Honors General Exam
Part 2: Macroeconomics
Solutions

Harvard University

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Question 1. (30 points)

This question asks you to use a Solow Model to analyze what happens to an economy when a government imposes a proportional tax on output.

(A) **(8 pts)** To start off, assume that there is no government in the economy, so the Solow setup is completely standard. Specifically, the economy has a production function $Y = K^{\alpha}(EL)^{1-\alpha}$, where K is physical capital, L is labor input, E is labor-augmenting technical progress, and α is an exogenous constant. The exogenous growth rates of E and L are g and n, respectively. Every period, a fraction δ between 0 and 1 of the physical capital stock deteriorates. The exogenous savings rate is a constant rate s. Under these conditions, what are the steady-state values of the following quantities in terms of the exogenous variables? (Note: You do not need to draw any graphs for this part of the question.)

- The capital-output ratio $\frac{K}{Y}$?
- The ratio of capital per effective unit of labor $k = \frac{K}{EL}$?
- The growth rate of output Y?
- The growth rate of output per worker $\frac{Y}{L}$?
- The growth rate of the marginal product of capital MPK?
- The growth rate of the marginal product of labor MPL?

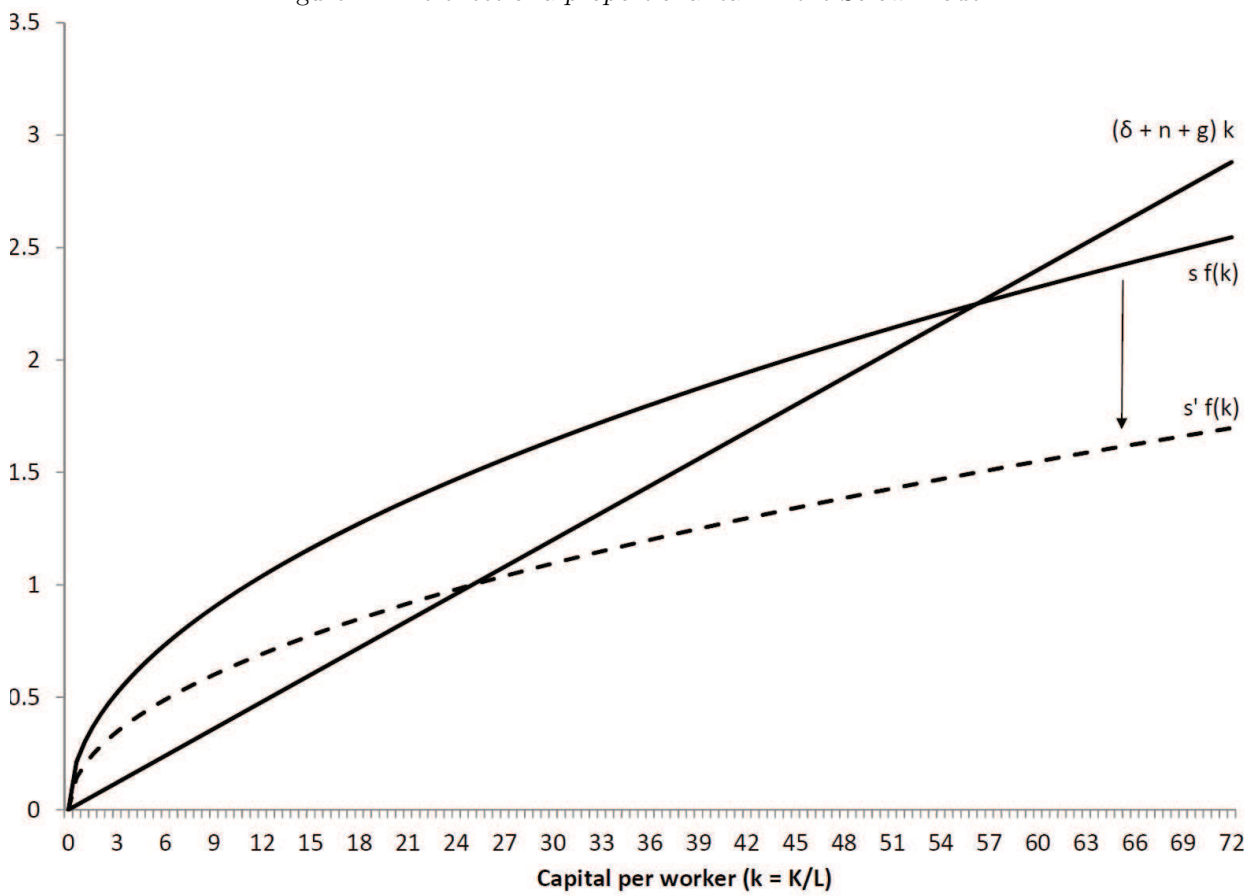
Solution: Consult standard treatments of the Solow Model for the logic behind these answers.

