Harvard University
Department of Economics

General Examination in Macroeconomic Theory

Spring 2007

PLEASE USE A SEPARATE BLUE BOOK FOR EACH PART AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

For those taking the GENERAL EXAM in macroeconomic theory:

1. You have FOUR hours.

2. Answer ALL QUESTIONS in Parts I, II, III, IV, and V.

3. Time allotted for each part:
   I.  48 minutes
   II. 48 minutes
   III. 48 minutes
   IV. 48 minutes
   V.  48 minutes
Problem 1 (Growth Model – 24 minutes): Consider a representative agent model in which $k$ is capital, the production function is $k^a$, the discount factor is $\delta$, and the utility function is the natural log of consumption. We can express the Bellman Equation as

$$v(k) = \sup_{0 \leq y \leq k^a} \ln(k^a - y) + \delta v(y). \quad (1)$$

a. Interpret all of the terms in the Bellman Equation.

b. What is the sequence problem that corresponds to this Bellman Equation?

c. Explain why the function $v$ appears on both the right and left sides of equation (1). Would the same $v$ appear on both sides of the equation if the representative agent died with certainty in period $T$?

d. Consider the functional operator, $B$, defined by

$$(Bf)(k) = \sup_{y \in F(k)} \ln(k^a - y) + \delta f(y).$$

Let $\phi(k) = \frac{\alpha \ln(k)}{1 - \alpha \delta}$. Show that

$$(B^n \phi)(k) = \frac{1 - \delta^n}{1 - \delta} \left[ \ln(1 - \alpha \delta) + \frac{\alpha \delta}{1 - \alpha \delta} \ln(\alpha \delta) \right] + \frac{\alpha \ln(k)}{1 - \alpha \delta}.$$ 

e. Let,

$$v(k) \equiv \lim_{n \to \infty} (B^n \phi)(k).$$

Confirm that $v(k)$ is a solution to the Bellman equation.
Problem 2 (True/False/Uncertain – 24 minutes): This question is graded on the quality of your explanation.

a. In theory, predictable changes in income should not generate changes in consumption as long as consumers can freely borrow and lend using labor income as collateral.

b. If $X(t)$ is Brownian motion, than the random variable $X(t) - X(0)$ is correlated with the random variable $X(t - 1) - X(0)$.

c. In the quasi-hyperbolic discounting model, the long-run slope of the equilibrium consumption path is not affected by the short-run discount factor $\beta$.

d. A value function exhibits smooth pasting at an adjustment boundary if the value function is continuous at the boundary.

e. In the optimal solution to a discrete adjustment Ss model, an agent will jump to different points depending on whether the agent hits the left-hand boundary or the right-hand boundary of the continuation region.
Consider the neoclassical growth model, discussed, for example, in Barro & Sala-i-Martin, *Economic Growth*, Ch. 2. Households maximize utility, $U$, over an infinite horizon, using a constant rate of time preference, $\rho$. Each period’s utility, $u(c)$, is isoelastic with curvature parameter $\theta$ (c is consumption per person). The production function is neoclassical. The rate of growth of population and the labor force is $n$.

**Assume that the rate of technological progress is zero.** Suppose that the economy starts with capital per worker, $k$, equal to $k(0)$, which is less than the steady-state value, $k^\ast$.

1. Write down the two dynamic equations for $c$ and $k$. Briefly sketch where these two equations come from.

2. Show the phase diagram—involving pairs of $(k, c)$ that generate $\dot{c} = 0$ and $\dot{k} = 0$. Briefly sketch where these curves come from. Use the phase diagram to discuss the steady-state values, $k^\ast$ and $c^\ast$. Use the diagram to discuss the dynamic path of $(k, c)$. Why do $k$ and $c$ approach $k^\ast$ and $c^\ast$, respectively, over time? That is, how do you rule out dynamic paths that lead asymptotically away from the steady-state position?

3. Suppose that the economy starts from its steady state. Assume now that the rate of population growth, $n$, declines once-and-for-all. How does this change affect the growth rate of $k$ in the short run and the long run?

4. Suppose now that a government exists and has a path of government purchases $G(t)$. The government levies lump-sum taxes $T(t)$. The initial quantity of real government debt is $B(0) > 0$. The real interest rate, $r(t)$, on these (short-term) bonds is the same as that on privately issued bonds. The government can increase or decrease its debt, $B(t)$, over time.

   a. What is the government’s budget constraint at each date, $t$?

   b. How does the existence of the government affect the representative household’s budget constraint at each date, $t$?

   c. For a given path of purchases, $G(t)$, how do differences in $B(0)$ and the path of $B(t)$ affect the equilibrium path of $k(t)$ and $c(t)$? Explain your answer.
Part III
For each of the following claims, state whether it is True or False (or Partly True).
Explain. Explanation determines grade.

1. In the IS-LM model with a fixed price level, a $1 increase in both government purchases and taxes, holding the money supply constant, increases equilibrium income by exactly $1 if the demand for money is infinitely interest elastic. If the interest elasticity is finite, equilibrium income increases by more than $1.

2. In the Lucas misperceptions model, an increase in the variance of idiosyncratic shocks increases the impact of monetary shocks on aggregate output.

