HARVARD UNIVERSITY
DEPARTMENT OF ECONOMICS

# General Examination in Macroeconomic Theory

SPRING 2013

You have **FOUR** hours. Answer all questions

Part A (Prof. Laibson): 48 minutes
Part B (Prof. Aghion): 48 minutes
Part C (Prof. Farhi): 72 minutes
Part D (Prof. Rogoff): 72 minutes

**PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.**

**PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.**

**PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.**

Instructor: David Laibson
Spring General Exam

**Problem 1 (Growth Model; 24 minutes):** Consider the following growth model:

$$v(k_0) = \sup_{\{k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln(k_t^\alpha - k_{t+1})$$

subject to the constraint

$$k_{t+1} \in [0, k_t^\alpha] \equiv \Gamma(k_t).$$

Finally, note that $0 \leq \alpha < 1$.

**a.** Write down the Bellman Equation.

**b.** Solve the Bellman Equation by "guessing" a solution. Specifically, start by guessing that the form of the solution is

$$v(k) = \psi + \phi \ln(k).$$

Solve for parameters $\psi$ and $\phi$.

**c.** Derive the optimal policy function.

**Problem 2 (True/False/Uncertain; 24 minutes):** This question is graded on the quality of your explanation (not on the one-word answer itself). So explain each answer. When a statement is false, you'll get full credit only if you provide a counter-example.

**a.** Assume that (i) consumers have rational expectations, and, (ii) the product of the gross interest rate and the discount factor is unity. Then marginal utility is a random walk.

**b.** If the flow payoff function is bounded, then there exists a unique bounded solution to the Bellman Equation.

**c.** In a discrete adjustment model, the region of inaction gets larger as the variance of background noise gets smaller.

**d.** The variance of an Ito Process monotonically increases with the forecasting horizon.

Philippe Aghion Question
Part B (48 minutes)
Macro Spring Generals

1) The Malthusian trap: how is it explained and how one may get out of it?

2) Discuss the relationship between competition and growth.

# 1 Short Questions

Answer all the following questions. Some are True/False/Uncertain and are explicitly denoted as such. The others are direct questions. Explain and detail your answers VERY carefully. The QUALITY of your explanation determines your grade.

1. TRUE/FALSE/UNCERTAIN. In order to match the data, the RBC model requires a large elasticity of labor supply. This is consistent with the microeconomic evidence on the elasticity of labor supply.

2. Explain how the impulse responses for consumption and labor in the RBC model change when the persistence of the productivity shock increases. How is the amount of amplification of productivity fluctuations on output affected?

3. Explain what the the employment-lottery model is. Can this model generate an aggregate elasticity of labor supply which is larger than the individual elasticity of labor supply?

4. Explain what happens when the RBC model is extended to incorporate endogenous capital utilization.

5. TRUE/FALSE/UNCERTAIN (for each of the following three statements). In the New-Keynesian model, real interest rates are counter-cyclical when monetary policy shocks are driving fluctuations. In the RBC model, real interest rates are pro-cyclical when productivity shocks are driving fluctuations. In both cases, real wages are pro-cyclical. All these properties are consistent with the data.

6. Consider the RBC model with money in the utility function. Imagine a Taylor rule of the form $i_t = \rho + \phi_\pi \pi_t$. What does local determinacy mean? What conditions on the parameters $(\rho, \phi_\pi)$ of the Taylor rule guarantee local determinacy? Consider alternatively a money supply rule $M_t = \bar{M}$ (where $M_t$ is nominal money supply and $\bar{M}$ is a constant) yield local determinacy? How would your answer change for the New Keynesian model?

7. Consider the New-Keynesian model with productivity shocks. Assume first that the labor tax is set to offset monopoly power. Characterize optimal monetary policy. Is commitment required to implement this outcome? Assume now that the labor tax is zero. Characterize optimal monetary policy under commitment and under no commitment, and explain why the two differ.

8. Consider the New-Keynesian model with cost-push shocks. Explain how to interpret cost-push shocks. Characterize optimal monetary policy under commitment and under no commitment, and explain why the two differ.

9. According to the Ricardian-Equivalence hypothesis, the purchase by the government of long-term government bonds financed by selling short term government bonds reduces long term real interest rates and raises short term real interest rates.

## 2 Problem

Consider the New-Keynesian model and let $\bar{r}_t$ be the natural interest rate.