- The capital-output ratio $\frac{K}{Y}$? $\frac{s}{\delta+n+g}$
- The ratio of capital per effective unit of labor $k = \frac{K}{EL}$? $\left(\frac{s}{\delta+n+g}\right)^{\frac{1}{1-\alpha}}$
- The growth rate of output Y? **n+g**
- The growth rate of output per worker $\frac{Y}{L}$? **g**
- The growth rate of the marginal product of capital MPK? **Zero**
- The growth rate of the marginal product of labor MPL? **g**

(B) **(5 pts)** Now assume that there is a government that collects a small positive fraction τ of real output Y each period as a tax. After the government collects this tax, you can assume that the output simply disappears and it not used to purchase any goods or services, or to pay any government employees. (In other words, you can assume that the government collects a constant fraction of real GDP τ each period and throws it in the ocean.) Draw a graph to show how the standard Solow graph that has $k = \frac{K}{EL}$ on the horizontal axis is affected by this change, under the assumption that the fraction of output allocated to consumption C is the same before and after the tax.

Solution: The key to answering this question is to understand how the tax affects capital accumulation. Before, output Y was divided into investment I and consumption C. The amount of output available for investment (that is, capital accumulation) was sY and the fraction allocated to consumption was $(1-s)Y$. Now, the same fraction $1-s$ is allocated to consumption, but a fraction τ must also be allocated to the government. This new allocation causes the share of real output left over for capital accumulation to fall to $s' = s - \tau$. The effect on the Solow graph is to shift the "savings" function down to $s'f(k)$, as seen in Figure 1.

Figure 1: *The effect of a proportional tax in the Solow Model.*



(C) **(2 pts)** How does the government tax affect your answers to part (A)? That is, what are the new steady-state values of

- The capital-output ratio $\frac{K}{Y}$?
- The ratio of capital per effective unit of labor $k = \frac{K}{EL}$?
- The growth rate of output $Y = \frac{Y}{L}$?
- The growth rate of output per worker $\frac{Y}{L}$?
- The growth rate of the marginal product of capital MPK?
- The growth rate of the marginal product of labor MPL?

Solution: The only effect of the government tax is to reduce the effective "savings rate" in the economy from s to s', where $s' = s - \tau$. Thus, the only differences from part (A) are as follows:

- The capital-output ratio $\frac{K}{Y}$ is now $\frac{s'}{\delta+n+g}$
- The ratio of capital per effective unit of labor $k = \frac{K}{EL}$ is now $\left(\frac{s'}{\delta+n+g}\right)^{\frac{1}{1-\alpha}}$

