General Examination in Microeconomic Theory

FALL 2013

You have **FOUR** hours. Answer all questions.

**PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.**

**PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.**

**PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.**
It has been suggested that the ability to deter crime with large penalties may be limited by the willingness of police to accept bribes that are below the penalty amount.

(1) (1) Craft a simple model of crime and punishment in which police will let criminals go if they are offered a bribe over a certain level. How does the willingness to accept bribes impact the optimal penalty and the amount of crime?

(2) (2) How does the distribution of bargaining power, between policeman and criminal, impact the optimal penalty and the amount of crime?

(3) (3) If the government can hire more police, and thereby increase the probability of the detection, how will the optimal number of police change with their willingness to accept bribes, if the penalty for crime is fixed? How will the optimal number of police change with their willingness to accept bribes, if the penalty for crime can be changed?
Consider the following normal-form game:

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<th>L</th>
<th>C</th>
<th>R</th>
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<tbody>
<tr>
<td>T</td>
<td>-2,2</td>
<td>-1,1</td>
<td>0,0</td>
</tr>
<tr>
<td>M</td>
<td>1,-1</td>
<td>3,5</td>
<td>3,4</td>
</tr>
<tr>
<td>B</td>
<td>0,0</td>
<td>4,2</td>
<td>2,4</td>
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Table 1

(A) Find all Nash equilibria (including those in mixed-strategies) of the game in Table 1.

(B) Now suppose that the game of Table 1 is repeated infinitely many times and that players maximize the sum of their discounted payoffs. Find the set $V^*$ of payoff pairs such that, for any $(v_1,v_2) \in V^*$, there exists a discount rate $r > 0$ and a subgame perfect equilibrium of the repeated game for which the discounted average payoffs are $(v_1,v_2)$ when players discount at rate $r$.

(C) For the repeated game of part (B), sketch subgame-perfect equilibrium strategies that attain a typical point $(v_1,v_2) \in V^*$.

(D) Now consider the game in which, with probability .5, the payoffs are as in Table 1 but with probability the payoffs are as follows:

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<tbody>
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<td>1,0</td>
<td>4,4</td>
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</tbody>
</table>

Table 2

Suppose that player 1 (but not player 2) knows whether the actual game is given by Table 1 or Table 2. Find all Bayesian equilibria of this game of incomplete information.
1. (i) State and prove the first theorem of welfare economics.

(ii) Consider a two date, one good exchange economy with consumption only at the second date. There are \( n \) states of the world, where \( \pi_j > 0 \) is the (objective) probability of state \( j \) (\( \sum_j \pi_j = 1 \)). There are \( m \) consumers, where consumer \( i \)'s (Bernoulli) utility function is \( u_i(c_i), u'_i > 0, u''_i < 0 \). Consumer \( i \) has endowment \( w^j_i \) in state \( j \). The aggregate endowment in state \( j \) is \( \bar{w}^j = \sum_i w^j_i \).

Write down the Arrow-Debreu equilibrium for this economy. Denote \( i \)'s equilibrium consumption in state \( j \) by \( c^j_i \). Establish the following "coinsurance" result: \( \bar{w}^{j_1} > \bar{w}^{j_2} \implies c^{j_1}_i > c^{j_2}_i \) for all \( i \).

2. An entrepreneur with no wealth needs to raise \( I \) dollars to undertake an investment. The return is given by \( y=20 \) or \( 30 \) with equal probability (this is common knowledge ex ante). The entrepreneur observes \( y \) ex post, but the investor must incur an inspection cost \( 4 \) to do so. Both parties are risk neutral and the interest rate is zero.

Consider a debt contract of the form: if the entrepreneur pays \( d \), no inspection occurs; if the entrepreneur does not pay \( d \), inspection occurs and the investor receives everything. (Assume that the entrepreneur pays \( d \) when he is indifferent.)

Suppose that the entrepreneur has all the bargaining power ex ante (so the only constraint is that the investor breaks even).

What is the optimal contract for different values of \( I \)? How does your answer change if the investor has all the bargaining power?

Return to the case where the entrepreneur has all the bargaining power, and suppose \( I = 22 \). Consider the following contract: if the entrepreneur pays \( d \) no inspection occurs; if the entrepreneur pays \( 20 \), inspection occurs with probability \( 0 < \pi < 1 \); if inspection occur, the investor receives everything. Show that \( d \) and \( \pi \) can be chosen so that this contract is superior to the optimal deterministic contract.
**Question D1**  
"Independence of Irrelevant Alternatives"

An assumption with the name "Independence of Irrelevant Alternatives" is used both in Nash Bargaining Theory and in Social Choice Theory.

Discuss, in a brief essay, what these assumptions actually say when applied to these two models.

Explain how they are different, what the motivations behind them are, and, most importantly, why is it the case that in Nash Bargaining Theory one develops a particular solution to bargaining problems whereas in Social Choice theory the main result is that no solution exists.

**Question D2**  
**Distributing the Interim Payoff in an Auction**

There are three bidders whose valuation for a single indivisible object are $\theta_i$. The $\theta_i$ are uniformly and independently distributed on $[0, 1]$. As usual, assume that the bidders are risk-neutral and have utility functions that are quasi-linear in money.

A mechanism is constructed to allocate the object efficiently at a Bayesian Nash equilibrium. That is, the highest evaluator always gets the good. This mechanism is symmetric across the bidders and returns all the revenue collected to the bidders, on average. That is the net expected revenue to the seller is zero when the mechanism is played in equilibrium, although it may not be zero for every realization of the $\theta_i$.

a) What is the interim utility $W(\theta_i)$ achieved for each bidder by this mechanism. (You may make reference to any result covered in class or in MWG. You should offer a full explanation of it.)

b) What is $W(\frac{1}{2})$ for this mechanism?

c) The remainder of this question concerns mechanisms that might not always achieve the efficient allocation, but which might be useful in getting to more equitable allocations of expected welfare for different $\theta$.

The mechanism designer, who psychologically identifies himself with the median evaluation of $\theta = \frac{1}{2}$, is unhappy with the result of part b) above and consults you about whether there might be a more equitable way to arrange for interim welfare outcomes, by which he means a way that has a higher value of $W(\frac{1}{2})$. He asks you about the following interim welfare functions. For each one, state whether there is any Bayesian mechanism that can achieve it without an external infusion of money on average.

(i) $W(\theta_i) = \frac{1}{3}\theta_i$ (which produces $W(\frac{1}{2}) = \frac{1}{6}$)
(ii) $W(\theta_i) = \min(\frac{1}{2}\theta_i, 0)$ (which produces $W(\frac{1}{2}) = \frac{1}{4}$)