HARVARD UNIVERSITY
DEPARTMENT OF ECONOMICS

General Examination in Macroeconomic Theory

Spring 2012

You have **FOUR** hours. Answer all questions
Part A(Laibson: 48 min)
Part B (Barro: 48 min)
Part C (Farhi: 48 min)
Part D (Rogoff: 96 min)

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Laibson questions

Laibson Part I (15 minutes) True/False/Uncertain. Explain your answer. Explanation determines grade. **Please write legibly.**

1. The Euler Equation predicts that predictable changes in income should not predict changes in consumption.

2. In an investment model with a discrete cost of adjustment, both firm level investment and aggregate investment tend to be lumpy.

3. A temporary investment tax credit will cause investment to fall below its steady state level.

Laibson Part II (33 minutes)

Assume that the price of oil, P, is arithmetic Brownian motion: $dP = adt + \sigma dz$. The parameters a and σ are fixed. Suppose that you own an oil field – a geological formation that contains oil deposits. You can't sell the oil field, but you can decide how much oil to pump every instant of time. You choose an instantaneous oil flow of $F(t)$ units per period, where $F(t) \in [0,1]$. For this problem you are allowed to adjust the oil flow as often as you want and you are allowed to restart the flow even if the flow was previously set to zero. Your firm's instantaneous profit is $F(t)*(P(t) - C)$, where $P(t)$ is the per unit price and C is the (non-stochastic) per unit extraction cost. Your firm's objective is to maximize the discounted value of this profit stream. The profit function implies that you pay no costs and receive no revenue when the flow is zero. (To simplify the problem, the oil field is not depletable.)

Explain which of the following statements are true and which are false. Correct the false statements (change them in a way that makes them into true statements). Explanation determines grade. Intuitive arguments or heuristic proofs will suffice. **Please write legibly.**

1. The price of oil is unbounded above and below.

2. The value of the oil field can be expressed as a function $V(P, a, \sigma, C)$. In other words, the value function can be expressed solely in terms of the price P and the fixed parameters a, σ, and C. Time is not an argument in the value function.

3. V is concave with respect to P.

4. The partial derivative of V with respect to σ is negative.

5. If the interest rate is r, then the Bellman Equation for this problem is:
   $rV = P - C + aV_P + \frac{\sigma^2}{2} V_{PP}$.

6. If the price of oil is currently $P < C$, then the value of the oil field is $V = 0$.

7. Assume $a = 0$. If the price of oil is currently $P \geq C$, then the value of the oil field is $V = \frac{P - C}{r}$.

Barro questions (8 minutes each, total of 48 minutes)

True-False-Uncertain

Are the following statements true, false, or uncertain? Explain **briefly**, but your explanation determines your grade.

1. In the varieties model of endogenous economic growth, a permanent reduction in monopoly power for inventors of new types of products leads to an increase in the long-run growth rate of real GDP.

2. If a war destroys a lot of physical capital but comparatively little human capital, the growth rate of real GDP following the war tends to be particularly high (for example, in the Mankiw-Romer-Weil type of model, in which production uses physical and human capital).

3. In the Blanchard-Weil "finite-horizons" model, a once-and-for-all rise in the mortality rate leads in the long run to a lower capital-labor ratio.

4. Ricardian Equivalence for public debt can fail in the standard neoclassical growth model (Ramsey-Cass-Koopmans model) because the government can finance its debt forever by always issuing enough new debt.

5. "U.S. Treasury Bills are actually the riskiest form of asset because they guarantee a very low real rate of return" (a paraphrasing of a recent statement from Warren Buffett).

6. If a government learns that a war is coming, it should raise tax rates (say on labor income) before the war and then lower tax rates after the war is finished.

Answer all the following questions. Explain and detail your answers VERY carefully. The QUALITY of the explanations that your provide determines your grade. Keep your answers succinct.