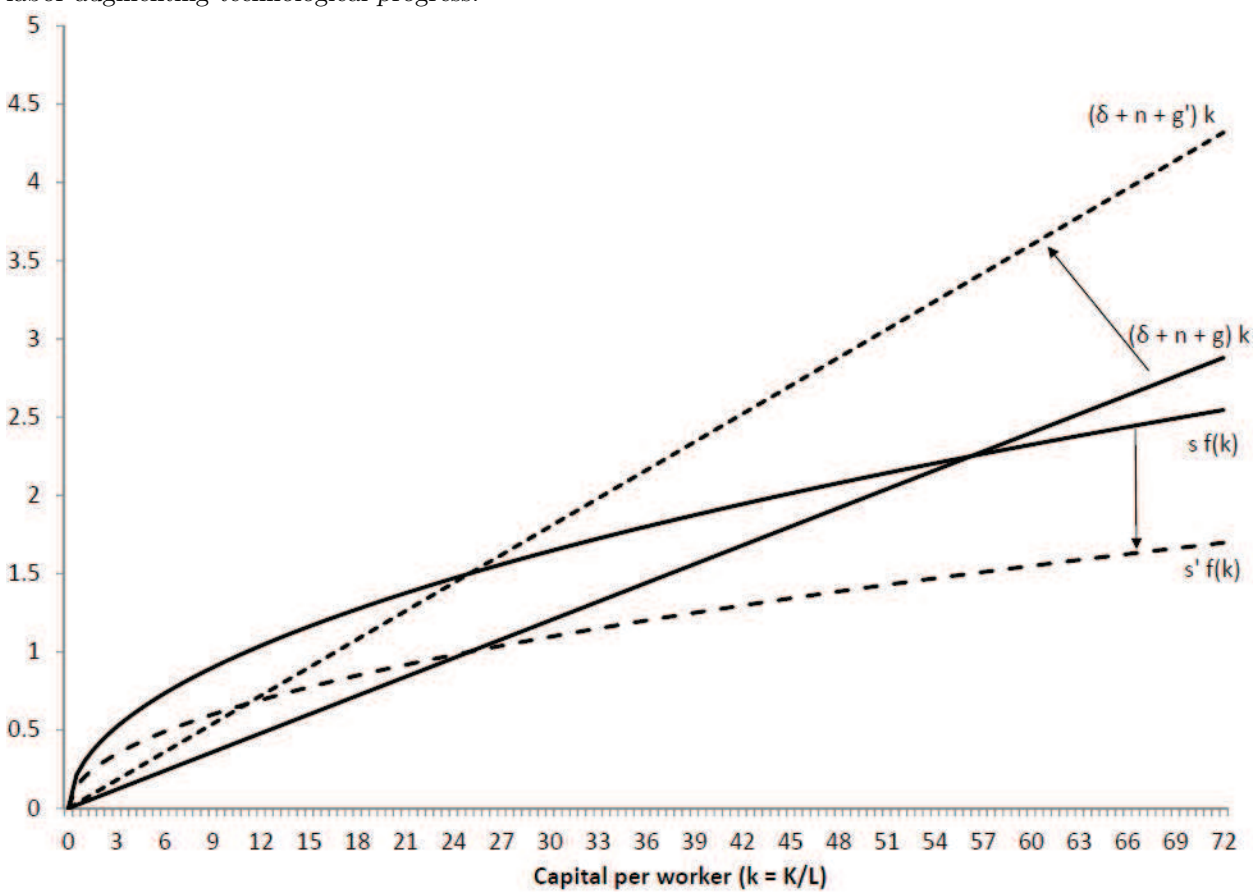
(D) **(5 pts)** Describe what would happen to output per worker and living standards $\frac{C}{L}$ if the tax were suddenly levied on an economy that did not have one before. In your answer, describe how these variables would change after the new tax (if at all) in both the short and long runs. You do not need to draw a graph.

Solution: The tax will reduce output per worker and standards of living, relative to levels that those variables would have reached without the tax. However, this reduction plays out over time and does not occur the instant that the tax is imposed. The tax leaves fewer resources available for capital accumulation, so as Figure 1 suggests, the levels of capital and output per worker will shrink toward a lower steady state. Immediately after the tax is imposed, however, there is no instantaneous change in output, labor, capital, or the share of output allocated to consumption. Hence, there is no instantaneous change in output per worker or consumption per worker. Yet once the tax is imposed, the growth rates of $\frac{Y}{L}$ and $\frac{C}{L}$ fall below their steady-state values, g. Once the new steady state is reached, growth in living standards and output per worker will be back at g, but the relatively low growth rates *during the intervening transition period* will leave output per worker and living standards at lower levels than they would have reached had the tax never been imposed.

(E) **(5 pts)** Now assume that the tax allows the government to create a set of blueprints that causes the growth rate of technological progress to rise to g', a rate that is higher than the original technological growth rate, g. Draw a standard Solow model graph to show how this type of tax would affect the steady-state value of capital per effective unit of labor k (if at all). (In your answer, you can ignore the effect of any government payments to the creators of the blueprints. Just pretend that as before, the output absorbed by the tax get thrown in the ocean, but in this process the rate of technical progress magically rises from g to g'.)

