Economics Honors Exam 2008 Solutions Question 5

(a) (12 points) Output can be decomposed as

$$Y = C + I + G.$$

And we can solve for it by substituting in equations given in the question,

$$\begin{aligned} Y &= C + I + G \\ &= c_0 + c_1 Y_D + I + G \\ &= c_0 + c_1(Y - T) + I + G \\ &= c_0 + c_1(Y - t_0 - t_1 Y) + I + G \\ \Rightarrow \qquad (1 - c_1 + c_1 t_1)Y &= c_0 - c_1 t_0 + I + G. \end{aligned}$$

Therefore the equilibrium output is,

$$Y = \frac{c_0 - c_1 t_0 + I + G}{1 - c_1 + c_1 t_1}.$$

Partial credit awards

3 points: for realizing $Y = C + I + G$
2 points: for substituting in $C = c_0 + c_1 Y_D$
2 points: for substituting in $T = t_0 + t_1 Y$
2 points: for substituting in $Y_D = Y - T$
3 points: for getting the final answer correct

(b) (12 points) The multiplier is

$$\frac{1}{1 - C_1 + c_1 t_1} < \frac{1}{1 - c_1}.$$

The economy responds less to changes in autonomous spending when t_1 is positive.

After a positive change in autonomous spending, the increase in total taxes (because of the increase in income) reduces consumption and tends to lessen the increase in output.

Partial credit awards

4 points: for getting the multiplier correct
4 points: for realizing that the economy responds less when t_1 is positive
0-4 points: depending on the quality of the explanation

(c) (6 points) Because of the automatic effect of taxes on the economy, the economy responds less to changes in autonomous spending than in the case

where taxes are independent of income. So output tends to vary less and fiscal policy is called an automatic stabilizer.

Partial credit awards

0-6 points: depending on the quality of the explanation

Economics Honors Exam 2008 Solutions Question 6

(a) (10 points) Since the rate of growth of E is 0, E is constant. Letting $A \equiv E^{1-\alpha}$, we can rewrite the aggregate production function as

$$\begin{aligned} Y &= E^{1-\alpha} \cdot K^{\alpha} \cdot L^{1-\alpha} \\ &= A \cdot K^{\alpha} \cdot L^{1-\alpha}. \end{aligned}$$

Thus income per worker, y, can be written as.

$$\begin{aligned} y &= \frac{Y}{L} \\ &= A \cdot \left(\frac{K}{L}\right)^{\alpha} \\ &= A \cdot k^{\alpha}. \end{aligned}$$

In steady state, we have

$$\Delta k = sy - (\delta + n)k.$$

Substituting in the expression for k, we can get

$$\begin{aligned} & s \cdot Ak^{\alpha} - (\delta + n)k &= 0 \\ \Rightarrow \quad & k^{1-\alpha} &= \tfrac{sA}{\delta+n} \\ \Rightarrow \quad & k^{*} &= \left(\tfrac{sA}{\delta+n}\right)^{\frac{1}{1-\alpha}}. \end{aligned}$$

Substituting into the expression for y, we get

$$\begin{aligned} y^{*} &= A \cdot A^{\frac{\alpha}{1-\alpha}} \cdot \left(\frac{s}{\delta + n}\right)^{\frac{\alpha}{1-\alpha}} \\ &= A^{\frac{1}{1-\alpha}} \cdot \left(\frac{s}{\delta + n}\right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Partial credit awards

1 point: for realizing that E is constant
2 points: for getting the expression $y = A \cdot k^{\alpha}$ correct
1 point: for realizing that in steady state, $\Delta k = 0$
2 points: for getting the expression $sy - (\delta + n)k = 0$ correct
2 points: for getting the steady-state k^{*} correct
2 points: for getting the steady-state y^{*} correct

(b) (3 points) Consumption per worker is

$$\begin{aligned} c^* &= (1-s)\cdot y^* \\ &= (1-s)\cdot A^{\frac{1}{1-\alpha}}\cdot\left(\frac{s}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Partial credit awards

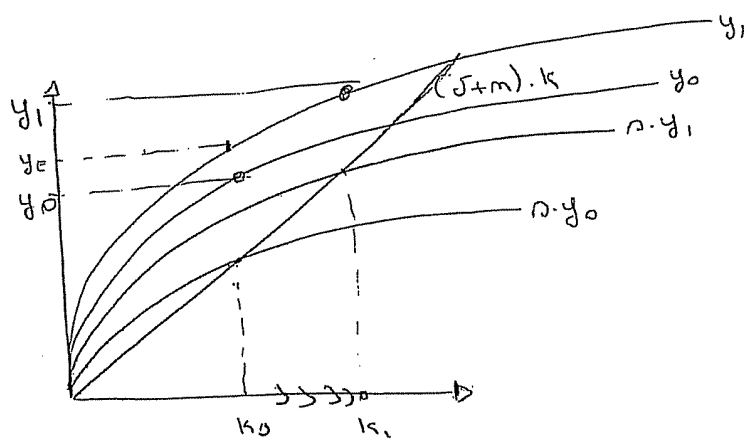
2 points: for getting $c^* = (1-s)\cdot y^*$ correct
1 point: for getting final answer correct

(c) (3 points) We know that $y = Ak^\alpha$, while $A \equiv E^{1-\alpha}$. Therefore if E increases, output per worker would increase as well.