1. Write down the two key equations for the linearized model and define the variables and parameters.

2. Imagine first that $\bar{r}_t \geq 0$ for all $t \geq 0$. Explain why optimal monetary policy can be described as a Taylor rule of the form $i_t = \bar{r}_t + \phi_\pi \pi_t$ with $\phi_\pi > 1$?

3. Imagine now that the natural interest rate $\bar{r}_0 < 0$ is negative, but $\bar{r}_t > 0$ for $t = 1, 2, \ldots$ There is a zero lower bound on nominal interest rates $i_t \geq 0$ because agents can substitute away from bonds and into cash. Show that if the central bank follows the optimal Taylor rule for $t \geq 1$ and sets $i_t = 0$ then $x_0 < 0$ and $\pi_0 < 0$.

4. Suppose the central bank deviates from the previous policy at $t = 1$ (but does exactly as before in all other periods) in an effort to ensure that $x_0 = 0$. Derive the four equations determining $\pi_0, \pi_1, x_1$ and $i_1$.

5. Assume the solution to the previous four equations has $i_1 > 0$. Show that $\pi_1, \pi_0, x_1 > 0$ and that $i_1 < r_1$.

# Part IV ROGOFF

## May 9, 2012

**Instructions:** Please answer all three questions in this section. You do not need to give every intermediate step. All three parts count 24 points. Please put your answer to question (3) in a separate bluebook. If you are having trouble with analytics in part 1 or part 2, try to give an intuitive answer.

**1. (24 points).** *Morris and Shin Speculative Exchange Rate Attacks*

This example is based on Morris and Shin (2001). Imagine that the Central Bank holds reserves (fundamentals) of

$$\theta \in [0, 1]$$

The level of reserves, however, are not common knowledge. We assume instead that there are a continuum of individual traders of mass 1, each of whom is endowed with one unit of domestic currency, and receives a private signal $x_i$ about the quality of the fundamentals,

$$x_i = \theta + \varepsilon_i \tag{1}$$

with $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$. Define the precision of information instead of the noise, $\lambda_\varepsilon \equiv \sigma_\varepsilon^{-2}$.

The only information that is common knowledge among traders is the prior distribution of $\theta$ (that is, the primitive distribution that generates $\theta$). Assume it is *uninformative*; in this case, this means all traders have the prior $\theta \sim N(\bar{\theta}, \lambda_\theta^{-1})$ with $\lambda_\theta \to 0$.

If a trader $i$ decides to attack ($I_i = 1$) she gets

$$\begin{aligned} -c, \text{ if } R &= 0 \\ 1 - c, \text{ if } R &= 1 \end{aligned} \tag{2}$$

where $R = 1$ stands for "regime change" (abandon the peg). The Central Bank abandons the peg whenever the size of the attack, or mass of agents that decide to sell their unit of domestic currency for 1 unit of reserves, is larger than reserves,

$$\begin{aligned} R &= 1 \Leftrightarrow A = \int_0^1 I_i di > \theta \\ R &= 0 \ \text{ otherwise} \end{aligned} \tag{3}$$

Since the only heterogeneity among individual traders is the signal they receive, we can characterize "threshold strategies" by

$$\begin{aligned} I_i &= 1 \Leftrightarrow x_i \leq x^* \\ I_i &= 0 \text{ otherwise} \end{aligned} \tag{4}$$

The game therefore consists in solving for the threshold signal $x^*$, and the (possibly multiple) equilibria of the game ($R = 1$ or $0$), as a function of the fundamentals. In order to solve for these, it is useful to remember that the Bayesian posterior expectation of $\theta$ given the normally distributed signal $x_i$ and uninformative prior is given by

$$\theta | x_i \sim N(x_i, \lambda_\varepsilon^{-1})$$

(a) Show that the size of the attack is given by

$$A = \int_0^1 I_i di = \Phi\left(\sqrt{\lambda_\varepsilon}(x^* - \theta)\right) \tag{5}$$

and therefore there exists a cutoff fundamental $\theta^*$ defined by

$$\theta^* = \Phi\left(\sqrt{\lambda_\varepsilon}(x^* - \theta^*)\right) \tag{6}$$

for which $R = 1$ (the peg is abandoned) when $\theta < \theta^*$ and $R = 0$ (the peg is maintained) when $\theta \geq \theta^*$.

(b) Characterize the indifference condition of the "marginal trader" (who receives $x_i = x^*$). Show it satisfies

$$\Phi\left(\sqrt{\lambda_\varepsilon}(x^* - \theta^*)\right) = 1 - c \tag{7}$$

(c) THIS NEXT PART ONLY REQUIRES A BRIEF INTUITIVE ANSWER.

