HARVARD UNIVERSITY
DEPARTMENT OF ECONOMICS

General Examination in Macroeconomic Theory

SPRING 2014

You have **FOUR** hours. Answer all questions

Part A (Prof. Laibson): 48 minutes
Part B (Prof. Aghion): 48 minutes
Part C (Prof. Basu): 72 minutes
Part D (Prof. Rogoff): 72 minutes

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Macro General Exam
May 2014

Problem 1 (13 minutes): State a set of assumptions on beliefs, preferences, and technology, which jointly imply that marginal utility is a random walk. The weaker your assumptions the more credit you get.

Problem 2 (20 minutes): State Blackwell's Theorem. Then apply Blackwell's Theorem to show that the following functional operator, B, is a contraction mapping with respect to the supremum metric and the space of bounded functions on $[0, \infty)$:

$$Bf(x) \equiv \sup_{0 \leq c \leq x} u(c) + \delta f((x - c)R).$$

Here u is a bounded function on $[0, \infty)$ and δ and R are non-negative constants. Explain what assumptions, if any, you needed to make on δ and R. [Note that I am not asking you to prove Blackwell's Theorem, just to state and apply it.]

Problem 3 (15 minutes; True, False, or Partially True): Please explain whether the following statements are True, False, or Partially True. You will be graded on the quality of your *explanation*.

a. Smooth pasting is sufficient for value matching.

b. If (i) agents are rational, and, (ii) liquidity constraints never bind in equilibrium, then consumption growth at date t should not be correlated with the expectation at date $t - 1$ of income growth at date t.

c. Assume that a functional operator, D, is a contraction mapping on S. Then D has a unique fixed point on S.

Philippe Aghion Question
Part B (48 minutes)
Macro Spring Generals

1) Discuss the various effects of competition on innovation.

I. (52 minutes)

Take a closed economy with no government purchases, so

$$Y_t = C_t + I_t.$$

The law of motion for capital is

$$K_{t+1} = (1-\delta)K_t + I_t.$$

Firms have the production function

$$Y_t = K_t^{\alpha}(Z_t H_t)^{1-\alpha}.$$

There is no steady-state growth, so the steady-state level of Z is a constant, Z^*. Fluctuations in this economy are driven by persistent but transitory shocks to technology:

$$\hat{Z}_t = \rho \hat{Z}_{t-1} + \varepsilon_t,$$

where a ^ denotes a log deviation from steady state.

A. Suppose the model is closed by assuming that the representative household maximizes:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{C_{t+j}^{1-\chi}}{1-\chi} + \log\left(\bar{L} - L_{t+j}\right) \right],$$

subject to a standard budget constraint. Assume $\chi > 0$ and $\beta < 1$. Firms' labor input equals lost leisure: $H_t = L_t$. Assume that a positive shock to technology ($\varepsilon > 0$) will lead to increases in Y, C and I. Does L definitely increase when Z improves? Comment on the roles of the parameters χ and ρ in determining the outcome for L.

B. Now suppose we build a different model, by assuming different consumption and labor supply behavior. We assume $C_t = (1-s)Y_t$. Workers supply $\bar{L}$ units of labor inelastically every period. However, labor input to production is $H_t = L_t\, e(W_t)$, where W is the real wage, $e' > 0$, and L_t is the number of hours. Firms maximize over the real wage they pay, as well as their choices of L and K. (Assume that all relevant second-order conditions for maximization are satisfied.)

Assume that in equilibrium the profit-maximizing real wage for firms is always higher than the wage that would equate labor supply and labor demand in this economy. Call this model, which draws on Solow (1956) and Solow (1979), the Solow-Solow model.

What are the effects of a positive shock to technology ($\varepsilon > 0$) on Y, C, L and I in this model? Discuss the effects on impact and over time.