Partial credit awards

1 point: for realizing that y increase on the day of the change
0-2 points: depending on the quality of the reasoning

(d) (7 points) An increase in E is equivalent to improved efficiency in the production function:



As could be seen from the graph, in the new steady state, both capital per worker (k) and output/income per worker (y) are higher, therefore the transition path is illustrated overleaf:

Partial credit awards

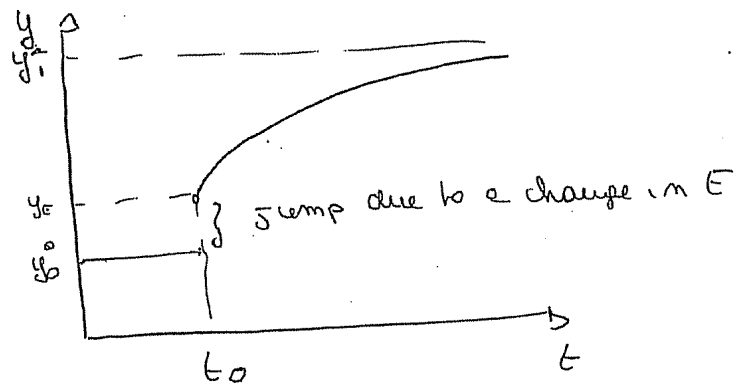
2 points: for realizing that income per worker keeps on increasing over the transition path

0-2 points: depending on the quality of the reasoning (does not necessarily have to draw the first graph)

1 point: for drawing a discrete jump at t_0 in the second graph

1 point: for showing that y increases over time after t_0 in the second graph

1 point: for showing that y converges to the new steady state value in the second graph



(e) (7 points) Immediately after the shock, there are two competing effects: E increases but capital stock is destroyed, hence the efficiency gain is offset by the capital loss. The net effect on initial output per worker is ambiguous. If the drop in capital stock dominates the increase in E, output per worker would actually drop on the day of the change. Otherwise, output per worker would still jump up on the day of the change, though to a lesser extent than in part (c).

Over time, output per worker is going to be higher since k^* is higher in the new steady state. The new graphs are shown below:

Partial credit awards

1 point: for realizing that the immediate effect of destruction of capital stock is to reduce y

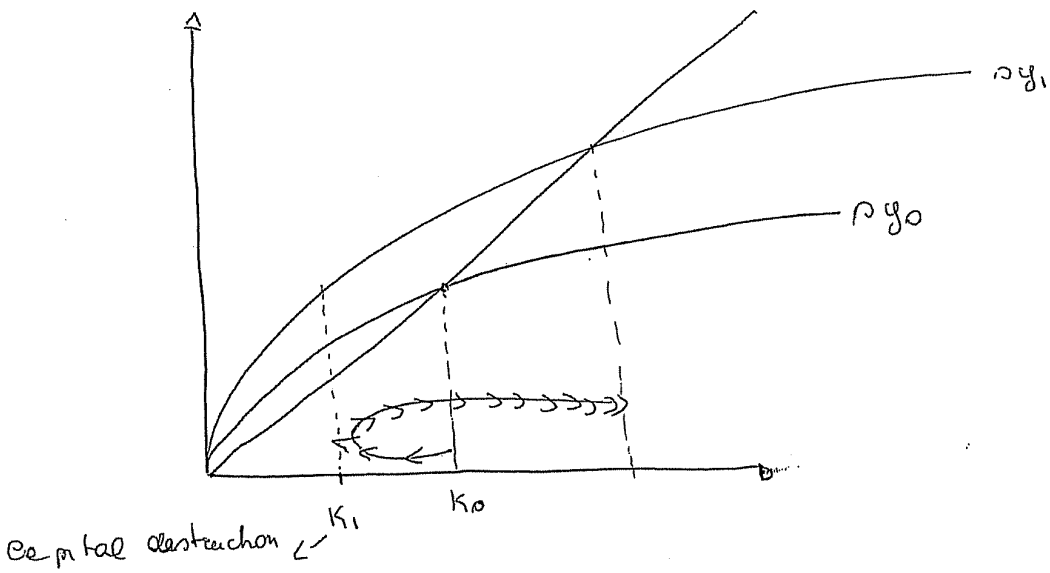
2 points: for realizing that the net effect on income per worker at t_0 is ambiguous