Solution: See Figure 2. The higher rate of technological program g' rotates the breakeven investment schedule up to $(\delta + n + g')k$. This line has a steeper slope than the previous schedule, because $g' > g$. At the intersection of the dotted lines, steady-state k is lower than the case with no tax. The lower value of steady-state k arises because the tax lowers the share of physical capital allocated for investment each period, as before, and also because the higher growth rate of technological progress E means that any given savings rate results in a lower steady-state level of capital *per effective unit of labor*: $k = \frac{K}{EL}$.

Figure 2: *The effect of a proportional tax in the Solow Model, combined with a higher growth rate of labor-augmenting technological progress.*



(F) **(5 pts)** How would the combination of the tax and the higher rate of technological progress that is described in part (E) affect output per worker $\frac{Y}{L}$ and living standards, if at all? Discuss both the short and long runs. You do not need to draw a graph.

Solution: The key to understanding this question is the difference between *levels* and *growth rates*. The government tax lowers the steady-state *levels* of output-per-worker and living standards, as before, the tax now raises the steady-state growth rates of these variables from g to g'. The combined short-run effects on output and consumption per worker, which occur during the transition to the new steady state, are impossible to know without knowing the relative sizes of τ and g'. However, in the long run, we can be sure that citizens will <u>eventually</u> be better off than they would have been without the tax. People eventually benefit in terms of higher living standards $\frac{C}{L}$ and output per worker $\frac{Y}{L}$ because growth-rate effects accumulate over time, while levels effects do not. Consequently, no matter how large the levels reduction in output per worker that arises from the tax's effect on the physical capital stock, the positive growth rate effect arising through higher technological growth will win out in the end. Of course, this beneficial effect will takes longer to occur the larger the size of the tax τ and the smaller the increment in growth technological progress $g' - g$.

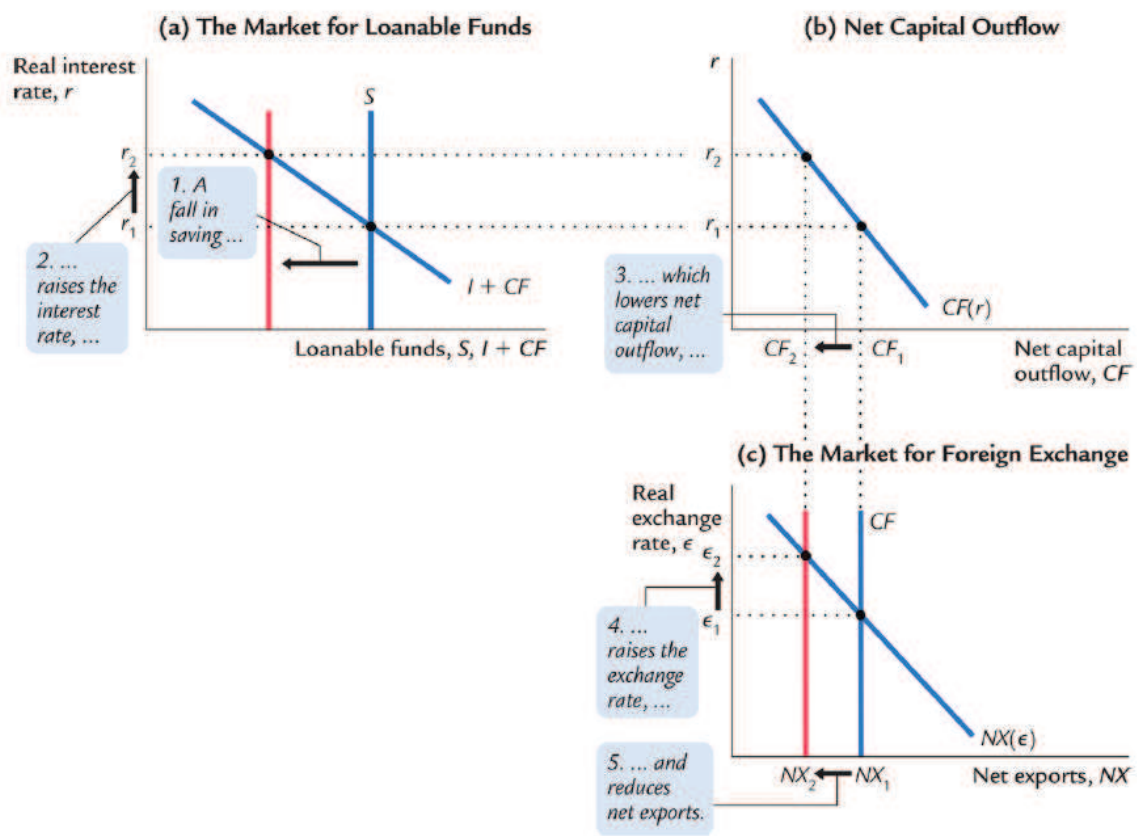
Question 2. (30 points)