C. How does a positive technology shock affect the real wage, *W*, in the Solow-Solow model? Relate your answer to the Barro-King argument that a procyclical real wage is a necessary condition for consumption and work hours to both be procyclical.

D. Suppose we are interested in matching the following basic facts of business cycles, where σ denotes a standard deviation:

i) $\sigma_{\hat{I}} > \sigma_{\hat{Y}} > \sigma_{\hat{C}}$,

ii) $\sigma_{\hat{Y}} \approx \sigma_{\hat{L}}$,

iii) $\hat{Y}$, $\hat{C}$, $\hat{I}$, and $\hat{L}$ commove positively, and

iv) the unemployment rate, u_t, is countercyclical.

(Note that the first three facts relate to *log* deviations from the steady state, while the fourth is about the *level* of the unemployment rate.)

How well does the neoclassical model of Part A match each of the four facts? How well does the Solow-Solow model of Part B match each of the four facts? Comment on the reasons for the strengths and weaknesses of each model.

In your answer, be sure to define what you mean by "unemployment" when you discuss each model.

E. Suppose you want to improve the ability of the neoclassical model in part A to match the four sets of facts in part D. Can you change parts of the model, while keeping the setting neoclassical (no efficiency wages, as in part B!), that would enable the model to match these facts better? Say what you would change, and why your modifications would improve the model. Again, note the concept of unemployment that you are using in this context.

II. (20 minutes)

Suppose that households maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \log\left(C_{t+j}\right)$$

subject to a standard budget constraint. Households supply one unit of labor inelastically in every period, and receive a wage W_t. $0 < \beta < 1$.

A. First assume that the real interest rate is fixed, and $\beta(1+r)=1$. Suppose there is an unexpected increase in the wage of size ΔW at time t, and at the same time households are told that the wage will decline by $\Delta W(1 + r)$ at time t+1. What is the effect on the time path of consumption starting from t?

B. Now, with the same preferences for the household, assume that output is produced by a neoclassical firm with the technology $Y_t = Z_t L_t$. (However, labor supply is still inelastic.) There is no capital or investment, so $C_t = Y_t$. In this setting, suppose there is an unexpected increase in Z of size ΔZ at time t, and at the same time everyone knows that Z will decline by ΔZ at time t+1. What is the effect on the time path of consumption starting from t?

C. Reconcile your answers in Parts A and B with the intuition of the Permanent Income Hypothesis.

Part IV ROGOFF

May 15 2014

Instructions: Please answer all four questions in this section. You do not need to give every intermediate step. All four parts have the same weight. You do not need to use a separate blue book for each question, but **PLEASE USE A SEPARATE BLUE BOOK FOR QUESTION 4.**

1. Escaping from a Liquidity trap

Consider a closed cash-in-advance endowment economy, with flexible prices, in which there is no investment and the representative agent has exogenous income stream Y_t and has utility given by

$$U = \sum_{s=t}^{\infty} \beta^{s-t} \, \mathsf{log}(C_s) \tag{1}$$

The transaction technology in this economy is governed by the cash-in-advance constraint $P_t C_t \leq M_t$ where M_t is the money supply at time t. *For the moment, we will make the conventional assumption that money does not pay interest.*

(a) Making use of the Euler condition for the intertemporal maximization problem and goods market clearing condition, write the equation that relates the nominal interest rate i_{t+1} (on a one-period bond that pays off a non-indexed cash amount in period $t+1$) in terms of Y_t, P_t, Y_{t+1}, P_{t+1}.

(b) Assume that M is fixed for all periods after the present period $\left(M_s = \bar{M} : \forall\, s > t\right)$, and that income $Y_s = \overline{Y}$ is constant $\forall$ $s > t$, but initial income $Y_t > \overline{Y}$. Under what conditions can the economy be in a liquidity trap where $i_{t+1} = 0$?