(6) and (7) clearly define unique cutoffs,

$$\begin{aligned} \theta^* &= 1 - c \\ x^* &= \frac{\Phi^{-1}(1-c)}{\sqrt{\lambda_\varepsilon}} + (1 - c) \end{aligned}$$

Does the equilibrium remain unique as we approach the common knowledge benchmark ($\lambda_\varepsilon \to \infty$)? Is uniqueness guaranteed if we add a public signal of the fundamentals

$$z = \theta + \delta$$

where $\delta \sim N(0, \lambda_z^{-1})$ which all traders observe? No analytical work is needed to obtain full credit; a brief explanation is sufficient for this part.

(d) Now suppose that instead of observing a normally distributed signal, agents observed a uniformly distributed signal. That is, suppose

$$\varepsilon_i \sim U[-\bar{\varepsilon}, \bar{\varepsilon}].$$

Assuming that agents follow threshold strategies with a fixed $x^* \in (-\bar{\varepsilon}, 1+\bar{\varepsilon})$, how does this change your answer to part (a)?
*Hint: after computing $A(\theta)$ in this new setting, plot $A(\theta)$ against $\theta$ over the interval $\theta \in [0,1]$ to identify the condition that must characterize $\theta^*$.*

**2. (24 points)** Consider a one-good stochastic $M$-country endowment economy where the representative agent in country $m$ lives for two periods and has utility function

$$U_m = \left(\frac{C_{m,1}^{1-\rho_m}}{1-\rho_m}\right) + E\left(\frac{\beta C_{m,2}^{1-\rho_m}}{1-\rho_m}\right).$$

First-period endowment income in country $m$ is given by $Y$, and in the second period by $Y_m(s)$, where $s$ is the state of nature. Assume that there are complete state contingent markets in this global economy.

(a) Characterize the Euler equation in country $m$ for a state-$s$ security - that is, a security which pays off one unit in date 2, period $s$. Denote the price of this bond in terms of date 1 consumption $\frac{p(s)}{1+r}$, and denote the probability state $s$ occurs $\pi(s)$.

(b) What relationship does the model predict between cross-country consumption growth rates if $\rho_m$ is the same for all countries? What if $\rho_m$ can differ across countries?

You can receive full credit by directly providing the answers without derivation, but if you choose to derive them, it will be helpful to use your answer from part (a) and the fact that all countries can buy the same set of state-contingent securities.

(c) Suppose $\rho_m$ differs across countries. In what sense can one think of the world riskless interest rate being determined in this model by a standard bond Euler equation of the form

$$U'(C_1) = \beta(1+r)EU'(C_2)$$

where $r$ is the interest rate on a riskless bond, $C_1$ and $C_2$ are aggregate world measures of consumption, and $U(\cdot)$ is an appropriately defined utility function over aggregate consumption? If you cannot answer this question analytically, you can get almost full credit by giving a brief intuitive discussion.
*Hint for analytical argument: show that the above Euler equation holds when aggregate consumption is defined using the geometric mean of country consumptions*

$$C \equiv \Pi_{m=1}^{M}(C_m)^{\frac{1}{M}}$$

*and* $U(C) = \frac{C^{1-\rho}}{1-\rho}$ *is CRRA with relative risk aversion that is the harmonic mean of country relative risk aversions*

$$\rho \equiv \frac{1}{\frac{1}{M}\sum_{m=1}^{M}\frac{1}{\rho_m}}$$

(d) Now suppose there are a large number of agents in each of the $m$ countries, and that within each country $m$, everyone has an identical utility function with the same $\rho_m$. The risk aversion parameter $\rho_m$ can still differ across countries. Very briefly, with no analytics necessary, does this change anything in part (c)?

**3. (24 points) Please give SHORT answers to ANY THREE of the following four short-answer essay questions (8 points each). Please use a separate blue book for this question.**

(a) If a country is on a fixed exchange rate, how can fiscal policy be used to effectively devalue a country's real exchange rate?

(b) Very briefly (no math needed), what are three possible explanations of why world long-term real interest rates have fallen in the past couple decades? What changes might take place in the next ten years that might make real interest rates rise back up to their long-term averages?

(c) The gravity model of trade has been widely used to test for whether currency unions lead to an increase in trade. Explain why there might be a causality issue here, using the Euro as an example.

(d) Explain two alternative rationales of why foreign creditors might be able to collect repayments from sovereign nations. Can you think of any empirical predictions that might differ across the different approaches?