Harvard University
Department of Economics

General Examination in Macroeconomic Theory

Fall 2004

PLEASE USE A SEPARATE BLUE BOOK FOR EACH PART AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

For those taking the **GENERAL EXAM** in macroeconomic theory:

1. You have **FOUR** hours.

2. Answer **ALL QUESTIONS** in Parts I, II, III, IV, and V.

3. Time allotted for each part:

I.	30 minutes
II.	30 minutes
III.	30 minutes
IV.	40 minutes
V.	80 minutes

Consider an economy described by the following supply function:

$$y_t = \pi_t - \pi_t^e + \theta_t$$

where y is output $\pi(\pi^e)$ is actual (expected) inflation and θ_t is an i.i.d. shock with mean zero and variance σ_θ^2. The monetary authority controls inflation directly and minimizes the following cost function:

$$C = 1/2(\pi_t)^2 + a/2(y_t)^2$$

with $a > 0$.In each period the timing of event is as follows. First expectations are set; then the shock θ_t occurs and it is observed then the monetary authority chooses π_t.

Questions:

1) What model of wage/price setting would be compatible with this supply function?

2) What inflation policy would the monetary authority choose? Characterize the average and variance of output and inflation.

3) Is monetary policy neutral in this model?

4) What is the effect of an increase in the parameter a on average and variance of inflation and output and how do you interpret that?

Question for fall 2004 macro theory generals

In an economy where (a) aggregate supply behavior is consistent with the natural rate property, so that there is no long-run trade-off between the mean rate of inflation and the mean rate of output relative to potential, but (b) price setting is subject to short-run stickiness, what determines whether the monetary policy authority faces a trade-off between the variability of inflation and the variability of output? If there is a trade-off between the variability of inflation and the variability of output (again, relative to potential), what determines its shape?

What would be the consequences for inflation if monetary policy sought to achieve the minimum possible variability of output?

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Be as explicit as you can in making clear the assumptions on which your answers rely.

Part III

Please answer all three equally-weighted questions in this section.

1. Consider the following model of moral hazard and lending

A small country is populated by entrepreneurs with utility function $U = C_2$, who can borrow abroad at world (net) interest rate r. Each entrepreneur has initial wealth Y_1, which can be used to invest in a project that yields output Z with probability $\pi(I)$, and yields nothing with probability $1-\pi(I)$; $\pi'(I) > 0$, $\pi''(I) < 0$. Initial wealth Y_1, however, is insufficient to achieve the efficient level of investment $\bar{I}$, defined implicitly by $\pi'(\bar{I})Z = 1+r$. Thus entrepreneurs would like to borrow $D = \bar{I} - Y_1$, but they are constrained by the fact that foreign creditors can only observe whether the project actually succeeds or fails, and cannot observe investment I directly. Potential creditors worry that once the entrepreneur has been given funds, she will sneak them into secret foreign bank accounts rather than invest.

Under this setup, including the information constraints, equilibrium investment is governed by the following two equations:

$$\pi(I)P(Z) = (1+r)(I - Y_1) \tag{1}$$

$$\pi'(I)[Z - P(Z)] = 1 + r \tag{2}$$

where $I - Y_1 = D$ gives the amount the entrepreneur borrows, and $P(Z)$ is the payment to the creditor if the project succeeds (obviously, $P(Z) < Z$).

Equations (1) and (2) govern the determination of I and $P(Z)$.

a. Can you show analytically or graphically, why the rate of return on investment is higher in a country with a low initial endowment than a country with a relatively high one?

b. Suppose the same small country is also inhabited by domestic "savers" who have the same first-period endowment as entrepreneurs and are otherwise identical except that they can only achieve second-period consumption by either lending funds abroad or lending to domestic firms. Is there any way for the small country government to design a *Pareto improving* scheme whereby domestic "savers" are taxed in the first period – with funds going to subsidize domestic entrepreneurs – then repaid in full with interest in the second period?

c. Continuing from the setting in part b, now assume that there are two large countries each inhabited by representative savers and investors. Preferences and technologies are the same, but iniital endowments may differ. Can

you show, graphically, the determination of the real interest rate? If one country is richer than the other, will investment be the same in both countries? Is it possible, in principle, for capital to flow from the poor country to the rich country?

Neoclassical growth model with land

Suppose that the production function takes a Cobb-Douglas form in capital, K, labor, L, and land, D:

$$Y = Ae^{xt} \cdot K^{\alpha} \cdot L^{\beta} \cdot D^{\gamma},$$

where A>0 is constant; x>0 is the rate of (Hicks-neutral) technological progress; $\alpha>0$, $\beta>0$, $\gamma>0$; and $\alpha+\beta+\gamma=1$ (constant returns to scale in the three factors of production). The quantity of land, D, is constant.

