would the interim utilities of all types necessarily remain the same? (**5**)

d) Suppose that the mechanism designer would like to arrange for asymmetric interim utilities while still preserving the efficiency of the social decision that is taken. For example, by using a Bayesian mechanism different from the EEM, he might like to favor those players who prefer a rather than b by inducing interim utilities such that $\bar{U}_i(z) > \bar{U}_i(-z)$ for all $z > 0$. Is this possible? Explain. (**5**)

e) Now consider a distribution $G(\theta_i)$ concentrated on only two points $-x^*$ and x^* each with probability $\frac{1}{2}$. What Bayesian mechanism will achieve an efficient decision for this G using no transfers of money at all? What are the interim utilities produced by this mechanism? Are the Bayesian strategies of the players in this mechanism actually dominant strategies? How does your answer to this part of the question relate to the Gibbard-Satterthwaite Theorem (that no ex post efficient dominant strategy mechanisms exist, MWG Proposition 23.C.3) and the Green-Laffont Theorem (that for quasi-linear environments no dominant strategy mechanism with zero net transfers exists, MWG Proposition 23.C.6) (**5**)?