In early 2016, several large economies around the world experienced changes in international capital flows. This question investigates the causes and consequences of movements in capital flows for large open economies in some detail.

(A) **(8 pts)** How would an increase in government purchases G affect (a) output, (b) the net capital outflow, (c) investment and (d) the real exchange rate in the standard long run large open economy (LOE) model? Use graphs to illustrate your answer, and be sure to explain your economic reasoning in words. Also explain how the answer to this question would be different if the economy were a small open economy (SOE) instead. (You not have to draw a graph for the SOE case, just explain the difference between the LOE case and the SOE case for output, the net capital outflow, investment, and the real exchange rate.)

Solution: See Figure 3. For an LOE, an increase in G reduces national savings S and raises the real interest rate r. This higher r reduces both investment I and the net capital outflow CF. The lower CF arises because higher interest rates encourage as a higher share of domestic savings to be invested at home while a higher fraction of savings from abroad is invested in the LOE. The reduction in the net capital outflow reduces the net supply of dollars to be invested abroad, so the real exchange rate rises and reduces net exports NX. In the SOE case, the interest rate r is pinned down by capital flows at r^*. Thus, there is no increase in r, and no reduction in I. All of the adjustment to the higher G must come about via lower CF and lower NX. Note that output Y does not change in either the SOE or the LOE case, due to our use of a long-run model.

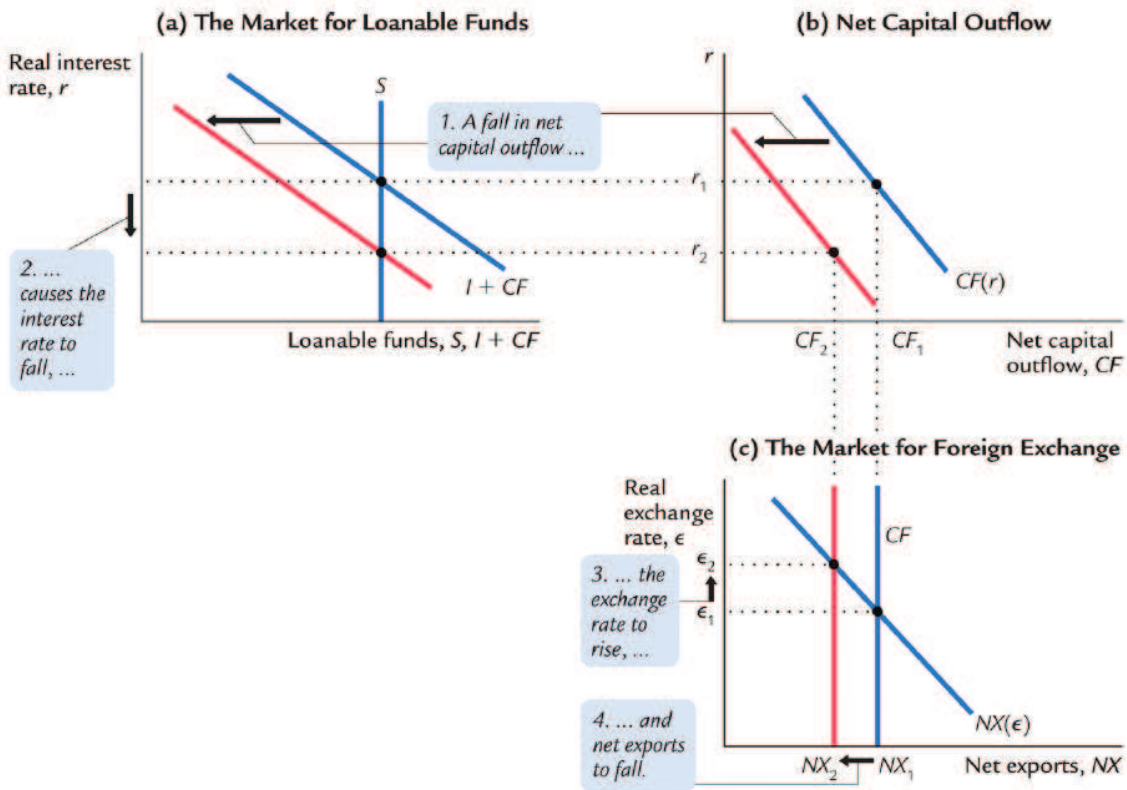
Figure 3: *A fiscal expansion in a large open economy.*



