Harvard University
Department of Economics

Economics 2010d: Final Examination and General Examination in Macroeconomic Theory

Spring Term 2003

PLEASE USE A SEPARATE BLUE BOOK FOR EACH PART AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

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For those taking the GENERAL EXAM in macroeconomic theory:

1. You have FOUR hours.

2. Answer ALL QUESTIONS in Parts I, II, III, and IV.

3. Time allotted for each part:
   
<table>
<thead>
<tr>
<th>Part</th>
<th>Time Allotted</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>30 minutes</td>
</tr>
<tr>
<td>II.</td>
<td>30 minutes</td>
</tr>
<tr>
<td>IIIA</td>
<td>40 minutes</td>
</tr>
<tr>
<td>IIIB</td>
<td>50 minutes</td>
</tr>
<tr>
<td>IVA</td>
<td>60 minutes</td>
</tr>
<tr>
<td>IVB</td>
<td>30 minutes</td>
</tr>
</tbody>
</table>

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For those taking the FINAL EXAMINATION in Economics 2010d (not the General Examination):

1. You have THREE hours.

Answer ALL QUESTIONS in Parts III and IV.

3. DO NOT ANSWER the questions in Parts I and II.
Question for spring 2003 macro theory generals

In an economy where (a) aggregate supply behavior is consistent with the natural rate property, so that there is no long-run trade-off between the mean rate of inflation and the mean rate of output relative to potential, but (b) price setting is subject to short-run stickiness, what determines whether the monetary policy authority faces a trade-off between the variability of inflation and the variability of output? If there is a trade-off between the variability of inflation and the variability of output (again, relative to potential), what determines its shape?

What would be the consequences for inflation if monetary policy sought to achieve the minimum possible variability of output?

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Be as explicit as you can in making clear the assumptions on which your answers rely.
Inventory sales problem: You hold an inventory of one bushel of wheat. The bushel of wheat costs you $s$ to store per period. The price per bushel, $p$, randomly varies over time and $p$ is iid. Your goal is to sell the wheat to maximize your expected discounted profits. The discount rate for this problem is $r$.

1. Explain why the Bellman Equation for this problem is given by

$$V(p) = \max \{p, \delta \cdot [-s + EV(p')]\},$$

where

$$\delta = \frac{1}{1+r}$$

2. Using intuition, explain why the solution to this problem is a ‘threshold rule’:

- sell if $p > p^*$
- wait if $p \leq p^*$

3. Plot the value function (with $p$ on the x-axis).

4. Explain why the threshold rule satisfies the equality

$$p^* = \delta \cdot [-s + EV(p)]$$

5. Assuming that $p$ is distributed uniformly between 0 and 1, solve for the threshold rule. You should be able to show that

$$p^* = \frac{\delta^{-1}}{\sqrt{\delta^{-2} + (2s - 1)}}.$$

6. Intuitively explain a) the direction in which the threshold rule changes when $\delta$ increases and b) the direction in which the threshold rule changes when $s$ increases.
Question A

A.1) Exhibit 1 shows the time series behavior of a calibrated "real" (i.e. perfectly competitive, with flexible prices) economy, when the initial capital stock per efficiency unit of labor is set at 1 percent below its nonstochastic BGP value. Two versions of the model are compared: one with inelastic labor supply (as in the Ramsey model), and one with endogenous labor supply determined by a leisure-consumption trade-off. The symbol "x" is associated with one of these two models, white circles with the other. The dashed line in the mid-left panel represents labor's response in the elastic labor-supply version. Which symbol denotes the endogenous-labor economy: "x" or circles? Why? Briefly comment on each of the six panels and rationalize the behavior of the two economies individually, and in comparison with each other.

A.2) Exhibit 2 reports some statistics from the simulation of a version of the RBC model. In particular, the authors have estimated the historical time series of the (standard) Solow residual, and have then "fed" to the model economy this estimated time series, as the exogenous path of technology. Based on your knowledge of business cycle "facts," would you say that based on this Table this RBC model performs reasonably well? Where is it especially successful, and where is it especially deficient (if anywhere)?