(c) What if the government finds a way to charge a tax on money (or equivalently pays a negative interest rate)? Assume tax on money is set permanently at τ, and the proceeds are rebated in lump-sum form. How does a tax on money affect the zero bound on interest rates as a constraint on current-period inflation? *IT IS SUFFICIENT TO EXPLAIN WHAT HAPPENS CONCEPTUALLY.*

(d) Suppose that in the current period t, the government institutes a TEMPORARY consumption subsidy at rate σ, paid for by a lump-sum tax T_t. In real terms, individuals now face a period-t budget constraint

$$C_t(1-\sigma) = Y_t - T_t$$

Since $Y_t = C_t$ in equilibrium, then in equilibrium $T_t = \sigma Y_t$. The subsidy is temporary and in all future periods $\sigma = 0$ and $T = 0$. Under these circumstances, it can be shown that because agents do not internalize the effect of their consumption on the tax, the real interest rate is defined by

$$\frac{u'(C_t)}{1-\sigma} = \beta(1+r_{t+1})u'(C_{t+1})$$

while the cash-in-advance constraint $P_t C_t \leq M_t$ is unchanged. Can this consumption subsidy help escape the liquidity trap? What do you think would happen if the subsidy is permanent and equally affects the current period and all future periods?

2. Avoiding a first-generation speculative attack

Consider the first-generation speculative attack model of Krugman (1979). Suppose that the demand for money in a small open economy is given by

$$\mathsf{m}_t - \mathsf{e}_t = -\eta\dot{\mathsf{e}}_t + \mathsf{y}_t \tag{2}$$

where m is the log of the money supply, e is the log of the exchange rate, y is the log of output (assumed to follow an exogenous process), and $\dot{\mathsf{e}}_t \equiv \mathrm{d}\mathsf{e}_t/\mathrm{d}t$.

The country seeks to maintain a fixed (log) exchange rate at $\bar{\mathsf{e}}$, so the (log) money supply must be set at

$$\bar{\mathsf{m}}_t = \bar{\mathsf{e}} + \mathsf{y}_t \tag{3}$$

Until part (d), you may assume $\mathsf{y}_t = 0\ \forall t$.

A simplified version of the central bank's balance sheet (in levels) is

$$M_t = B_{H,t} + \overline{\mathcal{E}}B_{F,t}$$

where $B_{H,t}$ and $\overline{\mathcal{E}}B_{F,t}$ are stocks of domestic bonds and foreign bonds (reserves), respectively.

Suppose that starting at date 0, the fiscal authority of the country in question requires the central bank to expand its holdings of domestic bonds at rate μ. Thus,

$$\frac{\dot{B}_H}{B_H} = \dot{\mathsf{b}}_H = \mu$$

where $\mathsf{b}_H \equiv \log B_H$. The central bank attempts to maintain the peg in (3) as long as it can, but we assume that its mandate to absorb the supply of domestic bonds takes precedence.

(a) Define the shadow exchange rate $\tilde{\mathsf{e}}_t$ (the rate that would prevail at time t should a run clean out central bank reserves at time t). Show that this is given by

$$\tilde{\mathsf{e}}_t = \mathsf{b}_{H,t} + \eta\mu$$

by arguing that it is consistent with the money demand equation in (2) once the peg is broken.

(b) Using the expression for the shadow exchange rate above, provide a condition which implicitly defines the time T at which the speculative attack will occur. What is the reason for this condition? Solve for T in terms of $\bar{\mathsf{e}}, \mathsf{b}_{H,0}, \eta$, and μ.

The following parts of the question consider how a country might avoid the speculative attack you have analyzed above.

(c) Fiscal discipline. Suppose that ex-ante (at date 0), the country's Finance Minister chooses μ. For what values of $\mu \geq 0$ can the speculative attack be avoided at all t? Why?