Assume, as in Solow, that the gross saving rate, s, is a positive constant. Capital, K, depreciates at the constant rate $\delta>0$. Labor, L, grows at the constant rate n>0.

Define $\kappa \equiv K/Y$, the capital-output ratio.

1. Work out an expression for the growth rate of the capital stock, $\dot{K}/K$, as a function of κ.

2. Work out an expression for the growth rate of output, $\dot{Y}/Y$, as a function of κ.

3. Use the results in (1) and (2) to work out an expression for the growth rate of the capital-output ratio, $\dot{\kappa}/\kappa$, as a function of κ.

4. Define a steady state to be a situation where κ is constant over time. What is the steady-state value of κ?

5. What is the steady-state growth rate of output, $\dot{Y}/Y$? Is this value positive? Explain.

6. What is the steady-state growth rate of output per worker, $\dot{Y}/Y$ - n? Is this value positive? Explain how this growth rate depends on the rate of technological progress, x, and the presence of land.

7. In the steady state (where κ is constant), what happens over time to the rental price of land, the real wage rate, and the rental price of capital?

Problem 1 (25 minutes): Evaluate each of these assertions as either *True*, *False*, or *Sometimes* (true). You will graded on the strength of your explanation:

a. Brownian motion is not differentiable but an Ito process is differentiable.

b. If a Bellman Operator is a contraction mapping, then it has one (and only one) fixed point.

c. If a tax cut is scheduled to occur at date t, then consumption should jump at t and then show no more predictable movements.

d. If an investment tax credit ends as scheduled at date t, then investment should return to its steady state level no later than date $t+1$.

e. In an Ss investment model, investment occurs in lumps but the capital stock changes continuously.

Problem 2 (35 minutes): Consider a person who lives for three periods, $t = 1, 2, 3$. To simplify exposition, I'll refer to three "selves" of this individual. These selves are indexed by their respective periods of control over the individual's consumption decision. At $t = 1$, "self one" chooses c_1. At $t = 2$, "self two" chooses c_2. At $t = 3$, "self three" chooses c_3. At time $t = 1$ the agent has the following utility function:

$$\ln c_1 + \beta\delta \ln c_2 + \beta\delta^2 \ln c_3.$$

These preferences would be "standard" if we set $\beta = 1$, in which case the discount structure would be "exponential." However, we'll assume $0 < \beta < 1$, to approximate the hyperbolic discount structure observed by experimental psychologists. Setting $\beta < 1$ captures the idea that, from the perspective of the current self, the discount between today and tomorrow is sharper then the discount between some future period and the day after that future period. At time $t = 2$ the agent has the following utility function:

$$\ln c_2 + \beta\delta \ln c_3.$$

Again, this utilityfunction reflects the sharp discount between the new current period (i.e., period two) and the new tomorrow (i.e., period three). Finally, at time $t = 3$ the agent has the following utility function:

$$\ln c_3.$$

a. Prove that if $\beta = 1$, then the utility function at $t = 2$ is a linear transformation of the last two terms of the utility function at $t = 1$.

b. Suppose self one could choose c_1, c_2, c_3, subject to the constraint, $c_1 + c_2 + c_3 \leq A_1$, where A_1 is the starting asset stock of the agent, and the gross interest rate is implicitly assumed to be equal to one. To simplify notation, assume $\delta = 1$, for this question and all remaining questions in the problem set. Show that under this "precommitment" scenario, self one would choose consumption levels such that

$$\beta c_1 = c_2 = c_3.$$

c. Now suppose that self two is given the chance to revise self one's consumption program for periods two and three. Intuitively, explain why self two has an incentive to implement such a revision (iff $\beta \neq 1$). Assume that self two "inherits" an asset stock of $A_2 = A_1 - c_1$ and is asked to pick c_2 and c_3. Show that self two will choose consumption levels such that

$$\beta c_2 = c_3.$$

d. Now construct the rational backwards induction solution to this problem. Assume $c_3 = A_3 = A_1 - c_1 - c_2$. You should find that

$$c_3 = \frac{\beta(A_1 - c_1)}{1 + \beta}$$

$$c_2 = \frac{A_1 - c_1}{1 + \beta}$$

$$c_1 = \frac{A_1}{1 + 2\beta}.$$

e. Derive the following Euler Equation, which applies to a standard consumption problem (i.e., with $\beta = 1$) :

$$u'(c_t) = R\delta u'(c_{t+1}).$$

Does this Euler Equation apply to the quasi-hyperbolic problem (with δ replaced by $\beta\delta$)? Why or why not?

f. The equilibrium time path of consumption is pareto-inefficient. I.e., the equilibrium path can be perturbed in a way which makes all three selves strictly better off. Find such a perturbation. Hint: perturb c_1 by -Δ, and perturb c_3 by Δ. Note that this perturbation represents an increase in "thriftiness" in the sense that consumption is being postponed to the future.