Problem 1

Consider an economy consisting of a constant population of infinitely-lived individuals. The representative individual maximizes $\sum_{t=0}^{\infty} \beta^t u(C_t)$ where $u(C_t) = C_t - \theta C_t^2$ (assume that C_t is always in the range where $u'(C_t)$ is positive). Output is linear in capital, plus an additive disturbance $Y_t = RK_t + e_t$ and there is no depreciation so that $K_{t+1} = K_t + Y_t - C_t$. Assume that $\beta R = 1$. FInally, the disturbance follows an AR(1) process $e_t = \rho e_{t-1} + \epsilon_t$ where $\rho \in [0, 1]$ and ϵ_t is i.i.d. with mean zero.

1. Derive the Euler equation for consumption.
2. Guess and verify that the policy function for consumption is of the form $C_t = \alpha_0 + \alpha_K K_t + \alpha_e e_t$ and derive the values of the coefficients α_0, α_K and α_e.
3. Derive the policy function for capital.
4. What are the effect of a one-time shock to ϵ on the paths of Y, K and C?
5. Explain how the effect of a one time shock on capital accumulation depends on the persistence ρ of the shock.

Problem 2

1. Consider the basic New-Keynesian model with a labor tax τ set to offset the monopoly distortion. The New-Keynesian Philips Curve is $\pi_t = \beta \pi_{t+1} + \kappa x_t + u_t$ and the Dynamic IS equation is $x_{t+1} - x_t = \sigma^{-1}(i_t - E_t \pi_{t+1} - \bar{r}_t)$ where $\bar{r}_t$ is the natural interest rate. There are two disturbances, natural interest rate shocks ($\bar{r}_t$) and cost push shocks (u_t). Explain how the output gap x_t defined. Explain what fundamental shocks could be behind the disturbances. Explain the intuition behind these equations.
2. The loss function can be written as $L = \sum_{t=0}^{\infty} \frac{1}{2}[\alpha_x x_t^2 + \pi_t^2]$. Explain the intuition behind the two terms in the period loss function. Set-up the optimal monetary policy planning problem.
3. Show that in the absence of cost push shocks ($u_t = 0$), the solution features perfect stabilization $x_t = \pi_t = 0$. How is the optimal nominal interest rate determined? Does this solution require commitment?
4. Assume that $\bar{r}_t = \rho$ and that u_t is an AR(1) with $u_t = \phi u_{t-1} + \epsilon_t$. Characterize optimal monetary policy under commitment and under no commitment. How do the two solutions differ? Explain the origin of the gains from commitment.
5. How does the solutions characterized in the previous question change when prices become more flexible?
6. Now assume that the labor tax τ is set to zero so that we have a steady monopoly distortion. Explain why the loss function can now be written as $L = \sum_{t=0}^{\infty} \frac{1}{2}[\alpha_x x_t^2 + \pi_t^2 - \Lambda x_t]$. Set-up the optimal monetary policy problem assuming that there are no disturbances ($\bar{r}_t = \rho$ and $u_t = 0$). Characterize optimal monetary policy under commitment and under no commitment. How do the two solutions differ? Explain the origin of the gains from commitment.

Part IV ROGOFF

May 9, 2012

Instructions: Please answer all four questions in this section. You do not need to give every intermediate step. Note that the third essay question is weighted more heavily (34 points) than the other three questions (22 points each). You do not need to use a separate blue book for each question, but **PLEASE USE A SEPARATE BLUE BOOK FOR QUESTION 4.**

1. (22 points) CONSUMPTION SMOOTHING ACROSS COUNTRIES

Consider a stochastic two-country world, two-period endowment economy, in which agents in the home country have utility function

$$U = \left(\frac{C_1^{1-\rho}}{1-\rho}\right) + E\left(\frac{\beta C_2^{1-\rho}}{1-\rho}\right) ,$$

and agents in the foreign country have the following preferences

$$U = \left(\frac{(C_1^*)^{1-\rho^*}}{1-\rho^*}\right) + E\left(\frac{\beta (C_2^*)^{1-\rho^*}}{1-\rho^*}\right).$$

Agents in the foreign country have an identical utility function. Home income in the first period is given by Y, and in the second period by $Y(s)$, where s is the state of nature. Agents abroad receive income stream Y^* and $Y^*(s)$.