(B) **(7 pts)** Now use a long-run model to explain what would happen if the LOE experienced no change in G, but were instead judged to be a better place for foreigners to invest at any given rate of return. (For example, foreigners might increasingly view the LOE as a safe haven in a turbulent world market.) Use a graph to explain how the change in foreigners' views would affect (a) the domestic real interest rate, (b) output, (c) investment, and (d) the net capital outflow, and (e) the trade balance. You do <u>not</u> have to compare your answers to a small open economy case.

Solution: See Figure 4. The increase in capital inflows from foreigners reduces the net capital outflow CF, so both the $I + CF$ curve and CF curves shift to the left by an equal amount. This reduces the domestic interest rate r and therefore increases investment I. The reduction also causes CF to fall by less than leftward horizontal shift in the CF and $I + CF$ curves. However, there is some reduction in CF, so there is some increase in the real exchange rate and some reduction in NX. Output does not change.

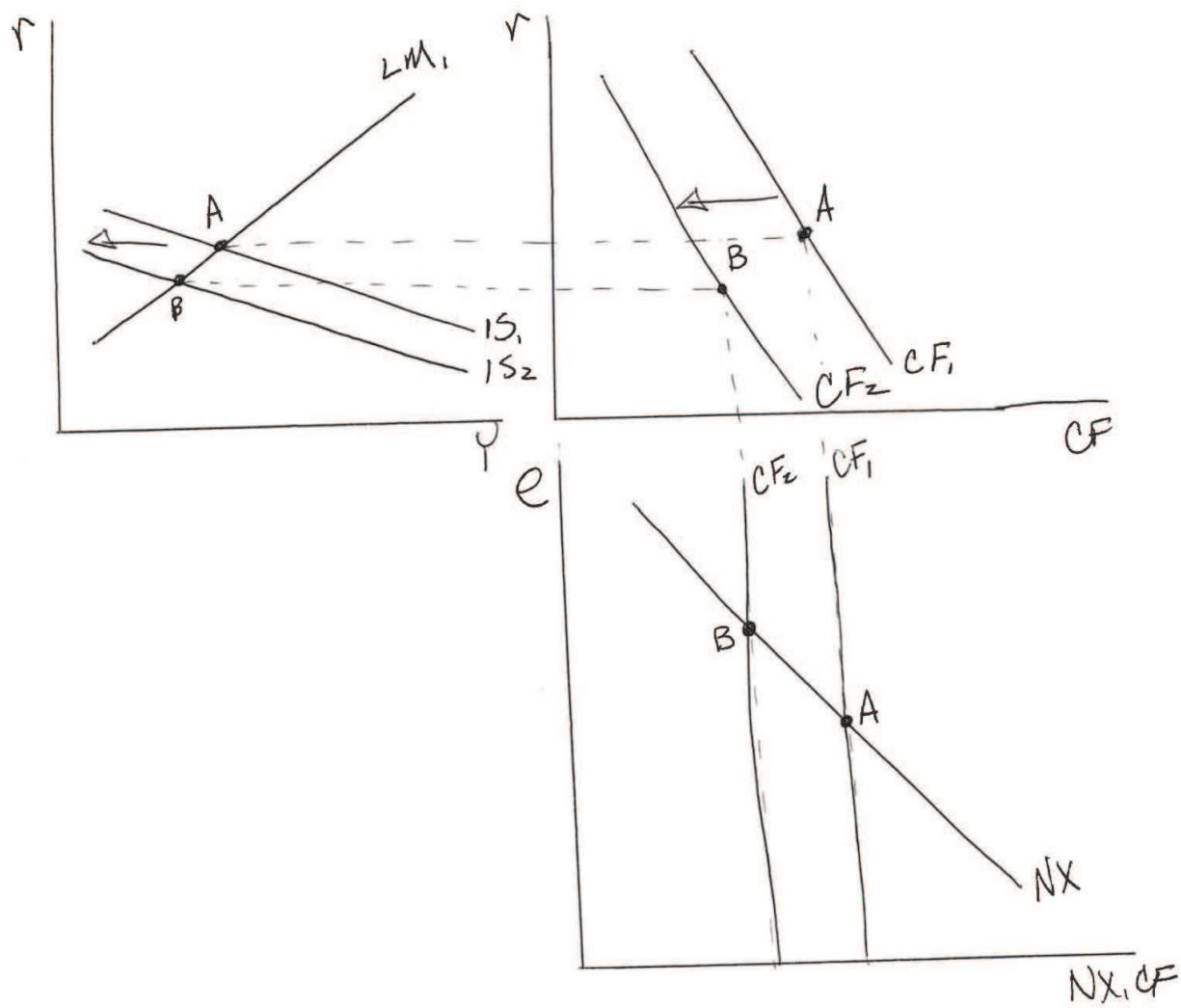
Figure 4: *An decline in net capital outflow in a large open economy.*