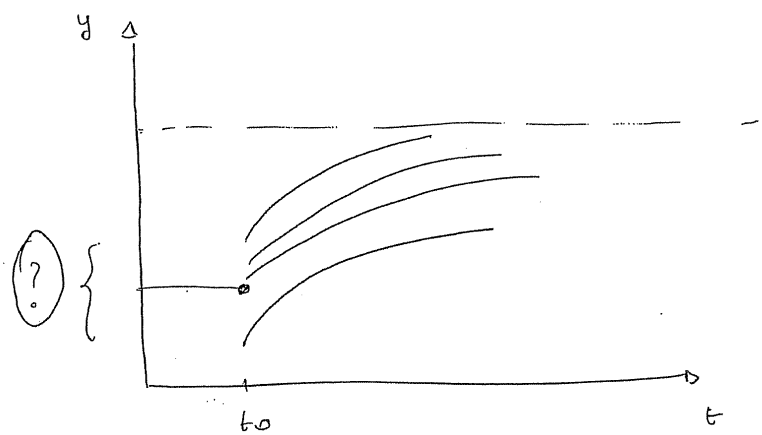
1 point: for realizing that y is going to be higher in the new steady state (does not necessarily have to draw the first graph)

1 point: for drawing at least two curves in the second graph, with a discrete jump up or down respectively at t_0

1 point: for showing that y increases over time after t_0 in the second graph

1 point: for showing that y converges to the new steady state value in the second graph


sy_1
sy_0
K_1
K_0
capital destruction


y
?
t_0
t

Economics Honors Exam 2008 Solutions Question 7

(a) (6 points) Consumption is:

$$\begin{aligned} C_t &= Y_t - G - K_{t+1} \\ &= G^{\frac{1}{2}} K_t^{\frac{1}{4}} - G - K_{t+1} \end{aligned}$$

Differentiating this with respect to G gives:

$$\frac{dC_t}{dG} = \frac{1}{2} G^{-\frac{1}{2}} K_t^{\frac{1}{4}} - 1$$

Setting this equal to zero gives:

$$G = \frac{1}{4} K_t^{\frac{1}{2}}$$

Partial credit awards

2 points: for getting the expression for consumption C_t correct
1 point: for differentiating C_t with respect to G and setting this to zero
2 points: for solving the differentiation problem correctly
1 point: for getting the final answer $G = \frac{1}{4} K_t^{\frac{1}{2}}$ correct

(b) (12 points) The household's budget constraint is:

$$C_t = G^{\frac{1}{2}} K_t^{\frac{1}{4}} - G - K_{t+1}$$

Substituting this into the utility function of the representative agent gives:

$$U = \sum_{t=0}^{\infty} \beta^t u(G^{\frac{1}{2}} K_t^{\frac{1}{4}} - G - K_{t+1})$$

The consumer does not take government spending as given, therefore G must be substituted out of this expression, i.e.:

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \beta^t u(\frac{1}{2} K_t^{\frac{1}{4}} K_t^{\frac{1}{4}} - K_{t+1} - \frac{1}{4} K_t^{\frac{1}{2}}) \\ &= \sum_{t=0}^{\infty} \beta^t u(\frac{1}{4} K_t^{\frac{1}{2}} - K_{t+1}) \end{aligned}$$

Taking first order conditions with respect to K_t gives:

$$0 = -u'(\frac{1}{4} K_{t-1}^{\frac{1}{2}} - K_t) + \beta \frac{1}{8} K_t^{-\frac{1}{2}} u'(\frac{1}{4} K_t^{\frac{1}{2}} - K_{t+1})$$

Therefore the first order condition gives:

$$1 = \beta \frac{1}{8} K_t^{-\frac{1}{2}}$$

$$\Rightarrow K_t = (\frac{8}{\beta})^{-2}$$

Partial credit awards

2 points: for substituting the expression for C_t into the lifetime utility function
2 points: for substituting $G = \frac{1}{4} K_t^{\frac{1}{2}}$ in
2 points: for getting $U = \sum_{t=0}^{\infty} \beta^t u(\frac{1}{4} K_t^{\frac{1}{2}} - K_{t+1})$ correct
2 points: for taking first order condition with respect to K_t
2 points: for solving the differentiation problem correctly
2 points: for getting the final answer $K_t = (\frac{8}{\beta})^{-2}$ correct

(c) (9 points) The household's budget constraint is:

$$C_t = G_t^{\frac{1}{2}} K_t^{\frac{1}{4}} - G_{t+1} - K_{t+1}$$

Substituting this into the utility function of the representative agent gives:

$$U = \sum_{t=0}^{\infty} \beta^t u(G_t^{\frac{1}{2}} K_t^{\frac{1}{4}} - G_{t+1} - K_{t+1})$$