(a) Assume that there are complete state-contingent markets in this global economy. Show, analytically, in what sense consumption risk is shared. It is sufficient to show your result in terms of relating consumption growth rates across countries ($\log(C_2/C_1)$ and $\log(C_2^*/C_1^*)$).

(b) Now assume that $\rho = \rho^*$; suppose that there are incomplete markets—agents can trade only a risk-free bond that pays a fixed interest rate r (where the interest rate is determined endogenously in equilibrium). Derive an equation that relates relative consumption across the two countries. How does your answer here relate to your answer in part (a) where there were complete markets?

2. (22 points) **THE COSTS OF FINANCIAL AUTARKY AND REPUTATIONAL INSURANCE CONTRACTS**

Consider the problem of a small endowment economy inhabited by a representive agent with utility function

$$U_t = \mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\rho}}{1-\rho} \right\} .$$

$Y_s = \overline{Y} \exp\left[\epsilon_s - \frac{1}{2}\sigma_\epsilon^2\right]$, where ϵ_s is normal i.i.d. over time with mean zero and variance σ_ϵ^2 . Note that there is no trend growth, so $\mathrm{E}_t\left\{C_s^{1-\rho}\right\}$ is a constant over time. Assume that $\rho > 1$ and $\beta(1+r) = 1$, where r is the world real interest rate.

(a) Assuming there is no investment or government spending, confirm that under financial autarky

$$E_t\left\{C_s^{1-\rho}\right\} = \overline{Y}^{1-\rho} \exp\left[-\frac{1}{2}(1-\rho)\rho\sigma_\epsilon^2\right] .$$

(Hint: recall that if $X \backsim N(\mu_x, \sigma_x^2)$, then $\exp X$ is distributed lognormally with mean

$$E \exp(X) = \exp\left(\mu_x + \frac{1}{2}\sigma_x^2\right) .$$

Note: You should be able to answer parts (b) - (f) even if you do not get this part.)

(b) Given your answer to (a), what is $\mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\rho}}{1-\rho} \right\}$ in terms of $\overline{Y}$ and σ_ϵ^2 ?

(c) Now assume that the country has access to reputational insurance contracts that allow it to completely eliminate all risk and offer it $C_s = \overline{Y}$ $\forall s$. What is $\mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\rho}}{1-\rho} \right\}$ in this case?

(d) Assume that the reputational insurance contracts involve a trigger strategy equilibrium where as long as the country pays out $Y - \overline{Y}$ in all states of nature where $Y - \overline{Y} > 0$, it maintains its reputation. It receives $Y - \overline{Y}$ in all states of nature where $Y - \overline{Y} < 0$.

If, however, the country ever fails to make a payout in full, it will lose its reputation and be forced permanently to return to autarky in all future periods. What condition on

$$\lim_{Y_t \to \infty} \left[u(Y_t) - u(\overline{Y})\right]$$

must hold for the full insurance contract to be sustainable in a reputational equilibrium? (If you cannot give an analytical answer, try to give a qualitative answer.)

(e) What factors make it more likely that the reputational equilibrium with full insurance is feasible? [Hint: It's enough to just verbally discuss the comparative statics with respect to relevant exogenenous parameters in the model; no need to implicitly differentiate.]

(f) Assume now that the country has access to a second insurance option. In particular, it is allowed to invest money abroad in period t that allows it to purchase fully collateralized insurance contracts where its foreign deposits at time t are always at least enough to cover any positive payment it might be asked to make to its foreign insurers. Note that we are implicitly assuming that the foreign insurer can make legally binding commitments even though the small country cannot. Is the reputational equilibrium you analyzed in parts (d) and (e) still feasible?