(C) **(7 pts)** How would you change the answers you obtained for the change in foreigners views for a large open economy described in (B) if you had used a short-run model instead of a long-run model? Again, use a graph to explain how the change in foreigners views' would affect (a) the domestic real interest rate, (b) output, (c) investment, (d) the net capital outflow, and (e) the trade balance. Explain your answers in words as well.

Solution: See Figure 5. In a short-run model, the domestic real interest rate r is determined by an $IS-LM$ model, in which the IS curve is now flatter than the closed-economy case. The reason IS is flatter is that it now reflects the positive effect of a lower domestic interest rate r on <u>both</u> investment I and the net capital outflow CF. Hence, the IS curve is also flatter than the CF schedule. The same type of "safe haven" movement as described in part (B) causes the IS and CF schedules to shift to the left by equal amounts. This causes a reduction in r, an <u>increase</u> in I, a reduction in the net capital outflow, a higher nominal exchange rate e, and lower net exports NX. Note that output falls because the reduction in CF and NX are larger than the increase in I. (You did not have to explain the intuition for why this occurs to get full credit.)

Figure 5: *The short-run effect of a decline in net capital output for a large open economy.*



(D) **(8 pts)** In early 2016, some people have argued that the United States is undergoing capital flows of the type described in parts (B) and (C) of this question. How should the Fed respond to this movements in capital flows if it wants to stabilize short-run output, the price level, or both? How would the results of the Fed's actions relate to the long-run outcome for the large open economy described in part (B)? (As you answer this question, you can ignore the fact that the Fed is currently near the zero lower bound, and just pretend that the level of output Y before the change in foreigners' views was close to its long-run level.)

Solution: See Figure 6. The Fed can stabilize output Y if undertakes a monetary expansion, raising M and thereby moving the LM curve to LM^3. This movement raises Y and reduces r. The reduction in r raises the net capital outflow and thus net exports (that is, we move from point B to point C). However, one can show that because the IS curve is flatter than the CF schedule, the ensuing reduction in the real interest rates r is not enough to return the net capital outflow back to its level before the change in foreigners' views (that is, we do not move NX and CF all the way back to where they are at point A). Consequently, NX remains lower than before the views of foreigners changed. All in all, the Fed's stabilization policy moves the LOE more quickly to the long-run outcome, which was discussed in Part (B) of the question and is depicted in Figure 6 as point C. At this outcome, output is stable near $\overline{Y}$ (and thus there is no need for P to change); I is higher, and NX is lower.

Figure 6: *The short-run effect of a decline in net capital output for a large open economy combined with a monetary policy offset.* More here.