Harvard University
Department of Economics

Economics 2010d: Final Examination and General Examination in Macroeconomic Theory

Spring Term 2004

PLEASE USE A SEPARATE BLUE BOOK FOR EACH PART AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

For those taking the **GENERAL EXAM** in macroeconomic theory:

1. You have **FOUR** hours.

2. Answer **ALL QUESTIONS** in Parts I, II, III, IV, and V.

3. Time allotted for each part:

I.	30 minutes
II.	30 minutes
III.	30 minutes
IV.	40 minutes
V.	80 minutes

For those taking the **FINAL EXAMINATION** in Economics 2010d (**not the General Examination**):

1. You have **THREE** hours.

2. Answer **ALL QUESTIONS** in Parts IV and V.

3. **DO NOT ANSWER** the questions in Parts I, II and III.

Part 1

General Exam Question

May 2004

You should devote no more than one-half hour to this question.

What are the implications of the following models for the effects of monetary policy on output and why

a) The traditional backward looking Phillips curve.

b) The Lucas' island model.

c) The Fischer wage contract model.

d) The Calvo pricing model

e) The Mankiw Reiss pricing model

f) The Caplin Spulber Ss model.

Part II
Question for spring 2004 macro theory generals

In both Europe and America, governments are running unusually large fiscal deficits. France and Germany are currently violating the EU's Stability and Growth Pact by having deficits in excess of 3% of their respective gross domestic product. The latest estimate of the U.S. Government's deficit for this year, from the Congressional Budget office, is more than 4% of GDP. Under current tax and spending policies, deficits of roughly this magnitude will persist, in both Europe and America, for some years to come. At the same time, both the European Central Bank and the U.S. Federal Reserve System apparently remain committed to low rates of price inflation.

Under what circumstances can persistent fiscal deficits lead to inflation even while the central bank is continuing to pursue a non-inflationary monetary policy? Be explicit about the elements of economic behavior on which such an outcome depends, as well as about any underlying assumptions on which your answer relies.

Part III

Speculative attacks on the exchange rate. Suppose that the demand for money in a small open economy is characterized by

$$\mathsf{m}_t - \mathsf{e}_t = -\eta \mathsf{r}_t, \tag{1}$$

where m is the log of the money supply, e is the log of the exchange rate, and r is the nominal interest rate. Assume that

$$r_t = r^* + \dot{\mathsf{e}}_t,$$

where $\dot{\mathsf{e}}_t \equiv \mathrm{d}\mathsf{e}_t/\mathrm{d}t$ is the expected rate of change of the exchange rate. r^* is the constant foreign interest rate, assumed equal to zero.

The central bank's balance sheet (in levels) is

$$M_t = B_{H,t} + \overline{\mathcal{E}} B_{F,t}, \tag{2}$$

where B_H denotes the central bank's holdings of domestic currency bonds, and B_F denotes its holdings of foreign currency bonds.

Assume that

$$\frac{\dot{B}_H}{B_H} = \dot{\mathsf{b}}_H = \mu, \tag{3}$$

where $\mathsf{b}_\mathrm{H} \equiv \log B_H$, for as long as $B_{F,t} > 0$.

(a) Suppose that the central bank aims to keep the exchange rate fixed as long as it has adequate reserves. What does that imply about the path of the money supply?

(b) Denote the "shadow exchange rate" $\tilde{\mathsf{e}}_t$ by

$$\tilde{\mathsf{e}}_t = \mathsf{b}_{H,t} + \eta\mu. \tag{4}$$

Draw a graph with time on the horizontal axis, giving the shadow exchange rate and the fixed exchange rate. Briefly explain why a collapse must occur on date T when $\tilde{\mathsf{e}}_T = \bar{\mathsf{e}}$, and why it cannot occur before or after this time.

(c) Draw a graph of the path of the money supply, foreign exchange reserves, and the exchange rate.

(d) In solving for the shadow exchange rate, we assumed that there are no speculative bubbles in post-attack exchange rate so that the solution was unique. What if this is not the case?

(e) What are some empirical inadequacies of this model of speculative attacks, for example, with regard to interest rates and foreign exchange reserves?

Part IV

I. Convergence Rates (40 minutes)

Consider the neoclassical growth model (Solow model) with Cobb-Douglas production function $y = Ak^{\alpha}$ (where y is output per worker, k is capital per worker, and $0<\alpha<1$), constant saving rate s, and constant population growth rate n. The rate of technological progress, x, is zero.

1. What is the equation that determines the growth rate $(1/k)\cdot(dk/dt)$?

2. Derive the formula for the effect of log(k) on $(1/k)\cdot(dk/dt)$. In what sense does this formula determine the rate of convergence for k? Is the convergence rate constant? What is the formula for the convergence rate in the neighborhood of the steady state?

3. Suppose that the saving rate, s, rises with k. Why might this relationship hold? How does this behavior of s affect the convergence rate?

4. Suppose that the population growth rate, n, falls with k. Why might this relationship hold? How does this behavior of n affect the convergence rate?