3. (34 points) Please give SHORT answers to ANY THREE of the following four short-answer essay questions.

(a) What is the key friction in Bernanke–Gertler that generates amplification? Describe in detail how the amplification process works. What is the role of the monitoring cost in the model? What happens if the monitoring cost goes to zero?

(b) The United States has been running large trade and current account deficits for two decades now, ranging from 3–6% of GDP. What are some arguments that have been advanced by researchers to suggest that this sustained deficit, normally a problem, might be relatively benign today and into the immediate future?

(c) What is the gravity model of trade and how has it been used to test for the effect of currency unions? What are some critiques of this literature (among your critiques, be sure to include issues involving causality, missing observations, and the theoretical foundations of the model)?

(d) Why do economists consider it a puzzle that short-term real interest rates for the largest advanced economies have, on average, been quite low since World War II, particularly during 1950–80, and after 2000? What are a couple possible explanations?

PLEASE USE A SEPARATE BLUE BOOK FOR THIS QUESTION.

4. (22 points) SPECULATIVE ATTACKS AND INDETERMINACY OF EQUILIBRIA

Consider an economy where a government is trying to maintain a fixed exchange rate between pesos (the domestic currency) and the US dollar, at $e_0 = 0$. Speculators (a continuum, of mass 1) believe that this exchange rate is too low compared to the one that would prevail under a floating exchange, e (here the exchange rate is in pesos/dollar; $e > 0$ means the currency would depreciate).

The government is trying to defend this regime with a level of reserves θ, where $\theta \in [0, 1]$ is a magnitude well known by all speculators in the model; i.e., it is common knowledge. The uncertainty in the environment comes from what the exchange rate, e, would be if the regime were to fall (that is, e is the devaluation of the currency if the regime fails).

Agents do not observe e perfectly, but only through noisy, private signals. Specifically, individual agent i observes a private signal

$$e_i = e + \varepsilon_i$$

where ε_i is uniformly distributed; $\varepsilon_i \sim U(-1, 1)$.

Since bonds denominated in the domestic currency yield a higher interest rate, there is an opportunity cost k associated with the "attack strategy" of purchasing one unit of reserves. Speculators' payoffs if they do not attack, $a_i = 0$ (that is, they hold the domestic peso) are normalized at zero, while the payoffs of the attack strategy, $a_i = 1$, are given by

$$\begin{aligned} e - k, \text{ if } R &= 1 \\ -k, \text{ if } R &= 0 \end{aligned}$$

where $R = 1$ labels the event where the attack is successfull (the aggregate attack is large enough to cause the regime to fail (devaluation occurs)).

The regime is abandoned, i.e. $R = 1$, if $A \geq \theta$ and is not abandoned, $R = 0$, if $A < \theta$, where $A = \int_0^1 a_i di$ is the aggregate size of the attack.

(a) Imagine that individual traders follow "cutoff strategies" of the type

$$\begin{aligned} a_i &= 1 \Leftrightarrow e_i \geq e^* \\ a_i &= 0 \text{ otherwise.} \end{aligned}$$

Compute the aggregate size of the attack $A = \int_0^1 a_i di$ as a function of $\bar{e}$. Characterize the "threshold" level of $e = \bar{e}$ such that the regime fails whenever $e > \bar{e}$, also as a function of e^*.

(b) Of course, such cutoff strategies e^* need to be determined endogenously in equilibrium. Intuitively, what condition will give you an additional expression linking $\bar{e}$ to e^*? Provide such expression explicitly. Hint: the conditional distribution of e given e_i is:

$$e|e_i \sim U\left(e_i - 1, e_i + 1\right) \ .$$

(c) Solve for e^* and $\bar{e}$. [IF YOU CANNOT EXPLICITLY SOLVE FOR THEM, DESCRIBE HOW IT CAN BE DONE.] Do you get a unique solution? If yes, why?

(d) Briefly describe a slightly different set-up where there might not be a unique solution. Link your answer to the notion of "strategic uncertainty."