(d) Growth. Now suppose that rather than being constant, the exogenous output process follows $\dot{\mathsf{y}}_t = \theta$. If $\mu < \theta$, can the speculative attack be avoided at all t? Why? You may assume that if the country chooses to buy foreign bonds with money it prints, there is no upper bound on how many foreign bonds it can purchase. *(Hint): Given $\dot{\mathsf{y}}_t = \theta$, what money growth rate is consistent with a fixed exchange rate in the demand for money equation?*

3. The costs of financial autarky and reputational insurance contracts

Consider the problem of a small endowment economy where there is no investment or government spending. The economy is inhabited by a representive agent with utility function

$$U_t = \mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\rho}}{1-\rho} \right\}$$

and endowment $Y_s = \overline{Y} \exp\left[\epsilon_s - \frac{1}{2}\sigma_\epsilon^2\right]$, where ϵ_s is normal i.i.d. over time with mean zero and variance σ_ϵ^2 . Note that there is no trend growth, so $\mathrm{E}_t \left\{C_s^{1-\rho}\right\}$ is a constant over time. Assume that $\rho > 1$.

(a) Assuming there is no investment or government spending, confirm that under financial autarky

$$E_t \left\{C_s^{1-\rho}\right\} = \overline{Y}^{1-\rho} \exp\left[-\frac{1}{2}(1-\rho)\rho\sigma_\epsilon^2\right] .$$

(Hint: recall that if $X \backsim N(\mu_x, \sigma_x^2)$, then $\exp X$ is distributed lognormally with mean

$$E \exp(X) = \exp\left(\mu_x + \frac{1}{2}\sigma_x^2\right) .$$

Note: You should be able to answer parts (b) – (e) even if you do not get this part.)

(b) Given your answer to (a), what is $\mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\rho}}{1-\rho} \right\}$ in terms of $\overline{Y}$ and σ_ϵ^2 ?

(c) We continue to assume the country has no access to borrowing and lending (eg, risk-free bonds) but it does have access to reputational insurance contracts that allow the country to have $C_s = \overline{Y}$ in all periods where the contract is in force. What is $\mathrm{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\rho}}{1-\rho} \right\}$ in this case, and what is the difference between expected utility under autarky and expected utility under the perfect insurance contract?

(d) Assume that the reputational insurance contracts involve a trigger strategy equilibrium where as long as the country pays out as required in states of nature where its output shock ϵ_s is high, it maintains its reputation. If, however, the country ever fails to make a payout in full, it will lose its reputation and be forced permanently to return to autarky in all future periods. What condition involving β, ρ and σ_ϵ^2 must hold for the full insurance contract to be sustainable in a reputational equilibrium? (If you cannot give an analytical answer, try to give a qualitative answer.)

(e) *FOR THIS PART, A QUALITATIVE ANSWER IS COMPLETELY SUFFICIENT.* Now assume that the condition you solved for in part (d) does not hold. Can the country still achive partial insurance through reputation-based insurance contracts? If so, can it at least get perfect insurance across bad states of nature where the country is receiving payments from abroad?

4. Please give SHORT answers to ANY THREE of the following four short-answer essay questions. Each of your three answers is equally weighted.

(a) Real interest rates today appear to be very low by historical standards. Are there explanations that do not rely on market imperfections?

(b) Why is it a puzzle in the standard complete markets macro model that consumption growth rates are not more highly correlated across countries? What might be a couple explanations?

(c) In "first generation" models of exchange rate attacks, governments run deficits that ultimately have to be financed by money creation, creating an inconsistency that ultimately leads to an attack. However, if attacks are perfectly predictable, one would expect that long-term (say five-year) interest rate differentials would incorporate a significant premium for the expected depreciation. In fact, the premia are usually very small until perhaps a month before the attack. How do second-generaton multiple equilibrium models of speculative attack attempt to address this deficiency? Is it a problem that the multiple equilibrium models do not really explain why the economy would jump from one equilibrium to another?

(d) *Briefly*, in what sense does having complete financial markets help justify the representative agent assumption in standard macroeconomic models?