II. Money and Inflation in the Neoclassical Growth Model (40 minutes)

Consider the Sidrauski-type version of the neoclassical growth model, where households have infinite horizons. The utility function is u(c, m), where m is real per capita money holdings. Assume that technical progress, x, and population growth, n, are both zero. Assume that the economy is initially in a steady state where capital per worker is $k=k^*$. The nominal quantity of money, M, is constant. That is, the monetary growth rate, μ, is constant at zero.

1. What is the inflation rate and nominal interest rate?

Assume that the monetary authority shifts at time 0, all at once and unexpectedly, to a positive, constant monetary growth rate, μ^*. (The nominal quantity of money does not jump at time zero—it just starts to grow at a positive rate.)

2. What happens at time 0 and subsequently to k and the real interest rate, r?

3. What happens at time 0 and subsequently to real money balances, m?

4. What happens to the path of the price level, starting at time 0?

5. What happens at time 0 and subsequently to the flow of utility, u(c, m)?

6. Why is there no real-balance effect? Does this result relate to Ricardian equivalence?

Part V

This part of the exam has three questions designed to take 80 minutes in total: 30+25+25. Good luck!

Problem 1 (Equity model: 30 minutes): Assume that a consumer with only equity wealth must choose period by period consumption in a discrete-time dynamic optimization problem. Specifically, consider the sequence problem:

$$v(x_0) = \sup_{\{c_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \delta^t u(c_t)$$

subject to the constraints: $x_{t+1} = \exp(r + \sigma u_t - \sigma^2/2)(x_t - c_t)$, u_t iid and $u_t \sim N(0,1)$, $c_t \in [0, x_t]$, $x_0 > 0$. Here x_t represents wealth at period t and c_t represents consumption at period t. The consumer has discount factor $\delta = \exp(-\rho)$ and the consumer can only invest in a risky asset with expected return $\exp(r) = E\exp(r+\sigma u-\sigma^2/2)$. Finally, assume that the consumer has log utility:

$$u(c) = \ln(c).$$

a. Explain why the associated Bellman equation is given by

$$v(x) = \sup_{y \in [0,x]} u(x-y) + E\,\delta\, v(\exp(r + \sigma u - \sigma^2/2)y).$$

Explain all of the terms in the Bellman equation.

b. Guess that the value function takes the special form

$$v(x) = \psi + \phi \ln x.$$

Assuming that the value function guess is correct, derive the consumption function:

$$c = \phi^{-1} x.$$

Now verify that the Bellman Equation is satisfied only if

$$\phi = (1-\delta)^{-1}.$$

c. Show that

$$E\Delta \ln c_{t+1} = (r - \rho) + \frac{1}{2}\sigma^2 - \sigma^2.$$

d. Interpret the previous equation for the certainty case $\sigma = 0$. Explain why $\Delta \ln c_{t+1}$ increases in r and decreases in ρ.

Problem 2 (Investment Problem 25 minutes): Consider the decision to complete a late project.

- If the project is uncompleted at the beginning of a period, you pay a per-period cost (late fee) of l.
- You then draw an iid cost c (distributed DISCRETELY).
- With probability p, $c = 1/2$; and with probability $1 - p$, $c = 1$.
- If you undertake the project, you pay c, and complete the project.
- The game continues to the next period until you complete the project.
- You have a zero discount rate.

a. Why is it OK to assume no discounting in this problem?

b. Write down the Bellman Equation *assuming* that the agent *only* undertakes the problem when $c = 1/2$. Using the Bellman Equation, show that the expected value of total costs is

$$\frac{1}{2} + \frac{l}{p}.$$

Note that this is the expected value of total costs where the expectation is taken at the beginning of the period, just before *both* the current late fee, l, is paid *and* the current cost value, c, is drawn.

c. Prove that the agent will undertake investment only when $c = 1/2$ if and only if

$$l \leq \left(\frac{p}{2}\right)\left(\frac{1}{1-p}\right).$$

d. Interpret this condition.

Problem 3 (Asset Allocation Problem 25 minutes): Merton's consumption problem:

- Consumer has CRRA utility: $u(c) = \ln(c)$.
- Consumer has two assets.
- Risk free asset has rate of return r.
- Equity asset has rate of return $r + \pi$ and standard deviation σ.
- Let x represent wealth.
- Consumer invests share of wealth θ in equities and instantaneously consumes c :

$$dx = [(r + \theta\pi)x - c]dt + \theta\sigma x dz.$$

1. Derive and interpret the Bellman Equation:

$$V(x) = \max_{c,\theta} \left\{ u(c) + \left[\frac{dV}{dx}[(r + \theta\pi)x - c] + \frac{1}{2}\frac{d^2V}{dx^2}(\theta\sigma x)^2 \right] \right\}$$

2. Guess solution: $V(x) = \psi + \phi \ln x$. Derive and interpret the policy functions

$$\theta = \frac{\pi}{\sigma^2}$$
$$c = \rho x$$

3. Calibrate the model and interpret the calibrated policy functions.