A.3) Consider the following description of the production side of the economy:

\[
\begin{align*}
Y_t &= A_t(X_tK_t)^{1-\alpha}(N_t)^{\alpha} \\
K_{t+1} &= I_t + [1 - \delta(X_t)]K_t
\end{align*}
\]

A.3.1) What is the usual interpretation of \(X_t\) in DSGE models? (Hint: it's a choice variable).

A.3.2) Once the optimal choice of \(X_t\) is taken into account, the production function in log-linearized form can be written as:

\[ y_t = a_t + (1 - \alpha)k_t + \alpha n_t + \frac{(1 - \alpha)}{\alpha + \xi} (a_t - \alpha k_t + \alpha n_t), \]

where lower-case letters denote log-deviations from the nonstochastic BGP, and \(\xi = X_*^{(\alpha)}(X_t^*)/\delta'(X_t^*)\). Derive this result, and discuss the economic interpretation of \(\xi\).

A.3.4) Exhibit 3 presents the impulse response function of labor input to a one percent (and fairly persistent) productivity increase under three assumptions on \(\xi\):
\( \xi = \infty, \xi = 1/5; \) and \( \xi = 1/10. \) The model is otherwise the work-horse RBC model (with a large elasticity of labor supply). Say which of the three sets of impulse response functions corresponds to each value of \( \xi. \) Explain your reasoning.
Figure 6

EXHIBIT 1
Table 3
Business Cycle Statistics for Basic RBC Model

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Relative Standard Deviation</th>
<th>First Order Autocorrelation</th>
<th>Contemporaneous Correlation with Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1.39</td>
<td>1.00</td>
<td>0.72</td>
<td>1.00</td>
</tr>
<tr>
<td>C</td>
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<td>0.44</td>
<td>0.79</td>
<td>0.94</td>
</tr>
<tr>
<td>I</td>
<td>4.09</td>
<td>2.95</td>
<td>0.71</td>
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<tr>
<td>N</td>
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<td>0.48</td>
<td>0.71</td>
<td>0.97</td>
</tr>
<tr>
<td>Y/N</td>
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<td>0.54</td>
<td>0.76</td>
<td>0.98</td>
</tr>
<tr>
<td>w</td>
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<td>0.54</td>
<td>0.76</td>
<td>0.98</td>
</tr>
<tr>
<td>r</td>
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<td>0.95</td>
</tr>
<tr>
<td>A</td>
<td>0.94</td>
<td>0.68</td>
<td>0.72</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: All variables have been logged (with the exception of the real interest rate) and detrended with the HP filter.
EXHIBIT 3
Question B

Consider a World with $I$ countries and 2 goods, a traded good ($T$) and a non-traded one ($N$).

B.1) Suppose that in country $i$ preferences are given by

$$
C_i = \frac{(C_i^T)^\delta (C_i^N)^{1-\delta}}{\delta^\delta (1-\delta)^{1-\delta}}
$$

(1)

What is the consumption-based price index (the minimum cost of purchasing one unit of $C$)? (If you know it you don't need to show the algebra).

B.2) In country $i$ the two goods are produced according to the technologies

$$
Y_i^T = A_i^T L_i^T
$$

and

$$
Y_i^N = A_i^N L_i^N
$$

where $L_i^q$ is labor used in sector $q$, and the $A$s are exogenous country-specific parameters. Labor is immobile across countries but fully mobile across sectors. Both sectors are perfectly competitive. Find a formula for the real exchange rate between country $i$ and country $j$, in terms of the exogenous parameters characterizing these economies. State what your formula implies, and offer an intuition.

B.3) There have been many attempts at explaining real exchange rates using the theorem you proved (or at least hopefully stated) in part (2). Is the current consensus that this model works well to explain real exchange rates between developing and developed countries? Within developed countries? Over time in the long run? Over time in the short run?