Taking first order conditions with respect to G_t gives:

$$0 = -u'(G_{t-1}^{\frac{1}{2}} K_{t-1}^{\frac{1}{4}} - G_t - K_t) + \beta \frac{1}{2} G_t^{-\frac{1}{2}} K_t^{\frac{1}{4}} u'(G_t^{\frac{1}{2}} K_t^{\frac{1}{4}} - G_{t+1} - K_{t+1})$$

In steady state the arguments are the same, therefore:

$$G_t^{\frac{1}{2}} = \frac{\beta}{2} K_t^{\frac{1}{4}} \Rightarrow \; G_t = \frac{\beta^2}{4} K_t^{\frac{1}{2}}$$

Partial credit awards

2 points: for getting the new expression for C_t correct (note the subscripts of G)
1 point: for substituting C_t into the lifetime utility function
2 points: for taking first order condition with respect to G_t
2 points: for solving the differentiation problem correctly
1 point: for realizing that in steady state, G is constant
1 point: for getting the final answer $G_t = \frac{\beta^2}{4} K_t^{\frac{1}{2}}$ correct

(d) (3 points) This is less than the answer from part (a). It is because there is an extra cost to government spending now, in that it must be from savings. Because agents are impatient, this means that it is less desirable.

Partial credit awards

1 point: for realizing that the new government spending level is lower
0-2 points: depending on the quality of the reasoning

Economics Honors Exam 2008 Solutions Question 8

(a) (6 points)

$$\begin{aligned}\dot{K} &= I - \delta K \\ &= sY - \delta K \\ &= sA(t)(K(t))^{\alpha}(L(t))^{1-\alpha} - \delta K(t)\end{aligned}$$

$$\begin{aligned}\frac{\dot{k}}{k} &= \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \\ &= sA(t)(k(t))^{\alpha-1} - \delta - n\end{aligned}$$

Partial credit awards

1 point: for realizing that $\dot{K} = I - \delta K$
2 points: for getting the expression $\dot{K} = sA(t)(K(t))^{\alpha}(L(t))^{1-\alpha} - \delta K(t)$ correct
1 point: for realizing that $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$
2 points: for getting the final answer correct

(b) (6 points)

$$y_t = A(t)k(t)^{\alpha}$$

In steady state, it must be that y and k grow at the same rate, call it g. Therefore it must be that:

$$(1-\alpha)g = \frac{\dot{A}}{A}$$

i.e. the growth of A must be constant. The growth of A is given by:

$$\frac{\dot{A(t)}}{A(t)} = \frac{y(t)}{A(t)} = k(t)^{\alpha}$$

This must be constant, i.e. $k(t)$ must be constant. However, it is not because if it were then $\frac{\dot{k}}{k}$ would be growing constantly over time, which can be seen from part (a).

Partial credit awards

1 point: for acknowledging that this model does have such a steady state
1 point: for getting $y_t = A(t)k(t)^{\alpha}$ correct
1 point: for realizing that y and k must grow at the same rate in steady state

1 point: for getting $(1-\alpha)g = \frac{\dot{A}}{A}$ correct
1 point: for getting $\frac{\dot{A(t)}}{A(t)} = k(t)^{\alpha}$ correct
1 point: for realizing that the growth rate of A is constant

(c) (9 points)

$$(1-\alpha)g = \frac{\dot{A(t)}}{A(t)} = \frac{k(t)^{\beta}}{A(t)}$$

Therefore if the last part is constant then $k(t)$ grows at a rate $\frac{1}{\beta}$ the growth rate of A, which is consistent with $\frac{\dot{A(t)}}{A(t)} = (1-\alpha)g$ if $(1-\alpha) = \beta$.

Partial credit awards

2 points: for getting $\frac{\dot{A(t)}}{A(t)} = \frac{k(t)^{\beta}}{A(t)}$ correct
2 points: for equating this to $(1-\alpha)g$
3 points: for realizing that $\frac{\dot{A(t)}}{A(t)}$ is constant if $k(t)$ grows at a rate $\frac{1}{\beta}$ the growth rate of A
2 points: for realizing that this is satisfied if $(1-\alpha) = \beta$

(d) (6 points) Here, savings affect growth. It does not in the other case. The more patient people are, the higher the optimal s will be. The usual 'Golden Rule' does not depend on the time preference.

Partial credit awards

2 points: for realizing that savings affect growth here, while it does not in the other case
2 points: for realizing that the more patient people are, the higher the optimal s will be
2 points: for realizing that the usual 'Golden Rule' does not depend on the time preference

(e) (3 points) Learning by doing.