3. In the Fischer-Gray wage contracting model, increased indexation causes monetary shocks to have smaller short-run effects on output and inflation.

4. According to the new Keynesian Phillips curve, such as the Calvo model, a preannounced, fully credible, and gradual reduction in inflation causes output to rise above its natural rate.

5. In the Caplin-Spulber model of state-dependent price adjustment, the overall price level moves proportionately with increases or decreases in the money supply, leading to monetary neutrality, even though many individual prices are sticky.

6. In the empirical calibration of the Golosov-Lucas menu cost model, idiosyncratic shocks play a crucial role in matching observed price dynamics, and as a result, the real effects of nominal disturbances are similar to those found in time-dependent models.
In what way does an economy's wage-setting mechanism affect the trade-off that monetary policymakers face between stabilizing inflation at some desired rate and stabilizing real economic activity (say, the rate of output) at the welfare-maximizing level even in the short run — that is, wholly apart from the dynamic processes by which inflation in one time period carries over inertially to inflation in subsequent periods? More specifically,

A. First suppose that firms are monopolistically competitive, that they price the goods and services they sell not every period but only at randomly determined intervals (along the lines first suggested by Calvo), and that their production is subject to random technology shocks. Is the monetary policy that renders inflation today equal to whatever inflation is expected for the future (say, zero for simplicity) the same as the monetary policy that renders output today equal to the welfare-maximizing output level? Why or why not?

B. Now suppose instead that the wage-setting process is such that real wages are less responsive to the level of employment than ordinary competitive labor supply relationships would imply. Does monetary policy now face a short-run trade-off between stabilizing inflation at the desired level and stabilizing output at the welfare-maximizing level? Why or why not?

C. Which of the following two arguments do you find more persuasive? Why?

1. "In the presence of this kind of real wage rigidity (as in part B), monetary policy should take a more flexible approach to controlling inflation than would otherwise be the case. In particular, adverse supply shocks — an increase in oil prices, for example, or a productivity slowdown — should be accommodated with a larger transitory increase in inflation than would otherwise be optimal."

2. "A flexible approach to controlling inflation in such circumstances would only lead to real wages becoming yet more rigid. It's therefore better to be strict about controlling inflation, so that firms and employees will understand the implications of setting wages in too rigid a way."

In answering each part of the question, be as explicit as you can about your reasoning, as well as about the assumptions on which your reasoning relies.
Open Macroeconomics
2007

Trade and International Asset Prices

It is consistently argued in the contagion literature that two countries have to have strong trade relationships to experience high correlation in their real exchange rates and asset prices. This question explores this assertion.

Assume there are three countries on earth. The Center which is a large country (which we denote as C), and two Periphery countries (which we denote A and B; importantly, we call them periphery as opposed to crappy out of respect of the professor’s nationality).

We assume that each country in an endowment economy, with one tree, that produces a different listed good. Denote the good produced in the center with a C, and the goods produced in the periphery as A and B. We will assume that the preferences are such that the two countries in the periphery do not trade at all, but both trade with the Center. Preferences are given by

\[ U_c = \alpha \ln C_{c,c} + \frac{1-\alpha}{2} \ln C_{c,a} + \frac{1-\alpha}{2} \ln C_{c,b} \]
\[ U_a = \alpha \ln C_{a,a} + (1-\alpha) \ln C_{a,c} \]
\[ U_b = \alpha \ln C_{b,a} + (1-\alpha) \ln C_{b,c} \]

where \( C_{i,j} \) is the consumption of country \( i \) of the good produced in country \( j \). \( \alpha \) indicates the degree of home bias in consumption, and as can be seen, country \( a \) consumes only its own goods and those produced in the Center, while country \( b \) consumes its own goods and the ones produced by the center. In other words, there is no trade in goods across countries \( a \) and \( b \).

The output of each country is stochastic. Denote (as we did in class) output in state \( s \) at time \( t \) as \( y_a(s,t) \), \( y_b(s,t) \), and \( y_c(s,t) \), respectively, and the probability that state \( s \) is realized at time \( t \) to be \( \pi(s,t) \).

1. (8 points) Solve for the Pareto allocation where the weight on the Center is one, the weights on the utility of country \( a \) is \( \Psi_a \), and country \( b \) is \( \Psi_b \).

2. (8 points) Solve for the terms of trade of country \( a \) and \( b \) relative to the Center. In fact, from now on assume that the numeraire is the Center good (set its price to one).

3. (8 points) Are the terms of trade correlated across the periphery countries? In response to which shocks (if any) are they uncorrelated?

4. (8 points) Solve for the competitive equilibrium assuming complete financial markets. Find an expression for the two stock markets in the periphery. Please, be careful writing the budget constraint. You need Arrow-Debreu assets for all three outputs. Hint: use the fact that the goods from the center have a price of one. Write the budget constraint in terms of the Center good.
5. (8 points) Show that the stock markets of the two periphery countries are perfectly correlated. What is the intuition?

6. (8 points) Assume that instead of the lack of trade due to preferences, that we assume symmetric preferences for all countries (all of them look like the one from the Center) and the reason why there is so little trade between the two periphery countries is because there are very large transport costs. Using only your intuition, how does the result from the previous question change?