B.4) Suppose now that there are $I$ traded goods. In country $i$ preferences are still as in (1), but $C_i^T$ is now defined as

$$
C_i^T = \left\{ \sum_j [C(j)_i^T]^{\frac{n-1}{n}} \right\}^{\frac{n}{n-1}}
$$

Assume that each country produces one and only one of the $I$ traded goods, and that each of the traded goods is produced by a different country. Traded-sector producers
behave as monopolists, while the non-traded sector continues to operate under perfect competition. Assume that $I$ is (and is perceived to be) very large, and that price setters take other producers’ prices and wages as given when they set their prices. What is the demand curve faced by each individual tradeable producer? What price will they charge (as a function of the things they take as given)?

B.5) Assume that each country has balanced trade, i.e. the value of its consumption equals the value of its production. Also, each country’s labor endowment is an exogenous $L_i$. Write down the system of equations that pins down the equilibrium values of $W_i$, $Y_i^T$, $C_i^T$, $C_i^N$, $C_i$, $P(j)^T$ (the world price of the tradable good produced in country $i$), $P_i^N$ (the price of the nontradeables in country $i$), and $P_i$ (the consumption-based price index in country $i$), for $i = 1...I$. (Hint: it may be useful – though not strictly necessary – to introduce a “consumption-based” price index for tradable goods, $P_i^T = \left\{ \left[ \sum_j P(j)^T \right]^{1-\eta} \right\}^{\frac{1}{1-\eta}}$, which should make sense to you based on your work in the previous question. Your system may well involve additional endogenous variables, in which case you should also supply the additional equations).

B.6) Using the system of equations you have written down, it is possible to show that the real exchange rate between countries $i$ and $j$ is

$$
\frac{P_i}{P_k} = \left[ \left( \frac{L_i}{L_k} \right)^{\frac{1}{\eta}} \left( \frac{A_k^T}{A_i^T} \right)^{\frac{(1-\eta)}{\eta}} \frac{A_k^N}{A_i^N} \right]^{1-\delta}
$$

Provide an intuition for this result, and for its differences with the result in Part 2.
I. 60 minutes. A variant of the neoclassical growth model.

Consider the neoclassical growth model, where the production function is \( Y = F(K, L) \). People seek to maximize the integral of utility, \( u(c) \), over an infinite horizon, where \( c \) is consumption per person. Suppose that each person always works one unit per unit of time and that the level of population is constant at the value \( N \) (the population growth rate, \( n \), equals zero). There is no technological progress (so that the rate of labor-augmenting technological progress, \( x \), is zero).

Assume that people differ by productivity and that person \( i \) has the effective quantity of labor \( L_i \). Normalize so that the mean of \( L_i \) is one. The total of labor input, \( L \), equals the sum of the \( L_i \) and is therefore equal to the population, \( N \). Define the economy-wide wage rate, \( w \), so that it is measured per unit of effective labor. Let the economy-wide real interest rate be \( r \).

1. What is the flow budget constraint for person \( i \)?
2. What is the first-order condition for person \( i \)'s growth rate of consumption?
3. What are the dynamic equations for the economy-wide changes in capital per effective worker, \( k = K/L \), and consumption per person, \( c = C/L \)?
4. How do the economy-wide results differ from those of the standard neoclassical growth model? In what ways do persons with higher labor productivity differ from those with lower productivity? What do we learn by including heterogeneity of labor productivity into the model?
5. How would you introduce exogenous, labor-augmenting technological progress at the rate \( x \) into this model? Now how do the results differ from those of the standard model?
6. How would you introduce exogenous population growth at the rate \( n \) into this model? Now how do the results differ from those of the standard model?
II. 30 minutes. Questions about public debt.

1. Does Ricardian equivalence hold in the usual neoclassical growth model? Explain, being sure to include a definition of Ricardian equivalence. You should also discuss the roles of Ponzi schemes (chain letters run by the government), infinite horizons of households, and lump-sum taxes.

2. What is the idea of tax-rate smoothing? How does this concept relate to the behavior of the public debt? Why and when would you expect tax-rate smoothing to apply or not apply? How do the results relate to the existence of state-contingent public debt?

3. Discuss the idea of strategic debt. What does this theory imply about the behavior of public debt? When would these strategic considerations be important for the behavior of the debt?